

IMAGE COMPRESSION USING TWO DIMENSIONAL DUAL TREE COMPLEX WAVELET TRANSFORM

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ABSTRACT

Digital image compression is important due to the high storage and transmission requirements. Various compression techniques have been proposed in recent years to achieve good compression. By removing the redundant data, the image can be represented in a smaller number of bits and hence can be compressed. There are many different methods of image compression. This paper investigates a proposed form of compression based on two dimensional Tree Complex Wavelet Transform The wavelet analysis does not actually compress a signal. Therefore Huffman coding is used with a signal processed by the wavelet analysis in order to compress data. Wide range of threshold values is used in the proposed form. From the results the proposed form gives higher rate of compression and lower RMS error.

KEYWORDS : Digital image compression, Wavelet transforms, Wavelet analysis, data compression.

INTRODUCTION

Digital image compression is important due to the high storage and transmission requirements. Various compression techniques have been proposed in recent years to achieve good compression. By removing the redundant data, the image can be represented in a smaller number of bits and hence can be compressed. A still image contains a large amount of spatial redundancy in plain areas where adjacent picture elements have almost the same values. It means that the pixel values are highly correlated. The redundancy can be removed to achieve Compression of the image data. The basic measure for the performance of a compression algorithm is compression ratio. In a lossy compression scheme, the image compression algorithm should achieve a trade-off between compression ratio and image quality. Higher compression ratios will produce lower image quality and vice versa. Quality and compression can also vary according to input image characteristics and content [2]. In recent years; many studies have been made on wavelets. Image compression is one of the most visible applications of wavelets. The rapid increase in the range and use of electronic imaging justifies attention for systematic design of an image compression system and for providing the image quality needed in different applications[2]. Although the discrete wavelet transform (DWT) has dominated the field of image compression for well over a decade, DWTs in their traditional critically sampled form are known to be somewhat deficient in several characteristics, lacking such properties as shift invariance and significant directional selectivity [2].

COMPLEX WAVELET

The critically sampled discrete wavelet transform (DWT) has been successfully applied to a wide range of signal processing tasks. However, its performance is limited because of the following problems [3].

1. Oscillations of the coefficients at a singularity (zero crossings).
2. Shift variance when small changes in the input cause large changes in the output.
3. Aliasing due to down sampling and non-ideal filtering during the analysis, which is cancelled out by the synthesis filters unless the coefficients are not altered.
4. Lack of directional selectivity in higher dimensions, e.g.in ability to distinguish between + 45degree and – 45degree edge orientations.

To overcome the shift dependence problem, they can exploit the undecimated (over-complete) DWT, however, without solving the directional selectivity problem. Another approach is inspired by the Fourier transform, whose magnitude is shift invariant and the phase offset encodes the shift. In such a wavelet transform, a large magnitude of a coefficient implies the presence of a singularity while the phase signifies its position within the support of the wavelet. The complex wavelet transform (CWT) employs *analytic* or *quadrature* wavelets guaranteeing Magnitude phase representation, shift invariance and no aliasing [2].

Recently, complex-valued wavelet transforms CWT have been proposed to improve upon these DWT deficiencies, with the dual-tree CWT (DT-CWT) [3] becoming a preferred approach due to the ease of its implementation. In the DT-CWT, real valued wavelet filters produce the real and imaginary parts of the transform in parallel decomposition trees, permitting exploitation of well established real-valued wavelet Implementations and methodologies. A primary advantage of the DT-CWT lies in that it results in decomposition with a much higher degree of directionality than that possessed by the traditional DWT. However, since both trees of the DT-CWT are themselves orthonormal or biorthogonal decompositions, the DTCWT taken as a whole is a redundant tight frame [3].

An analytic wavelet $(t) c y$ is composed of two real wavelets $(t) r y$ and $(t) i y$ forming a Hilbert transform (HT) pair which means that they are orthogonal, i.e. shifted by $p / 2$ in the complex plain [4]

$$\begin{aligned} \Psi_c(t) &= \Psi_r(t) + j\Psi_i(t) \quad \dots (1) \\ \Psi_i(t) &= HT\{\Psi_r(t)\} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\Psi_r(\tau)}{t - \tau} d\tau = \Psi_r(t) \frac{1}{\pi t} \end{aligned}$$

And for their Fourier transform pairs

$$\begin{aligned} H_r(\omega) &\text{ and } H_i(\omega) . \\ H_i(\omega) &= FT\{HT\{\Psi_r(t)\}\} = -j.\text{sgn}(\omega)H_r(\omega) \end{aligned}$$

