

RELIABILITY TEST PLANS BASED ON BURR DISTRIBUTION FROM TRUNCATED LIFE TESTS

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ABSTRACT

In this paper, four parameter Burr distribution is considered as a life-testing model. The problem of acceptance sampling when the life test is truncated at a pre-assigned time is discussed with a known shape and scale parameters. For various acceptance number, various confidence level and various values of ratio of the fixed experimental time to the specified mean life, the minimum sample size necessary to assure a specified mean life time worked out. The operating characteristic functions of the sampling plans are obtained. Producer's risk is also discussed. A table for the ratio of true mean life to a specified means that ensures acceptance with a pre-assigned probability is provided. And finally we compare minimum sample size necessary to assert the average life to a given values specified average life.

KEYWORDS : Reliability test plans, Four parameter Burr distribution, Consumer's risk, Producer's risk, Operating characteristic function, Truncated life test.

1. INTRODUCTION

The Burr distribution was first introduced by Burr (1942). In this paper, we consider the four-parameter Burr-Type X distribution. Suppose that X follows Burr Distribution with four parameters. Then the pdf is given by:

$$f(x; \gamma, \beta, k, \alpha) = \left(\frac{\alpha k \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\beta \left(1 + \left(\frac{x-\gamma}{\beta}\right)^\alpha\right)^{k+1}} \right) \text{ for } \gamma, \beta, k, \alpha > 0; x > \gamma \quad (1.1)$$

Then the CDF of x is

$$F(x; \gamma, \beta, k, \alpha) = 1 - \left(1 + \left(\frac{x-\gamma}{\beta}\right)^\alpha \right)^{-k} \text{ for } \gamma, \beta, k, \alpha > 0; x > \gamma \quad (1.2)$$

Let $t = x - \gamma$, then the CDF becomes,

$$F(t; \beta, k, \alpha) = 1 - \left(1 + \left(\frac{t}{\beta}\right)^\alpha \right)^{-k} \text{ for } t, \beta, k, \alpha > 0 \quad (1.3)$$

In the following assumed that the distribution parameters γ, k, c are known. In this case the average life time of the product is depends only on β and it is easily seen that the average life time is monotonic increasing in β . Let β_0 represents the required minimum average life time, then the following holds:

$$F_{\frac{t}{\beta}}(t) \leq F_{\frac{t}{\beta_0}}(t) \iff \beta \geq \beta_0 \quad (1.4)$$

If t follows equation (1.3), the reliability function is given as,

$$R(t; \beta, k, \alpha) = \left(1 + \left(\frac{t}{\beta}\right)^\alpha \right)^{-k} \quad (1.5)$$

The hazard function corresponding to (1.3) is,

$$H(t; \beta, k, \alpha) = \frac{\left(\frac{\alpha k \left(\frac{t}{\beta}\right)^{\alpha-1}}{\beta \left(1 + \left(\frac{t}{\beta}\right)^{\alpha}\right)^{k+1}} \right)}{\left(1 + \left(\frac{t}{\beta}\right)^{\alpha}\right)^{-k}} \quad (1.6)$$

The main aim of this paper is to develop a sampling plan, under the assumption the Lifetime of a product follows four-parameter Burr distribution. And make a comparison of Burr distribution with six different probability distributions to identify the importance of four parameter Burr distribution among them.

2. ACCEPTANCE SAMPLING PLAN

Acceptance sampling plan is a essential quality control technique where a random sample is taken from a lot and based on the results of the sample taken, the lot will be sentenced (rejected or accepted). The purposes of an acceptance sampling plan are:

- 1) Determining the quality level of an incoming shipment or end product
- 2) Ensuring that the quality level is within the level that has been predetermined.

Acceptance sampling plan is applicable in the following situations:

- 1) When products in testing are destructive in nature
- 2) When the suppliers are new
- 3) When new products produced and
- 4) Testing whole lot could be expensive and time consuming.

We consider a reliability test plan or acceptance sampling plan dealing with the determination of minimum sample size required to assure a minimum average life Time, when the life test is terminated at a pre assigned time t such

that the observed number of failures does not exceed a given acceptance number c . If the number of failures exceeds c before the time t or at the end of time t , whichever is earlier leads to the rejection of hypothesis regarding the minimum average mean life. A sampling plan is associated with two types of errors which are producer's risk or consumer's risk. Producers risk is the probability that a good lot will be rejected or the probability that a process producing acceptable values of a particular quality characteristic will be rejected as performing unsatisfactorily and consumers risk is the probability of accepting a lot of poor quality product, or allowing a process that is operating in an unsatisfactory manner relative to some quality characteristic to continue in operation. The paper is organized as follows.

