REVERSE ENGINEERING OF STEERING SYSTEM WITH DEVELOPED EQUATION FOR STEER ANGLE

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ABSTRACT

This paper deals with the steering mechanism in detail. As the design component, various equations were derived from the fundamentals. A computer program was made on C language, which calculates all the data relating to steering i.e. (Ackermann angle, length of tie rod, steer angle). The program was simulated under different Ackermann angle, and various inferences were drawn from it. CAD models were also created to demonstrate the various components used in steering systems.

KEYWORDS: Steering, Rack and Pinion, Steering Column, Tie Rod, Gear System, Steering Wheel.

INTRODUCTION

The intention of the steering arrangement is to permit the driver to manipulation the association of the vehicle by curving the wheels. This is completed by way of steering wheel, a steering column that transmits the rotation of the steering wheel to the steering gears, the steering gears, that rise the rotational power of the steering wheel so as to send larger torque to the steering linkage, and the steering linkage that transmits the steering gear movement to the front wheels (See Fig 1). The steering arrangement configuration depends on vehicle design (the drive train and suspension utilized, whether it is a traveler car or a commercial vehicle). At present the majorly utilized arrangement is the rack –and-pinion kind and the recirculating–ball types.

Figure 1 : Cross section of steering system
PRINCIPLE

The relative motion between the wheels of a vehicle at the road surface should be of a pure rolling type so that wear of the tires is minimum and uniform. When the vehicle is moving on a curved path, as shown in Fig 2, the steering gear must be so designed that the paths of the point of contact of each wheel with the ground are concentric circular arcs. For proper steering the axis of rotation of all the wheels should meet at G i.e. the instantaneous center of rotation of the vehicle. To satisfy this, the inner wheels should be turned through a greater angle as shown in Fig in which $\theta_2$ is greater than $\theta_1$ when the vehicle is turning towards the right side.

![Figure 2: True rolling motion](image)

FUNCTIONS OF STEERING SYSTEM

The primary function of the steering system are as follows:

1. The primary function of the steering system is to achieve angular motion of the wheels to negotiate a turn.
2. To return to straight head position after a complete turn (Self Returning mechanism)
3. Absorb major part of road shocks thereby preventing them to get transmitted to the driver
4. Attain perfect rolling motion of the wheels at all times.

VARIOUS COMPONENTS USED IN STEERING SYSTEM

The steering system has three major subsystems: the steering linkage, steering gear, and steering column and wheel. As the driver turns the steering wheel, the steering gears transfer this motion to the steering linkage, which is responsible for the turning of the wheels. Although there may be many variations to this system, but you will find this assembly in all steering systems.

The following components are present

a. Steering Wheel
b. Steering Column
c. Gear System
d. Track Rod / Tie Rod
e. Steering Arm / Track Arm
f. Spindle

A. Steering column

The automotive steering column is a mechanism chiefly aimed for relating the steering wheel to the steering mechanism by input torque given by the driver from the steering wheel. The steering column is an extremely convoluted mechanism. It is projected in such a method that it collapses in an encounter so as to protect the driver.

B. Uses of adjustable steering column

1. In the event of a frontal encounter utilized as power dissipation management.
2. Provide climbing for assorted mechanism: the multi-function switch, column lock, column wiring, column shroud(s), transmission gear selector, gauges or supplementary instruments in addition to the electro motor and gear constituents.
3. Offers height/length adjustment to suit driver preference.

C. Steering Gears

The steering gearbox gives the driver of the vehicle alongside maximum impact to enable him to exert a colossal power at road wheel alongside minimum power, to manipulation the association of the wheels on the road.

D. Calculations for lock-to-lock steering

If a car has a steering ratio of 18:1 and the front wheels have a maximum deflection of 25°, then at 25°, the steering wheel has turned 25°x18, which is 450°. That's only to one side, so the entire steering goes from -25° to plus 25° giving a lock-to-lock angle at the steering wheel of 900°, or 2.5 turns (900° / 360).

This works the other way around too of course. If you know the lock-to-lock turns and the steering ratio, you can figure out the wheel deflection. For example if a car is advertised as having a 16:1 steering ratio and 3 turns lock-to-lock, then the steering wheel can turn 1.5x360° (540°) each way. At a ratio of 16:1 that means the front wheels deflect by 33.75° each way.