The same concept of analytic or quadrature formulation is applied to the filter bank structure of standard DWT to produce complex solutions and in turn the CWT. The real-valued filter coefficients are replaced by Complex-valued coefficients by proper design methodology that satisfies the required conditions for convergence. Then the complex filter can again be decomposed into two real-valued filters. Thus, two real-valued filters that give their respective impulse responses in quadrature will form the Hilbert transform pair. The combined pair of two such filters is termed as an analytic filter. The formulation and interpretation of the analytic filter is fig (1)

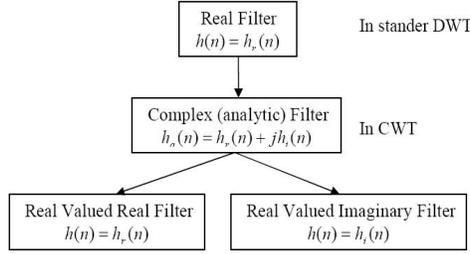


Figure 1 : Analytic Filter

2.1 2D DT-CWT

The 2D DT-CWT also more selectively discriminates features of various Orientations. While the critically decimated 2D DWT outputs three orientation selective sub-bands per level conveying image features oriented at the angles of 90° , $\pm 45^\circ$, and 0° , the 2D DT-CWT produces six directional sub bands per level to reveal the details of an image in $\pm 15^\circ$, $\pm 45^\circ$ and $\pm 75^\circ$ directions with 4:1 redundancy [6].

The implementation of 2-D DTCWT consists of two steps. **Firstly**, an input image is decomposed up to a desired level by two separable 2-DDWT branches, branch *a* and branch *b*, whose filters are specifically designed to meet the Hilbert pair requirement. Then six high-pass sub bands are generated at each level.

$$HL_a, LH_a, HH_a, HL_b, LH_b, \text{ and } HH_b$$

Secondly, every two corresponding sub bands which have the same pass-bands are linearly combined by either averaging or differencing. As a result, sub bands of 2-D DT-CWT at each level are obtained as

$$(LH_a + LH_b) / \sqrt{2}$$

$$\begin{aligned}
 & (LH_a - LH_b) / \sqrt{2}, \\
 & (HL_a + HL_b) / \sqrt{2}, \\
 & (HL_a - HL_b) / \sqrt{2}, \\
 & (HH_a + HH_b) / \sqrt{2}, \\
 & (HH_a - HH_b) / \sqrt{2}.
 \end{aligned}$$

The six wavelets defined by oriented, as shown in above. Because the sum/difference operation is orthonormal, this constitutes a perfect reconstruction wavelet transform. The imaginary part of 2D DT-CWT has similar basis function as the real part [8]. The 2-D DT-CWT structure has an extension of conjugate filtering in 2-D case. The filter bank structure of 2-D dual-tree is shown in figure (2).

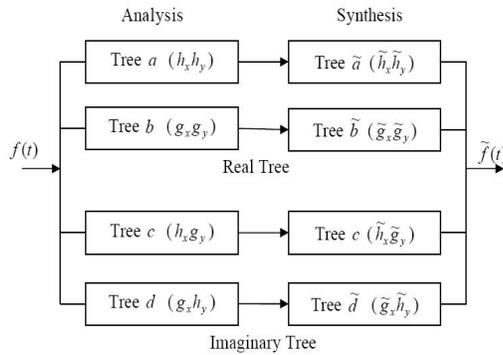


Figure 2 : Filter bank structure for 2D DT-CWT

2-D structure needs four trees for analysis as well as for synthesis. The pairs of conjugate Filters are applied to two dimensions (x and y) directions, which can be expressed as:

$$(h_x + jg_x)(h_y + jg_y) = (h_x h_y - g_x g_y) + j(h_x g_y + g_x h_y)$$

The filter bank structure of tree a, similar to standard 2-D DWT spanned over 3-level, is shown in figure (3).

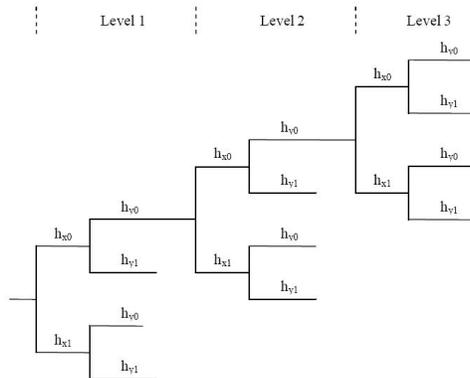


Figure 3 : Filter bank structure of figure (2)