In this paper we introduce reliability test plan and develop a sampling plan when the life time of a product is governed by four parameter Burr distribution. The operating characteristic values and the minimum value of the ratio of true average life to required average life for various sampling plans are tabulated. Then values from the tables in the light of the sampling plan that we have adopted is described. The new sampling plan is applied for a simulated data. Gupta and Kantam and Rosaiah (2005), Kantam and Rao (2009), Kantam and Rosaiah (2006), Kantam and Rosaiah(2009), Muhammad, Debasis and Munir (2010) developed reliability test plans when the life time of product follows various probability distributions.

3. RELIABILITY TEST PLAN

A sampling plan consists of the following quantities,

1. The number of units n on test,
2. The acceptance number c ,
3. The maximum test duration t and
4. The ratio t/β_0 where β_0 is the specified average life.

The consumer's risk, i.e., the probability of accepting a bad lot (the one for which the true average life is below the specified average life t/β_0) not to exceed $1-p^*$, so that p^* is a minimum confidence level with which a lot of true average life below β_0 is rejected, by the sampling plan. For a fixed p^* our sampling plan is characterized by $(n, c, t/\beta_0)$. Here we consider sufficiently large sized lots so that the binomial distribution can be applied. The problem is to determine for given values of p^* ($0 < p^* < 1$), β_0 and c , the smallest positive integer n such that:

$$L(p_0) = \sum_{i=0}^c \binom{n}{i} p_0^i (1-p_0)^{n-i} \leq 1-p^* \quad (3.1)$$

Where $p_0 = F_{t/\beta_0}(t)$ given by (1.3), which is the failure probability before time t which depends on the ratio t/β_0 it is sufficient to design the experiment.

The function $L(p)$ is operating characteristic function of the sampling plan. i.e the acceptance probability of the lot as function of failure probability. The average life time of the product is increasing with β and, therefore, the failure probability

$p_0 = F_{t/\beta_0}(t)$ decreases with increasing β implying that the operating characteristic function is increasing in β . For given values of p^* , t/β_0 and c the values of n is determined by means of the operating characteristic function. The minimum values of n satisfying the inequality (3.1) are obtained and displayed in the Table 1 for $p^* = 0.75, 0.90, 0.95, 0.99$ and $t/\beta_0 = 0.306, 0.413, 0.521, 0.841, 1.055, 1.483, 1.911$ for $\gamma = 500, k = 2, \alpha = 2$.

If $p_0 = F(t; \beta, k, \gamma)$ is small and n is large (as assumed in some cases of our present work), The Binomial probability may be approximated by Poisson probability with parameter $\lambda = np_0$. So that the approximation is

Table 1 : Minimum Sample Size for the specified ratio t/β_0 , confidence level p^* , acceptance number c , $k=2$ and $\alpha=2$ using binomial approximation

p^*	c	t/β_0							
		0.306	0.413	0.521	0.627	0.841	1.055	1.483	1.911
0.75	0	8	5	3	3	2	1	1	1
0.75	1	16	10	7	5	4	3	2	2
0.75	2	23	14	10	8	5	4	4	3
0.75	3	31	18	13	10	7	6	5	4
0.75	4	38	23	16	12	9	7	6	5
0.75	5	45	27	19	14	10	9	7	6
0.75	6	51	31	22	17	12	10	8	8
0.75	7	58	35	24	19	14	11	9	9
0.75	8	65	39	27	21	15	13	11	10
0.75	9	72	43	30	23	17	14	12	11
0.75	10	78	47	33	26	19	15	13	12
0.9	0	13	8	5	4	3	2	1	1
0.9	1	23	13	9	7	5	4	3	2
0.9	2	31	18	13	10	7	5	4	4
0.9	3	39	23	16	12	8	7	5	5
0.9	4	47	28	19	15	10	8	7	6
0.9	5	55	33	22	17	12	10	8	7
0.9	6	62	37	26	20	14	11	9	8
0.9	7	70	42	29	22	16	13	10	9
0.9	8	77	46	32	24	17	14	11	10
0.9	9	84	50	35	27	19	15	13	11
0.9	10	92	55	38	29	21	17	14	13
0.95	0	17	10	7	5	3	3	2	1
0.95	1	28	16	11	8	6	4	3	3
0.95	2	37	22	15	11	8	6	5	4
0.95	3	45	27	18	14	9	7	6	5
0.95	4	54	32	22	16	11	9	7	6
0.95	5	62	36	25	19	13	11	8	7
0.95	6	70	41	28	22	15	12	9	8
0.95	7	78	46	32	24	17	14	11	10
0.95	8	85	51	35	27	19	15	12	11
0.95	9	93	55	38	29	20	16	13	12
0.95	10	100	60	41	32	22	18	14	13
0.99	0	26	15	10	7	5	4	2	2
0.99	1	38	22	15	11	7	6	4	3
0.99	2	48	28	19	14	9	7	5	5
0.99	3	58	34	23	17	12	9	7	6
0.99	4	67	39	27	20	14	11	8	7
0.99	5	76	45	30	23	16	12	9	8
0.99	6	85	50	34	26	18	14	11	9
0.99	7	93	55	38	28	19	15	12	10
0.99	8	102	60	41	31	21	17	13	12
0.99	9	110	65	44	34	23	18	14	13
0.99	10	118	70	48	36	25	20	16	14

