GENERALIZED GAUSSIAN QUADRATURE RULES OVER TWO-DIMENSIONAL REGIONS WITH A CURVED EXPONENTIAL EDGE

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ABSTRACT

This paper presents a generalized Gaussian quadrature method for numerical integration over two-dimensional bounded regions with a curved exponential edge. A general formulae for numerical integration over the regions $R_1 = \{(x,y)|a \le x \le b, c \le y \le e^{kx}\}$ and $R_2 = \{(x,y)|a \le y \le b, c \le x \le e^{ky}\}$ are derived, which can be directly used for integrating any type of functions over such regions. In order to derive this formulae, a general transformation of these regions is given from (x,y) space to a square in (ξ,η) space, $S:\{(\xi,\eta) \mid 0 \le \xi \le 1 , 0 \le \eta \le 1\}$. Generlized Gaussian quadrature nodes and weights introduced by Ma *et.al.* in 1997 are used in the product formula presented in this paper to evaluate the integral over S, as it is proved to give more accurate results than the classical Gauss Legendre nodes and weights. The method can be used to integrate a wide class of functions including smooth functions and functions with end-point singularities. The performance of the method is illustrated for different functions over different regions with numerical examples.

Keywords: Numerical integration, Quadrature rules, Gaussian quadrature, Finite-element method