For racing cars, the steering ratio is normally much smaller than for passenger cars, as the racing drivers need to get fuller deflection into the steering as quickly as possible.
COMPUTER AIDED DESIGN (CAD) SIMULATION OF VARIOUS STEERING COMPONENTS

E. Steering wheel

This steering wheel is specially designed for BAJA SAE 2012 (See Fig 3). The dimensions are as per specified in the rulebook. The Diameter = 30 cm. The steering is specially designed this way so that it can act as a damper during rough terrain.

F. Steering Rack

G. Pinion Gear
H. Rack and Pinion Assembly

Figure 6: Rack and Pinion Assembly

As per the SAE BAJA handbook, Fig 6, is a 14 inch rack and pinion system. Fig 4 and Fig 5 illustrates the rack and pinion gear. Taking other features into account Rack and pinion also comes out to be cheap, easy to repair and reduces the overall weight of the car compared to other steering gearboxes. Hence it’s preferable for BAJA

STEERING GEOMETRY

I. Ackermann Geometry

Ackermann steering geometry (See Fig 7) is a geometric arrangement of linkages in the steering of a car or other vehicle designed to solve the problem of wheels on the inside and outside of a turn needing to trace out circles of different radius. The general equation to calculate the Ackermann angle is given by,

\[
\text{Cot } \Theta_o - \text{Cot } \Theta_i = \frac{L}{B}
\]

\(\Theta_o\) = turn angle of the wheel on the outside of the turn

\(\Theta_i\) = turn angle of the wheel on the inside of the turn

L = track width

B = wheel base

b = distance from rear axle to center of mass

Figure 7: Ackermann Principle
J. **Design Component of Ackermann - Deriving equation to calculate steering angle**

Determining the steering angle of the vehicle when experiencing a turn. That is to say that we have two components moving together – the left and right steering knuckles, but the relationship between their motions changes as we move them. It’s a bit like having a bowling ball in a dark room and spinning other bowling balls in an attempt to locate it by listening of its impact.

![Figure 8: Derivation of steer angle 1](image)

We have some mathematical formulas that can help us figure it out. Let’s take a look at the important angles and distances. The two most fundamental distances are the wheelbase of the car and the kingpin center-to-center distance, which can be seen from the Fig 8 above. Drawing two lines representing the wheelbase and the distance from the car’s centerline to one of the kingpins, we can make a triangle. By design, the line that connects through the centers of the Ackerman arm forms the hypotenuse of this triangle. See Fig9 below.

![Figure 9: Derivation of steer angle 2](image)

Note that the angle with its vertex at A is 90 degrees by design, unless the vehicle has been crashed. If this angle experiences an unplanned adjustment due to an impact, the car will dog track. This can be checked using a tape measure and comparing distances from side to side. Also note that the line that forms the Ackerman angle with the hypotenuse is parallel with the wheel base line, again by definition. Because of this, we can say that angle B and the Ackerman angle are similar, so if we know one angle, we know the other directly. But angle B isn’t too hard to calculate. Recall that the tan function gives the ratio between the opposite side and the adjacent side of the triangle.

\[
\tan(B) = \frac{\text{opposite}}{\text{adjacent}}
\]
we know the distances and we are trying to find angle B. We need the inverse function of \( \tan() \) to get angle B. So rearranging, we get:

\[
\tan^{-1}\left(\frac{\text{king pin center-to-center}}{\text{W}}\right) = \text{angle B}
\]

For example, let's choose a wheelbase of 76" and a kingpin to kingpin distance of 38". The formula would look as follows:

\[
\tan^{-1}\left(\frac{\text{king pin center-to-center}}{\text{W}}\right) = \text{angle B}
\]

angle B = 14.036°

So, the Ackerman Angle is 14.036°. Now we can use this to find the length of the tie rod.

To approach our next problem, which is to find the length of the tie rod, we can divide the trapezoid ABCD into a rectangle and two triangles. See Fig 10 below:

![Figure 10: Derivation of steer angle 3](image)

If you look through the diagram above carefully, you see the length of the tie rod segment BC is equal to the kingpin to kingpin center distance minus distance Y on each side. So, we have to calculate what is this distance Y? To find out, we have to decide an Ackerman Arm Radius. Purchasing a standard Ackerman arm out of a catalog can choose this, or we can design your own. Either way, this is basically just a parameter that the engineer chooses by his gut. Let's pick 6" (as per the catalog) to make calculations easy. So, what is the length of distance Y? If we recall that the sin of an angle is the ratio between the side opposite the angle and the hypotenuse.