$$L^*(p_0) = \sum_{i=0}^c \frac{\lambda^i}{i!} e^{-\lambda} \leq 1 - p^* \tag{3.2}$$

where $\lambda = np_0$ The minimum value of n satisfy Poisson approximation are obtained for the same combination of p^* and t/β_0 values as those used for Table 1. The results are given in Table 2.

Table 2 : Minimum Sample Size for the specified ratio t/β_0 , confidence level p^* , acceptance number c, k=2 and $\alpha=2$ using Poisson approximation

p^*	c	t/β_0							
		0.306	0.413	0.521	0.627	0.841	1.055	1.483	1.911
0.75	0	9	6	4	3	3	2	2	2
0.75	1	17	10	8	6	5	4	3	3
0.75	2	24	15	11	9	6	6	5	5
0.75	3	32	19	14	11	8	7	6	6
0.75	4	39	24	17	13	10	9	7	7
0.75	5	46	28	20	16	12	10	9	8
0.75	6	53	32	23	18	14	12	10	9
0.75	7	60	36	26	20	15	13	11	11
0.75	8	66	40	29	23	17	14	12	12
0.75	9	73	45	32	25	19	16	14	13
0.75	10	80	49	35	27	20	17	15	14
0.9	0	15	9	7	5	4	3	3	3
0.9	1	29	15	11	9	6	6	5	5
0.9	2	33	20	14	11	9	7	6	6
0.9	3	41	25	18	14	11	9	8	7
0.9	4	49	30	21	17	13	11	9	9
0.9	5	57	35	25	20	15	12	11	10
0.9	6	65	39	28	22	17	14	12	12
0.9	7	72	44	31	25	18	16	14	13
0.9	8	80	49	35	27	20	17	15	14
0.9	9	87	53	38	30	22	19	16	15
0.9	10	95	57	41	32	24	20	18	17
0.95	0	19	12	8	7	5	4	4	4
0.95	1	29	18	13	10	8	7	6	5
0.95	2	39	24	17	13	10	9	7	7
0.95	3	48	29	21	16	12	10	9	9
0.95	4	56	34	24	19	14	12	11	10
0.95	5	65	39	28	22	16	14	12	12
0.95	6	73	44	32	25	18	16	14	13
0.95	7	87	49	35	28	20	17	15	14
0.95	8	90	54	38	30	22	19	16	16
0.95	9	96	59	42	33	24	21	18	17
0.95	10	104	63	45	35	26	22	19	18
0.99	0	28	17	12	10	7	6	6	5
0.99	1	41	25	18	14	11	9	8	7
0.99	2	51	31	22	18	13	11	10	9
0.99	3	61	37	27	21	16	13	12	11
0.99	4	71	43	31	24	18	15	13	13
0.99	5	80	49	35	27	20	17	15	14
0.99	6	89	54	38	30	23	19	17	16
0.99	7	98	59	42	33	25	21	18	17
0.99	8	106	65	46	36	27	23	20	19
0.99	9	115	70	50	39	29	25	21	20
0.99	10	123	75	53	42	31	26	23	22

The producer’s risk is the probability of rejection a lot although $\beta \geq \beta_0$ holds. It is obtained by the operating characteristic function:

$$L(p(\beta)) = L(F_{\frac{t}{\beta_0}}(t)) \tag{3.3}$$

$$p(\beta) = F_{\frac{t}{\beta_0} \frac{\beta_0}{\beta}}(t) \tag{3.4}$$

For some sampling plans, the values of the operating characteristic function depending on β/β_0 are displayed in Table 3.