All other *trees-(b, c & d)* have similar structures with the appropriate combinations of filters for row- and column- filtering. The overall 2-D dual-tree structure is 4-times redundant (expensive) than the standard 2-DDWT. The *tree-a* and *tree-b* form the real pair, while the *tree-c* and *tree-d* form the imaginary pair of the analysis filter bank. *Trees-(a~, b~)* and *trees-(c~, d~)* are the real and imaginary pairs respectively in the synthesis filter bank similar to their corresponding analysis pairs [4]

IMAGE COMPRESSION

There are many different methods of data compression. This investigation will concentrate on transform coding and then more specifically on Wavelet Transforms. Image data can be represented by coefficients of discrete image transforms. Coefficients that make only small contributions to the information contents can be omitted. Usually the image is split into blocks (sub images) of 8x8 or 16x16 pixels, and then each block in discrete cosine transform is transformed separately. However this does not take into account any correlation between blocks, and creates "blocking artifacts", which are not good if a smooth image is required

However wavelets transform is applied to entire images, rather than sub images, so it produces no blocking artifacts. This is a major advantage of wavelet compression over other transform compression methods.

For some signals, many of the wavelet coefficients are close to or equal to zero. Thresholding can modify the coefficients to produce more zeros. In Hard thresholding any coefficient below a threshold λ , is set to zero. This should then produce many consecutive zero's which can be stored in much less space, and transmitted more quickly by using entropy coding compression. An important point to note about Wavelet compression is "The use of wavelets and thresholding serves to process the original signal, but, to this point, no actual compression of data has occurred".

This explains that the wavelet analysis does not actually compress a signal. It simply provides information about the signal which allows the data to be compressed by standard entropy coding techniques, such as Huffman coding. Huffman coding is good to use with a signal processed by analysis wavelet, because it relies on the fact that the data values are small and in particular zero, Dual Tree CWT to compress data. It works by giving large numbers more bits and small numbers fewer bits. Long strings of zeros can be encoded very efficiently using this scheme. Therefore an actual percentage compression value can only be stated in conjunction with an entropy coding technique. To compare different wavelets, the number of zeros is used. More zeros will allow a higher compression rate, if there are many consecutive zeros, this will give an excellent compression rate. One of the popular threshold is the *hard threshold* function is shown in figure (4)

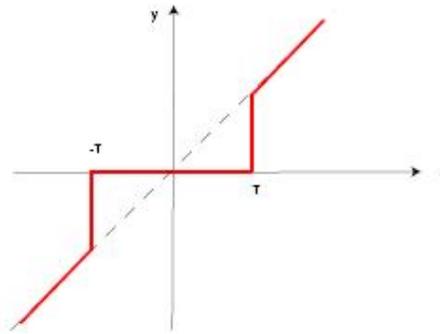


Figure 4 : Hard Thresholding function

$$\psi_T(x) = x \{ |x| > T \}$$

Which keeps the input if it is larger than the threshold T ; otherwise, it is set to zero. The wavelet thresholding procedure removes noise by thresholding *only* the wavelet coefficients of the detail sub bands, while keeping the low resolution coefficients unaltered [7].

Huffman Coding

Huffman coding is an entropy encoding algorithm used for lossless data compression. The term refers to the use of a variable-length code table for encoding a source symbol (such as a character in a file) where the variable-length code table has been derived in a particular way based on the estimated probability of occurrence for each possible value of the source symbol. It uses a specific method for choosing the representation for each symbol, resulting in a prefix code that expresses the most common source symbols using shorter strings of bits than are used for less common source symbols. The Huffman algorithm is based on statistical coding, which means that the probability of a symbol has a direct bearing on the length of its representation. The more probable the occurrence of a symbol is, the shorter will be its bit-size representation. In any file, certain characters are used more than others. Using binary representation, the number of bits required to represent each character depends upon the number of characters that have to be represented. Using one bit we can represent two characters, i.e., 0 represents the first character and 1 represents the second character. Using two bits we can represent four characters, and so on [8]. Unlike ASCII code, which is a fixed-length code using seven bits per character, Huffman compression is a variable-length coding system that assigns smaller codes for more frequently used characters and larger codes for less frequently used characters in order to reduce the size of files being compressed and transferred [9].

Compression Algorithm

The different method for compression, investigate differ only in the selection of the method. The basic procedure remains the same:

- (1) Digitize the source image into a signal.

- (2) Decompose the signals into wavelet w coefficients using DT-CWT.
- (3) Modify the coefficients from w, using thresholding.
- (4) Apply Huffman encoding to compress.
- (5) Reconstruct using the original approximation coefficients using Huffman decoding and inverse DT-CWT.

