SHEAR STRENGTH OF NORMAL AND LIGHT WEIGHT REINFORCED CONCRETE DEEP AND SHORT BEAMS WITHOUT WEB REINFORCEMENT

SHUAIB H. AHMAD, S.F.A.RAFEEQI AND SHAMSOON FAREED

1PhD
NED University of Engineering & Technology, Karachi-Pakistan
Telephone: (92-21) 99261261-8 Fax: (92-21) 99261255
sahmad@neduet.edu.pk

2PhD
NED University of Engineering & Technology, Karachi-Pakistan
Telephone: (92-21) 99261261-8 Fax: (92-21) 99261255
pvc2@neduet.edu.pk

3MEngg.
NED University of Engineering & Technology, Karachi-Pakistan
Telephone: (92-21) 99261261-8 Fax: (92-21) 99261255
sfareed@neduet.edu.pk

ABSTRACT

For deep reinforced concrete beams, the shear capacity can be predicted using empirical equations or the Strut-and-Tie Model Analysis as permitted in the Eurocode EC2 and ACI 318-08 Code. In this paper, a study is conducted to evaluate the predictive accuracy of Euro code EC2 empirical equation for shear capacity of deep and short reinforced concrete beams. The results indicate that for normal as well as high strength reinforced concrete deep beams, the Euro code EC2 predictions are overly conservative and Euro code EC2 equation proves to be more accurate in predicting the shear capacity of reinforced concrete short reinforced concrete beams. On the basis of experimental results of
reinforced concrete deep beams, an empirical equation is proposed, which applicability is over a larger number of deep beams and has better predictive capability signified by higher value of Coefficient of Correlation as compared to Euro code EC2 equation.

KEYWORDS: Empirical equations, shear strength, deep beams, short beams.

LIST OF ABBREVIATIONS

\[ V_c: \text{Shear capacity of concrete}; \]
\[ f'_c: \text{concrete compressive strength}; \]
\[ \frac{a}{d}: \text{shear span to depth ratio}; \]
\[ \zeta: \text{size effect or depth factor}; \]
\[ b: \text{width of beam}; \]
\[ d: \text{effective depth}; \]
\[ V_{\text{exp}}: \text{shear force observed}; \]
\[ V_{\text{pre}}: \text{shear force predicted}; \]
\[ \text{COR}: \text{coefficient of correlation}; \]
\[ V_p: \text{shear capacity predicted by proposed empirical equation}. \]

INTRODUCTION

The shear capacity (strength) of deep beams \( \left( \frac{a}{d} \leq 1 \right) \) can be predicted using empirical equations or the Strut-and-Tie Model Analysis as permitted in the ACI 318-08 (2008) and the Eurocode EC2 (2002). Recently Jung-woong Park et al (2007) used the Strut-and-Tie based method for predicting the shear strength of deep beams. The proposed method employed constitutive laws for cracked reinforced concrete, considered strain compatibility, and used a secant stiffness formulation and was used to calculate the capacity of 214 normal- and high-strength concrete deep beams with and without web reinforcement tested in laboratories. The proposed method provided more accurate estimates of capacity than the strut-and-tie provisions in either ACI code (2008) or the Canadian code (1999). The comparison shows that the proposed method consistently predicts the strengths of deep beams with a wide range of horizontal and vertical web reinforcement ratios, concrete strengths, and shear span-to-depth ratios \( (a/d) \) well. The use of strut-tie approach although seemingly more comprehensive, is iterative in nature and from the design perspective,
Shear Strength of Normal and Light Weight Reinforced Concrete Deep and Short Beams Without Web Reinforcement


Euro code EC2 (2002) provides empirical equation for computing the shear capacity of deep beams. Recently, a study was conducted [Shamssoon et al (2011)] to assess the predictive accuracy of empirical equation used in Euro code EC2 (2002) and those proposed in literature for evaluating the shear strength of reinforced concrete short \((2.5 \leq \frac{a}{d} \leq 1)\) beams. The study identified that the use of function seem to better reflect the effect of the concrete compressive strength \(f'_c\), when estimating the shear capacity of short reinforced concrete beams without web reinforcement.

In this study, the predictive accuracy of Euro code EC2 (2002) for predicting the shear capacity of reinforced concrete deep \((\frac{a}{d} \leq 1)\) and short \((\frac{a}{d} < 2.5)\) beams without web reinforcement is evaluated using the experimental data contained in ACCESS shear database [Rafeeqi et al (2011)]. The results indicate that for normal strength as well as high strength \((f'_c \geq 6000\) psi) reinforced concrete deep beams, the predictive accuracy of Euro code EC2 is overly conservative and Euro code EC2 (2002) equation proves to be more accurate in predicting the shear capacity of reinforced concrete short \((a/d < 2.5)\) as compare to reinforced concrete deep \((a/d \leq 1)\) beams.

