FILM THICKNESS EQUATION FOR ELASTOHYDRODYNAMIC LUBRICATION OF ISOTHERMAL SMOOTH LINE CONTACTS UNDER HEAVY LOADS FOR NEWTONIAN FLUIDS

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ABSTRACT

A new equation is proposed for predicting the film thickness at the Hertzian contact center in elastohydrodynamic lubrication (EHL) of isothermal smooth line contacts under heavy loads when the fluid is Newtonian. Results show that this film thickness equation has a satisfactory prediction precision for the dimensionless load to film thickness ratio \( W/H_c \) ranging from 0.01 to 200, especially when the load is heavy. In the heavy load condition, it has a much higher prediction precision than the Grubin formula and the Dowson-Higginson formula, both of which much overestimates the central EHL film thickness even in the magnitude of 1 to 2 orders. The maximum prediction error of the present film thickness equation is examined to be about 40% in the investigated wide range of the parameter value \( W/H_c \). It gives that the present film thickness equation reaches application value for heavy loads, which is unable to be accommodated by the previous EHL film thickness formula.

KEYWORDS: Elastohydrodynamic Lubrication; Film Thickness; Line Contacts; Heavy Loads; Newtonian Fluids.

INTRODUCTION

Elastohydrodynamic lubrication is the hydrodynamic lubrication occurring in line or point contacts. Its theories have been well established based on the condition of isothermal and smooth contacts and Newtonian fluids through persisting efforts for several decades. They can be represented by the Grubin film thickness formula[1], the Dowson-Higginson film thickness formula[2] and the Hamrock-Dowson film thickness formula [3]. In these theories, the contact surface elastic deformation and the lubricant viscosity variation due to film pressure both were considered.
It was recognized that the Grubin film thickness formula is suitable for heavy loads while the Dowson-Higginson film thickness formula and the Hamrock-Dowson film thickness formula are suitable for relatively light loads. Numerical computation and experiments in the past supported this recognition. The numerical calculation by the author [4] showed that the Dowson-Higginson film thickness formula prediction deviates from the accurate film thickness results and gives significant overestimations when the load is heavy enough. This phenomenon was also observed by Greenwood [5] and Venner [6].

For isothermal smooth line contacts, when the whole EHL contact is lubricated by Newtonian fluids, the dimensionless load \( W \) carried by the whole EHL contact satisfies the following equation when the carried load is heavy [1]:

\[
24.383 G_j U W^{13/4} H_r^{1/4} = 1
\]  

(1)

where 

\[
J_1 = \int_{-\infty}^{\infty} \left[ \frac{1}{2} \ln \left| x - (x^2 - 1)^{1/2} \right| \right] \left( \frac{1}{4} - \frac{1}{4} \ln \left| x - (x^2 - 1)^{1/2} \right| \right) \left( \frac{1}{4} - \frac{1}{4} \ln \left| x - (x^2 - 1)^{1/2} \right| \right) \frac{1}{\sqrt{\pi H_r}} \, dx
\]

The integration \( J_1 \) in Eq. (1) is the function of the variable \( W/H_c \). It can be found by numerical integration for given \( W/H_c \) values and be regressed out as a function of \( W/H_c \) for a range of the parameter value \( W/H_c \). Substituting this \( J_1 \) function into Eq. (1) gives the relations of the dimensionless film thickness \( H_c \) at the Hertzian contact center with the dimensionless load \( W \) and the dimensionless rolling speed \( U \) for the studied EHL when the load is heavy. In itself, equation (1) gives accurate solution of the film thickness \( H_c \) provided that \( J_1 \) is accurate. By using the sufficiently accurate function of \( J_1 \), equation (1) is able to correctly reveal the central film thickness variations with both load and rolling speed when the load is heavy. It then has prediction value for the central EHL film thickness for heavy loads and would be progressive than the previous EHL film thickness formula.