Figure (5) illustrates that the Compression algorithm using DTCWT.

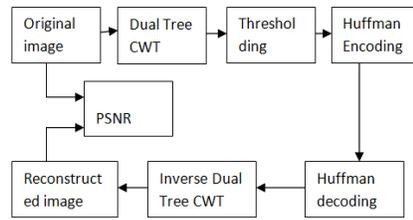


Figure 5 : Block diagram of compression algorithm using DTCWT

EXPERIMENTAL RESULTS AND DISCUSSIONS

Firstly, original image is applied to the 2D Dual tree Complex wavelet Transform, then the coefficient are applied to threshold function which is further compressed using Huffman Encoding. To reconstruct compressed image, compressed image is applied to decompression program, by which Huffman decoder and Inverse Dual Tree Complex wavelet is obtained. Compression Ratio (CR), RMS error, and Peak-Signal-to-Noise Ratio (PSNR) are obtained for the original and reconstructed images.

In the experiment image having size 256 x 256 (65,536 Bytes). The different statistical values of the image for Various Thresholds are summarized in the table (1)

Table 1 Image Size: 256*256

Parameter	TH=6	TH=10	TH=30	TH=60
Original File Size(bytes)	65240	65240	65240	65240
Compressed File Size (bytes)	10783	10114	9320	8125
Compression Ratio (CR)	6.05	6.45	7.39	7.98
Bits Per pixel (Bpp)	1.32	1.24	1.08	1.002
Peak-Signal-to-Noise Ratio(PSNR)	40.32	36.25	30.15	26.32
RMS error	2.43	3.9	8.93	12.81

Thus, it can be concluded that 2D Dual Tree Complex Wavelet and Huffman encoder gives excellent results. By choosing suitable threshold value compression ratio as high as 8 can be achieved.

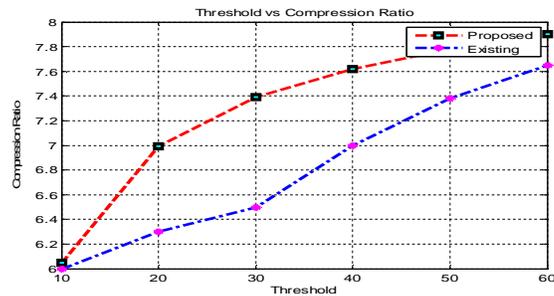


Figure 6.1 (a) : Threshold Vs compression Ratio

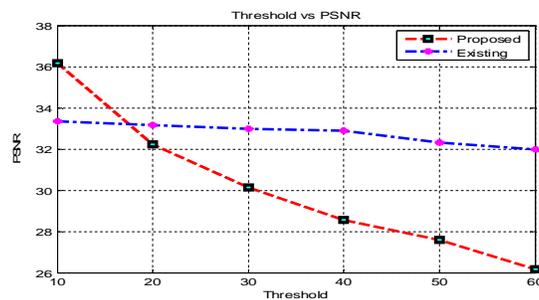


Figure 6.1 (b) : 'Threshold Vs PSNR

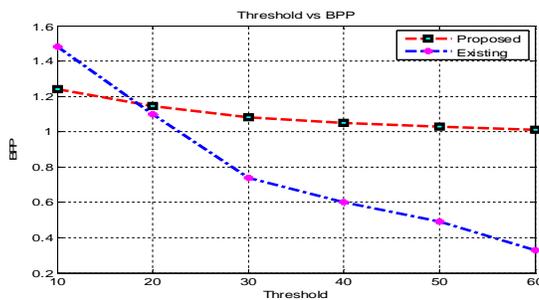


Figure 6.1 (c) : 'Threshold Vs BPP

The curves of 'Threshold Vs compression Ratio', 'Threshold Vs PSNR' and 'Threshold Vs BPP' and have been calculated and depicted in the fig 6.1 (a), (b) &(c) respectively. In which Image encoded and compressed using complex wavelet and only Huffman Encoder gives better Compression Ratio, BPP and PSNR values than Image Compressed by combining EZW Encoding with Huffman Encoder.

CONCLUSIONS

An Image compression Technique which uses the 2D Dual Tree Complex Wavelet Transform in combination with Huffman encoder is proposed here. Complex wavelet with Huffman encoder gives effective results in higher compression ratio, good BPP and better PSNR. The algorithm is tested on

different images, and it is observed that the Image Compression Based on 2D Dual Tree Complex Wavelet Transform (2D DT-CWT) performs consistent results compared to the results obtained by Still Image Compression by Combining EZW Encoding with Huffman Encoder. It is also observed that the results are better than these reported in [1]

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