Table 3 : Values of the operating characteristic function of the sampling plan $(n,c,t/\beta_0)$ for given confidence level p^* with $k=2$ and $\alpha = 2$

p*	n	c	t/β ₀	β/β ₀					
				2	4	6	8	10	12
0.75	23	2	0.306	0.9165	0.9977	0.9998	1	1	1
0.75	14	2	0.413	0.9038	0.9972	0.9997	0.9999	1	1
0.75	10	2	0.521	0.8848	0.9964	0.9996	0.9999	1	1
0.75	8	2	0.627	0.857	0.995	0.9995	0.9999	1	1
0.75	5	2	0.841	0.8649	0.995	0.9995	0.9999	1	1
0.75	4	2	1.055	0.8344	0.9928	0.9992	0.9998	1	1
0.75	4	2	1.483	0.5528	0.9611	0.9949	0.9989	0.9997	0.9999
0.75	3	2	1.911	0.6162	0.9612	0.9946	0.9988	0.9997	0.9999
0.9	31	2	0.306	0.8365	0.9945	0.9994	0.9999	1	1
0.9	18	2	0.413	0.8292	0.994	0.9994	0.9999	1	1
0.9	13	2	0.521	0.7909	0.9919	0.9992	0.9998	1	1
0.9	10	2	0.627	0.7631	0.9901	0.9989	0.9998	0.9999	1
0.9	7	2	0.841	0.6965	0.9846	0.9983	0.9997	0.9999	1
0.9	5	2	1.055	0.7031	0.9836	0.9981	0.9996	0.9999	1
0.9	4	2	1.483	0.5528	0.9611	0.9949	0.9989	0.9997	0.9999
0.9	4	2	1.911	0.3015	0.8855	0.9812	0.9957	0.9987	0.9996
0.95	37	2	0.306	0.7663	0.991	0.9991	0.9998	1	1
0.95	22	2	0.413	0.7435	0.9894	0.9989	0.9998	0.9999	1
0.95	15	2	0.521	0.7215	0.9878	0.9987	0.9997	0.9999	1
0.95	11	2	0.627	0.713	0.9868	0.9986	0.9997	0.9999	1
0.95	8	2	0.841	0.608	0.9768	0.9973	0.9995	0.9999	0.9999
0.95	6	2	1.055	0.5692	0.9703	0.9964	0.9993	0.9998	0.9999
0.95	5	2	1.483	0.346	0.9192	0.9883	0.9975	0.9993	0.9997
0.95	4	2	1.911	0.3015	0.8855	0.9812	0.9957	0.9987	0.9996
0.99	48	2	0.306	0.6295	0.9817	0.998	0.9996	0.9999	1
0.99	28	2	0.413	0.6085	0.9795	0.9877	0.9996	0.9999	1
0.99	19	2	0.521	0.5803	0.9764	0.9973	0.9995	0.9999	1
0.99	14	2	0.627	0.502	0.9739	0.997	0.9994	0.9998	0.9999
0.99	9	2	0.841	0.5228	0.9673	0.9961	0.9992	0.9998	0.9999
0.99	7	2	1.055	0.4463	0.9529	0.994	0.9988	0.9997	0.9999
0.99	5	2	1.483	0.346	0.9192	0.9883	0.9975	0.9993	0.9997
0.99	5	2	1.911	0.1296	0.7845	0.959	0.9902	0.997	0.9989

For a specified values of producer’s risk say 0.05 , only may be interested in knowing what value of β or β/β_0 will ensure a producer’s risk less than or equal to 0.05 for a given sampling plan. The value β and, hence the value of β/β_0 is the smallest positive number for which the following inequality holds:

$$L(p_0) = \sum_{i=0}^c \binom{n}{i} p(\beta)^i (1 - p(\beta))^{n-i} \geq 0.95 \tag{3.5}$$

For some sampling plans (n, c, t/β₀) and the values of p*, minimum values of β/β₀ satisfying (3.5) are given in Table 4.