\[
\sin(14.036°) = \frac{Y}{6}
\]

\[
Y = 6 \times \sin (14.036°)
\]

\[
Y = 6 \times 0.243
\]

\[
Y = 1.445
\]

Calculating this gives us the value that its 1.445 inches shorter at the bottom and 1.445 inches shorter on the top than the kingpin center-to-center distance. Expressed mathematically:
Reverse Engineering of Steering System with Developed Equation for Steer Angle

$L_T = D_{KC} - 2R_{AA} \cdot \sin (\text{Ackerman angle})$  \hspace{1cm} (i)

Where:

$L_T$ is the length of the tie rod

$D_{KC}$ is the distance between king pins center to center

$R_{AA}$ is the radius of the Ackerman Arm

Using equation (i),

$L_T = 38 - 2 \cdot 6 \cdot \sin 14.036^\circ$

$L_T = 38 - 2 \cdot 6 \cdot 0.243$

$L_T = 33.084$

So, for a car configured with this values, the tie rod length needs to be 33.084

Since we have figured out all of the basic values. Now the real fun begins. Let’s contemplate a turn as shown in the figure 11 in red below. Suppose that the Ackerman arm labeled AB steers 20°to the left as shown. What angle does the other Ackerman arm transect? You might think 20°, but this would result in the wheels being parallel in a turn, which would be unsatisfactory. In reality, because the car pictured is turning to the left, the right Ackerman arm (CD) needs to steer something less than 20°. But the problem is how much less?

**Figure 11 : Derivation of steer angle 4**

Let us consider a line drawn diagonally from point D to B (See Fig 12 below). This creates three angles that add together to give the angle of the wheel that pivots at point D. We’ll call the first angle $K$, the second angle $\gamma$, and the third angle is of course, the Ackerman angle, which is already been calculated.

Now we can start to work on calculating each of them. If you think about angle $K$, we can determine it because for any steer angle, we know the positions of the ends of the diagonal line. If we assigned point A the coordinate of 0,0(assume origin) then point D would have the coordinates (Kingpin Center to
Center Distance, 0). In our case specifically point D’s coordinates would be (38,0). Point B’s coordinates take a little bit more complexity to find.

![Figure 12: Derivation of steer angle 4](image)

We can calculate its co-ordinates with the following formulae:

- Point B’s X coordinate = \( R_{AA} \times \cos(\alpha + SA_L) \)
- Point B’s Y coordinate = \( R_{AA} \times \sin(\alpha + SA_L) \)

Where:
- \( R_{AA} \) is the Ackerman Arm Radius
- \( \alpha \) is the Ackerman Angle
- \( SA_L \) is the steering angle of the left wheel. Zero degrees is straight ahead.

Positive values are a left turn; negative values are a right turn.

Using our calculated values for a 20º left turn…

- Point B’s X coordinate = 6 * \( \cos(14.036º + 20º) \)
- Point B’s Y coordinate = 6 * \( \sin(14.036º + 20º) \)
- Point B’s X coordinate = 4.972
- Point B’s Y coordinate = 3.358

So, the coordinates of Point B at a 20º left turn are (4.972,3.358). Now we can project straight to the left of point B and straight up from point A to create a new point called point E (See Fig 13). Because we projected straight left and straight up, the angle at E is by definition 90º. Also, because point E falls on segment AD, we can calculate distance DE with the formula:

\[ DE = AD - AE, \]

Where,

\[ AD = \text{Distance between king pin center to center distance} \]
Reverse Engineering of Steering System with Developed Equation for Steer Angle

AE = Points B, Y-coordinate
DE = 38 - 3.358
DE = 34.642

Figure 13: Derivation of steer angle 5

Now that we know EB and ED, we can find the length of BD because it is a hypotenuse triangle formed. Using Pythagoras Theorem:

\[ BD = \sqrt{(EB^2) + (DE)^2} \]

\[ BD = \sqrt{(34.642)^2 + (4.972)^2} \]

\[ BD = 33.019 \]

Furthermore, because we know the sides of the triangle we can determine angle k in the following manner:

\[ \tan k = \frac{EB}{ED} \]

\[ \tan^{-1} \left( \frac{EB}{ED} \right) = k \]

\[ k = \tan^{-1} \left( \frac{4.972}{34.642} \right) \]

\[ k = 8.661^\circ \]