The present study attempts to do this work. It incorporates four parts of work: (1) Formulation of the function \( J_1 \) as the variable \( W/H_c \); (2) Establishment of the central film thickness equation for heavy loads by using the formulated \( J_1 \) function; (3) Comparisons of the present film thickness equation prediction with accurate film thickness results and the previous EHL film thickness formulae such as the Grubin film thickness formula and the Dowson-Higginson film thickness formula; (4) Evaluation of the prediction value of the present film thickness equation. These contents are respectively presented in the following sections.
SIMULATED CONTACT

The simulated EHL contact is shown in Fig.1. In this contact, Newtonian fluid film lubrication is assumed to occur in the whole contact area. The contact is assumed as a “heavy-load” EHL contact [7]. In this contact, the contact surface elastic deformation can be described by the Hertzian dry contact theory [1], [8]; The contact surfaces in the Hertzian contact zone are flattened as Fig.1 shows; The film thickness at the Hertzian contact center (hc) can represent the film thickness in the Hertzian contact zone.

Incorporating the contact surface elastic deformations and the dimensionless film thickness $H_c$ at the Hertzian contact center, the dimensionless film thickness in the inlet zone in the present contact is expressed as [1]:

$$H(X) = H_c - \frac{4}{\pi} \left[ X(X^2 - 1)^{0.5} + \ln[-X - (X^2 - 1)^{0.5}] \right] W \quad \text{for} \quad X \leq -1$$

(2)

For generality, the analysis in the following is in dimensionless form.

ANALYSIS

1. In the present analysis, the following assumptions are added to the condition in the inlet zone:

2. The fluid is continuous and homogeneous;

3. The flow is one-dimensional;

4. The pressure across film thickness is constant;

5. The fluid inertia is negligible;

6. The film is isothermal;

7. The mated surfaces are perfectly smooth;

8. The fluid operating condition is steady-state;

9. The fluid compressibility is negligible;

10. The fluid viscosity follows the Barus viscosity equation [9];

11. The fluid completely adheres to the contact surfaces.
Basic Equations

The Reynolds equation in the inlet zone is [10]:

$$\frac{dP}{dX} = \frac{12\lambda U(H - H_c)e^\epsilon}{H^3}$$

(3)

where $$\lambda = G\sqrt{W/H_c}$$.

Integrating Eq.(3) and rearranging yields:

$$19.15/W^{1/4}\left[\int_{-\infty}^{X} \frac{H - H_c}{H^3} dX \right] = \frac{1}{G}[1 - e^{-\gamma(X)}]$$

(4)

Assume that the film pressure at X=-1 is high enough so that $$e^{-\gamma(X)} \ll 1$$. Substituting Eq.(2) into Eq.(4) and rearranging then gives Eq.(1). As commented above, the formulation of the integration $J_1$ as function of the variable $W/H_c$ makes the solution of Eq.(1) feasible.

Function Formulation of the Integration $J_1$

Numerical calculation of the value of the integration $J_1$ was performed for the value of the variable $W/H_c$ ranging from 0.01 to 200. The calculation values are plotted in Fig.2. Curve fitting of these values gives the following function of $J_1$:

$$\lg J_1 = -0.2041(\lg W/H_c)^2 - 2.1124\lg (W/H_c) - 1.2725 \quad \text{for} \quad 0.01 \leq W/H_c \leq 200$$

(5)

Figure 2 compares Eq.(5) with the numerical calculation results and shows that Eq.(5) is satisfying in formulation of $J_1$ for $0.01 \leq W/H_c \leq 200$.

Central Film Thickness Equation

Substituting Eq.(5) into Eq.(1) and rearranging gives the following central film thickness equation:

$$-0.2041(\lg W/H_c)^2 + 0.8876\lg (W/H_c) + (\lg (24.3825UG) - 1.5\lg W - 1.2725) = 0 \quad \text{for} \quad 0.01 \leq W/H_c \leq 200$$

(6)

When the values of $G$, $U$ and $W$ are given, solving Eq.(6) gives the central film thickness $H_c$.