4. ILLUSTRATION OF TABLES AND GRAPHS

Assume that the life time distribution is Burr distribution with $\gamma = 500$, $k = 2$ and $\alpha = 2$ and the experimenter is interested in knowing that the true unknown average lifetime is at least 2421 hours. Let the consumers risk is set to be $1 - p^* = 0.05$, it is desired to stop the experiment at $t = 1000$ hours. Then for an acceptance number $c = 2$, the required sample size $n = 22$ is the entry in Table 1, corresponding to the value $p^* = 0.95$, $t/\beta_0 = 0.413$ and $c = 2$. Thus 22 units have to be put on test. If during 1000 hours, not more than 2 failures out of 22 are observed then the experimenter can assert with confidence level $p^* = 0.95$ that the average life is at least 2421 hours. If we use Poisson approximation to binomial the corresponding value of $n = 22$. For the sampling plan (n = 22, c = 2, t/β₀ = 0.521) with the consumer risk 0.01. For the sampling plan (n = 22, c = 2, t/β₀ = 0.413) and confidence level $p^* = 0.95$. Under the Burr distribution the operating characteristic values from Table 3 are,

β/β_0	2	4	6	8	10	12
$L(p)$	0.7435	0.9894	0.9989	0.9998	0.9999	1.000

This shows that if the true mean life is twice the required mean life time $\beta/\beta_0 = 2$, the producers risk is approximately 0.26. From Table 4, we get the value of β/β_0 for various values of c and t/β₀ in orders that the producer’s risk

may not exceed 0.05. Thus for the above discussion we obtain the value of $\beta/\beta_0 = 2.95$, i.e the product should have an average life time in order that under the above acceptance sampling plan (22, 2, 0.413), the product is accepted with probability of at least 0.95. The actual average life time necessary to accept 95% of the lots is provided in Table 4.

5. APPLICATION

Consider a simulated data of failure times generated from Burr distribution with $\gamma = 500$, $k = 2$ and $\alpha = 2$. The samples of size 19 with observation $(t_i, i = 1, 2, \dots, 19)$ where $(t_i = x_i - \gamma, i = 1, 2, \dots, 19)$ are 1060, 4856, 1463, 4959, 4514, 2830, 465, 1699, 3186, 1126, 1622, 2346, 1138, 1508, 2085, 2628, 1229, 58, 1758.

Let the required average life time be 1000 hours and the testing time be $t = 521$, this leads to ratio of $t/\beta_0 = 0.521$ with a corresponding sample size $n = 19$ and an acceptance number $c = 2$, which are obtained from Table 1 for $p^* = 0.99$. Therefore the sampling plan for the above sample data is $(n = 19, c = 2, t/\beta_0 = 0.521)$. Based on the observation we have to decide whether to accept the product or reject it. We accept the product only if the number failure before 521 hours is less than or equal 2. However, the confidence level is assumed by the sampling plan only if the given life time is Burr distribution. In order to confirm that the given sample is generated by life times following a least approximately the Burr distribution, we have compared the sample quantiles and the corresponding population quantiles and found satisfactory agreement. Thus, the adoption of the decision rule of the sampling plan seems to be justified. In sample of 19 failure time instant there are 2 failures at 58 and 465 hours before the termination $t = 521$ hours, and the acceptance number of the plan $c = 2$. Therefore we accept 95% of the lot is provided in Table 4.

6. COMPARISON

For the comparison purpose of test plan based on Burr distribution with other six test plans based on different probability distributions. First, determine the minimum sample size for the specified values of ratio of maximum test duration to specified average life using Binomial approximation, the minimum sample sizes are displayed in figure 1. Then, we can realize that test plan based on Burr distribution has less sample size compared to other test plans. And also determine minimum ratio of true average life time to required average life time for the acceptability of a lot with producer's risk of 0.05 for specified parameters, which are displayed in Table 5.

Figure 1: Plot for minimum sample size comparison

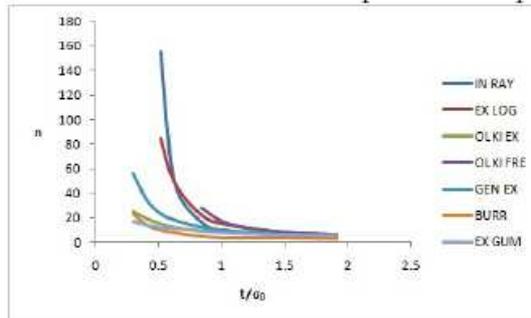
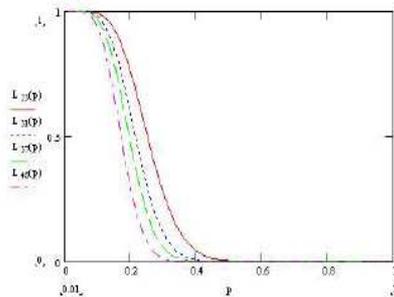


Figure 2: The plot for the operating characteristic as a function of $\frac{t}{\beta}$ and n for $k = 2$, $\alpha = 2$ and $c = 2$



CONCLUSION

The continuous improvement and review of acceptance sampling plan is important to improve the quality of the products and to ensure customer satisfaction.

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