So now that we know angle k and the Ackerman angle, the problem is two thirds solved. All we have left is to find angle \( \gamma \). Note that \( \gamma \) is included in triangle BDC (See Fig 12 and Fig 13). We know that side DC is the length of the Ackerman arm, which we chose to be 6. We know that side CB is the length of the tie rod, which we calculated earlier to be 33.084. And finally, we know the distance BD, which we determined using Pythagoras Theorem to be 33.019. So we have a triangle of which all three sides are known. Going through some math’s concepts I remembered the law of cosines that can be used here. It could be expressed a number of ways, but we will use the permutation shown below.
\[
\cos \gamma = \frac{(A^2 + B^2 - C^2)}{2AB}
\]

Since we need to find \( \gamma \), using the inverse function of cosine will give the following relation,

\[
\gamma = \cos^{-1}\left(\frac{A^2 + B^2 - C^2}{2AB}\right)
\]

\[
\gamma = 85.411^\circ
\]

Now if we add up angle \(k\), \(\gamma\) and the Ackerman angle, it will give the steer angle from the line that connects the two kingpins. To get the steer angle, we have to subtract 90\(^\circ\). The formula is:

Steer Angle = \(k + \gamma + \) Ackerman Angle - 90\(^\circ\)

Steer Angle = 8.661\(^\circ\) + 85.411\(^\circ\) + 14.036\(^\circ\) - 90\(^\circ\)

Steer Angle = 18.108\(^\circ\)

As per our assumption the car is executing a left turn. The left front wheel is steered 20\(^\circ\) to the left. The right wheel is tracing a larger arc, and therefore should have a lesser steer angle theoretically. In short the left side steering angle is 20\(^\circ\) and the right side should be something less than 20\(^\circ\). Comparing our answer, which is less than 20, so we have passed the test. Additionally, experienced on the real cars on alignment rack indicates that these numbers are reasonable.

**K. Design Component of Ackermann- C program to calculate steering angle**

The figure below is a screenshot of the output of the coded program.

**Figure 14 : Screenshot of program output**

**L. Program Simulation for different Ackermann Angle**

1. **Ackermann angle 12.90\(^\circ\)**

The calculation is based on the following assumption

Car Make - Honda Civic 2011 model*
Steering ratio-16
Lock-lock-turn-2.5
Lock-lock turn angle- -28.125 to 28.125
King-pin Center to Center distance- 55
Track width- 120
*values taken from Honda Civic handbook 2011 version 43

Now if the car is taking a turn, one wheel deflects more than the other (as per Ackermann principle) and which is also evident from the values we got from the program refer Table1.

Table 1 : Inner Wheel VS Outer wheel for Ackermann Angle 12.90°

<table>
<thead>
<tr>
<th>Inner Wheel (in Degrees)</th>
<th>Outer Wheel (in Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.932</td>
</tr>
<tr>
<td>2</td>
<td>1.960632</td>
</tr>
<tr>
<td>3</td>
<td>2.921877</td>
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<td>4</td>
<td>3.868224</td>
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<td>5</td>
<td>4.8000</td>
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<tr>
<td>6</td>
<td>5.7175</td>
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<tr>
<td>7</td>
<td>6.621226</td>
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<tr>
<td>8</td>
<td>7.5113</td>
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<tr>
<td>9</td>
<td>8.388092</td>
</tr>
<tr>
<td>10</td>
<td>9.251884</td>
</tr>
<tr>
<td>11</td>
<td>10.102979</td>
</tr>
<tr>
<td>12</td>
<td>10.941649</td>
</tr>
<tr>
<td>13</td>
<td>11.768167</td>
</tr>
<tr>
<td>14</td>
<td>12.582794</td>
</tr>
<tr>
<td>15</td>
<td>13.385757</td>
</tr>
<tr>
<td>16</td>
<td>14.177371</td>
</tr>
<tr>
<td>17</td>
<td>14.957809</td>
</tr>
<tr>
<td>18</td>
<td>15.727362</td>
</tr>
<tr>
<td>19</td>
<td>16.486193</td>
</tr>
</tbody>
</table>
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| 20  | 17.234596 |
| 21  | 17.972712 |
| 22  | 18.7008   |
| 23  | 19.49024  |
| 24  | 20.127617 |
| 25  | 20.826788 |
| 26  | 21.516701 |
| 27  | 22.197489 |
| 28  | 22.8695   |
| 28.125 | 22.952831 |

Figure 15 : Inner wheel Vs Outer wheel for Ackermann 12.9°

2. Ackermann angle 19.57°

The calculation is based on the following assumption

Steering ratio-18
Lock-lock-turn-3
Lock-lock turn angle- -30 to30
King-pin Center to Center distance- 64°
Track width- 90°
Length of Tie-rod- 63.86°

Now we have tabulated the various values of the Ackermann angle when the vehicle is encountering different turns.