Or, when the values of $G$, $U$ and $H_c$ are given, solving the following equation gives the load $W$:

$$-0.2041(\lg W/H_c)^2 - 0.6124\lg (W/H_c) + (\lg (24.3825UG) - 1.5\lg H_c - 1.2725) = 0 \quad \text{for} \quad 0.01 \leq W/H_c \leq 200$$

(7)
RESULTS

When the parameter $G$ has a typical value 4500, the central film thickness $H_c$ is solved from Eq.(6) for different $U$ and $W$ values. For these operational parameter values, the accurate values of $H_c$ are also solved directly from Eq.(1), in which $J_1$ is accurately calculated by numerical integration; The central film thickness $H_c$ is also respectively predicted by the Grubin film thickness formula and the Dowson-Higginson film thickness formula. These two formulae are respectively written as:

Grubin formula [1]:

$$H_c = 1.95 W^{1/11} (GU)^{8/11}$$  \hspace{1cm} (8)$$

Dowson-Higginson formula [2]:

$$H_c = 3.533 G^{0.7} U^{0.7} W^{-0.13}$$  \hspace{1cm} (9)$$

These central film thickness are compared and discussed in the following.

For $U$=1.2E-9

Figure 3 plots the accurate, present equation predicted, classical formula predicted central EHL film thickness in the present contact when $G$=4500, $U$=1.2E-9 and the load $W$ varies from 1.0E-4 to 1.6E-3. It is shown that the central EHL film thickness predicted by the present equation (Eq.(6)) well matches the accurate value, while the Grubin and Dowson-Higginson formulae both much overestimate the film thickness for these heavy loads even in the magnitude of 1 to 2 orders. Although it was recognized that the Grubin film thickness formula is suitable for heavy loads, the present results show that such a recognition should be justified and the Grubin film thickness formula is unable to be predictive when the load is heavy enough. The Dowson-Higginson film thickness formula is also unable to be predictive when the load is heavy enough. The present equation predicts that the sensitivity of the central EHL film thickness to load is increased with the increase of load and the central EHL film thickness is drastically reduced with the load increase when the load is heavy enough. This result is well supported by the numerical results obtained by the author [4]. However, it is contradicted to classical EHL theory.

For $W$=1.0E-3

Figure 4 plots the accurate, present equation predicted, classical formula predicted central EHL film thickness in the present contact when $G$=4500, $W$=1.0E-3 and the rolling speed $U$ varies from 5.85E-10 to 1.0E-7. For the investigated rolling speed range, the present equation prediction matches the accurate
value of the central film thickness very well. The Grubin and Dowson-Higginson film thickness formulae both have good predictions for relatively high rolling speeds but give much overestimation even in the magnitude of 1 to 2 orders when the rolling speed is relatively low. The present equation predicts that when the rolling speed is relatively high, the central film thickness follows classical EHL theory; When the rolling speed is low, the central film thickness is drastically reduced with the reduction of the rolling speed. When the rolling speed is low enough, classical EHL theory appears to fail.

**Prediction Accuracy of the Present Equation**

Figure 5 plots the relative errors of the present equation prediction of the central film thickness compared to the accurate central film thickness value respectively calculated from Figs.3 and 4. Figure 5 shows that the prediction error of the present equation in Fig.3 is no more than 25%. It also shows that the prediction error of the present equation in Fig.4 is often no more than 25%, except it reaches 40% for \( \frac{W}{H_c}=32 \). Figure 5 shows the application value of the present central film thickness equation, which is much better than that of classical EHL theory in the “heavy load” or “low rolling speed” conditions.

**CONCLUSIONS**

A central film thickness equation is derived for the elastohydrodynamic lubrication of isothermal smooth line contacts under heavy loads when the fluid is Newtonian. This film thickness equation is examined to have satisfactory prediction accuracy as compared to the accurate central EHL film thickness when the dimensionless load to film thickness ratio \( \frac{W}{H_c} \) ranges from 0.01 to 200. It is much better than classical EHL film thickness formulae for “heavy loads” or “low rolling speeds”. It predicts that the sensitivity of the central EHL film thickness to load is increased with the increase of load and the central EHL film thickness is drastically reduced with the load increase when the load is heavy enough. It also predicts that when the rolling speed is relatively high, the central EHL film thickness follows classical EHL theory; While, when the rolling speed is low enough, classical EHL theory fails.