Now if the car is taking a turn, one wheel deflects more than the other (as per Ackermann principle) and which is also evident from the values we got from the program referTable 2.
### Table 2: Inner Wheel VS Outer wheel for Ackermann Angle 19.57°

<table>
<thead>
<tr>
<th>Inner Wheel (in Degrees)</th>
<th>Outer Wheel (in Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.9754</td>
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<tr>
<td>3</td>
<td>2.88</td>
</tr>
<tr>
<td>5</td>
<td>4.6961</td>
</tr>
<tr>
<td>7</td>
<td>6.429</td>
</tr>
<tr>
<td>9</td>
<td>8.084</td>
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<td>11</td>
<td>9.66</td>
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<td>13</td>
<td>11.238</td>
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<td>25</td>
<td>19.070</td>
</tr>
<tr>
<td>30</td>
<td>21.8511</td>
</tr>
</tbody>
</table>

![Figure 16: Inner wheel Vs Outer wheel Ackermann 19.57°](image)

3. **Ackermann angle 8.348°**

The calculation is based on the following assumption:

- Steering ratio: 13
- Lock-lock-turn: 2
- Lock-lock turn angle: -27.6932 to 27.6932
- King-pin Center to Center distance: 22"
Track width- 75"

Length of Tie-rod- 21.81519"

Now we have tabulated the various values of the Ackermann angle when the vehicle is encountering different turns.

Now if the car is taking a turn, one wheel deflects more than the other (as per Ackermann principle) and which is also evident from the values we got from the program refer Table 3.

Table 3 : Inner Wheel VS Outer wheel for Ackermann Angle 8.34°

<table>
<thead>
<tr>
<th>Inner Wheel (in degrees)</th>
<th>Outer Wheel (in degrees)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.9898</td>
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<td>3</td>
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<td>11</td>
<td>10.4311</td>
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<td>13</td>
<td>12.1839</td>
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<td>13.925</td>
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<tr>
<td>25</td>
<td>22.158</td>
</tr>
<tr>
<td>27.6923</td>
<td>24.2418</td>
</tr>
</tbody>
</table>

Figure 17 : Inner wheel Vs Outer wheel Ackermann 8.348°
II. RESULT AND INFERENCE

From the above calculations it is very evident refer Table 4, that as the Ackermann angle increases the outer wheel angle decreases. As a result

a. Steering response will be much more smoother
b. Vehicle will react smoothly to any steering input

Table 4 : Comparing Outer wheel angle deflection for different Ackermann Angle

<table>
<thead>
<tr>
<th>Inner Wheel</th>
<th>Ackermann Angle- 8.343°</th>
<th>Ackermann Angle- 12.90°</th>
<th>Ackermann Angle- 19.57°</th>
</tr>
</thead>
<tbody>
<tr>
<td>20°</td>
<td>18.137°</td>
<td>17.23°</td>
<td>15.43°</td>
</tr>
</tbody>
</table>

It should be known that lesser Ackermann angle is preferred for:

a. When initial steering response will be more direct
b. Vehicle will react faster to any steering input.

Having a high Ackermann factor is useful in taking tight corners at a relatively lower speed. At higher speeds its utility is uncertain. In fact during high speed cornering the dynamic effects compensate for the Ackermann effect and sometimes its even suggested using a negative Ackermann angle, which is referred to as anti-Ackermann angle. Formula One car generally uses zero Ackermann factors, which is also known as Parallel Steering.
CONCLUSIONS

This paper gives detailed information about the steering system available today. As the design component of the paper, various mathematical formula was derived from the fundamental to calculate the exact steer angle under the Ackermann principle. And to make things even simpler a C program was coded for it, which calculates various steering values needed such as Ackermann angle, length of tie rod, steering ratio, steer angle etc. under assumptions of some basic values of the vehicle.

The program was simulated under different Ackermann angle, and it was inferred that as the Ackermann angle increases the outer wheel turns lesser, which gives the vehicle better steering response, and input variability increases hence giving smooth driving.

REFERENCES