The present central film thickness equation has obvious application values for “heavy loads” or “low rolling speeds” provided that the fluid in the EHL inlet zone is Newtonian. It is written as follows:
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\[-0.2041(\log \frac{W}{H_c})^{0.8876(\log(24.3825/G) - 1.5\log W - 1.2725}) = 0 \quad \text{for } 0.01 \leq \frac{W}{H_c} \leq 200 \quad (6)\]

When the values of G, U and W are given, solving Eq.(6) gives the central film thickness Hc. Or, when the values of G, U and Hc are given, solving the following equation gives the load W:

\[-0.2041(\log \frac{W}{H_c})^{0.6124(\log(24.3825/G) - 1.5\log H_c - 1.2725}) = 0 \quad \text{for } 0.01 \leq \frac{W}{H_c} \leq 200 \quad (7)\]

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REFERENCES


NOMENCLATURE

\[ b = \text{half Hertzian contact width} \]
\[ E' = \text{compound Young's modulus of elasticity of the contact surfaces (see ref.[7])} \]
\[ E_r = \text{relative error of the present equation prediction of the central EHL film thickness compared to the accurate central EHL film thickness} \]
\[ G = \text{dimensionless material parameter, } a E' \]
\[ h = \text{film thickness} \]
\[ h_c = \text{film thickness at the Hertzian contact center} \]
\[ H = \text{dimensionless film thickness, } h/R \]
\[ H_c = \text{dimensionless film thickness at the Hertzian contact center, } h_c/R \]
\[ J_1 = \text{integration, Eq.(1)} \]
\[ p = \text{pressure} \]
\[ P = \text{dimensionless pressure, } a p \]
\[ R = \text{compound curvature radius of the contact surfaces, } R_a R_b/(R_a+R_b) \]
\[ R_a, R_b = \text{curvature radii of the contact surfaces respectively} \]
\[ u = \text{rolling speed, } (u_a+u_b)/2 \]
\[ u_a, u_b = \text{speeds of the upper and lower contact surfaces respectively, Fig.1} \]
\[ U = \text{dimensionless rolling speed, } \eta u/(E' R) \]
\[ w = \text{load of the contact per unit contact width} \]
\[ W = \text{dimensionless load of the contact, } w/(E' R) \]
\[ x = \text{coordinate, Fig.1} \]
\[ X = \text{dimensionless coordinate, } x/b \]
\[ \eta = \text{lubricant viscosity at ambient pressure} \]
\[ \alpha = \text{viscosity-pressure index of the lubricant} \]
\[ \lambda = G \sqrt{8W/\pi} \]
Fig. 1 Simulated contact. A-Inlet zone; B-Hertzian contact zone.

Fig. 2: The values of the integration $J_1$ in Eq.(1) calculated from Eq.(5) for $W/H_c$ ranging from 0.01 to 200 and their comparisons with the corresponding accurate values of $J_1$
Fig. 3  Plots of the accurate, present equation predicted, classical formula predicted central EHL film thickness in the present contact against the dimensionless load when $G=4500$ and $U=1.2E-9$. Eq.(6)-Present equation; Eq.(9)-The Dowson-Higginson film thickness formula.

Fig. 4  Plots of the accurate, present equation predicted, classical formula predicted central EHL film thickness in the present contact against the dimensionless rolling speed when $G=4500$ and $W=1.0E-3$. Eq.(6)-Present equation; Eq.(9)-The Dowson-Higginson film thickness formula.
Fig. 5: Plots of the relative errors (Er) of the present equation prediction of the central EHL film thickness compared to the accurate central EHL film thickness against the dimensionless load to film thickness ratio W/Hc respectively calculated from Figs. 3 and 4, G=4500