**EVALUATION OF EUROCODE EC2 EQUATION**

Euro code EC2 (2002) uses the following equations for estimating the shear capacity of reinforced concrete beams;
For beams having shear span to depth ratio \((\frac{b}{d} \geq 1)\)

\[
\phi_c = \left( \frac{0.18}{f_c} K (100 f_{ck})^{1/2} + 0.15 \sigma_{cp} \right)
\]

\[
\phi_{2ax} = 0.035 k^{3/2} f_{ck}^{1/2}
\]

Where

\[
f_{ck} \leq 100 \text{ksi}
\]

\[
K = 1 + \sqrt{\frac{200}{d}} \leq 2
\]

\[
\rho_1 = \frac{A_s}{b_d d} \leq 0.02
\]

For beams having shear span to depth ratio \((\frac{b}{d} \leq 1)\)

\[
\phi_c = \left( \frac{4.2}{d} \right) \left( \frac{0.18}{f_c} K (100 f_{ck})^{1/2} + 0.13 \sigma_{cp} \right)
\]

\(\sigma_{cp}\) is the term that accounts for the effect of prestressing. For the case of reinforced concrete, this term is zero.

In order to evaluate the predictive accuracy of Euro code EC2 (2002) equation for reinforced concrete deep \((\frac{b}{d} \leq 1)\) and short beams having shear span to depth ratio \(\frac{b}{d} < 2.5\), a total of 114 test beams over which use of Euro code EC2 (2002) were applicable were selected from ACCESS shear database [Rafeeqi et al (2011)]. The test data was categorized into normal and high strength \((f'_c \geq 6000 \text{ psi})\) reinforced concrete beams as shown in Table 1 to study the effect of concrete compressive strength \(f'_c\) on the shear capacity of reinforced concrete deep and short beams.
Table 1: Summary of results showing the Average Margin of Safety \( \left( \frac{V_{\text{avg}}}{V_{\text{pre}}} \right)_{\text{avg}} \) with coefficient of correlation (COR) for NSC and HSC reinforced deep and short beams

<table>
<thead>
<tr>
<th>Code</th>
<th>No. of Deep Beams ( \frac{h}{L} \leq 1 ) used for Evaluation</th>
<th>Average Margin of Safety ( \left( \frac{V_{\text{avg}}}{V_{\text{pre}}} \right)_{\text{avg}} )</th>
<th>Coefficient of Correlation (COR)</th>
<th>Strength of concrete</th>
<th>Coefficient of Correlation (COR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro code EC2</td>
<td>114</td>
<td>6.73</td>
<td>0.396</td>
<td>91</td>
<td>6.99</td>
</tr>
<tr>
<td></td>
<td>273</td>
<td>2.86</td>
<td>0.828</td>
<td>23</td>
<td>5.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>215</td>
<td>2.86</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>58</td>
<td>0.72</td>
</tr>
</tbody>
</table>

For all 114 deep beams \( \frac{h}{L} \leq 1 \) of normal strength concrete (NSC) as well as high strength concrete (HSC), the Average Margin of Safety \( \left( \frac{V_{\text{avg}}}{V_{\text{pre}}} \right)_{\text{avg}} \) for over which the Euro code EC2 (2002) is applicable, comes out to be 6.73 and the coefficient of correlation (COR) comes out to be 0.396 (Table 1). This indicates that Euro code EC2 (2002) equation is overly conservative for predicting the shear capacity of deep beams and the accuracy of the predictions is not good.

For 91 NSC deep beams, the average Margin of Safety \( \left( \frac{V_{\text{avg}}}{V_{\text{pre}}} \right)_{\text{avg}} \) is 6.99, whereas for 23 HSC deep beams, the average Margin of Safety \( \left( \frac{V_{\text{avg}}}{V_{\text{pre}}} \right)_{\text{avg}} \) is 5.70 (Table 1) and the COR for both cases is 0.447.
For 273 short beams \( \left( \frac{h}{d} \leq 2.5 \right) \) of NSC as well as HSC, the average Margin of Safety for Euro code EC2 (2002) comes out to be 2.86 with COR of 0.828 (Table 1). This indicates that Euro code EC2 (2002) equation is adequately conservative for predicting the shear capacity of short beams \( \left( \frac{h}{d} \leq 2.5 \right) \) beams and the accuracy of the predictions is good. This shows that Euro code EC2 (2002) equation is more accurate in predicting the shear capacity of reinforced concrete short beams \( \left( \frac{h}{d} \leq 2.5 \right) \) as compared to reinforced concrete deep \( \left( \frac{h}{d} \leq 1 \right) \) beams.

For 215 NSC short beams \( \left( \frac{h}{d} \leq 2.5 \right) \), the average Margin of Safety \( \frac{\text{exp} \left( \frac{\tau_{\text{prf}c}}{\text{avg}} \right)}{\text{avg}} \) is 2.86, with a COR of 0.313, whereas for 58 HSC short beams \( \left( \frac{h}{d} \leq 2.5 \right) \), the average Margin of Safety \( \frac{\text{exp} \left( \frac{\tau_{\text{prf}c}}{\text{avg}} \right)}{\text{avg}} \) is 0.72 with a COR of 0.724 (Table 1). This indicates that Euro code EC2 (2002) equation may not be safe for predicting the shear capacity of HSC short beams \( \left( \frac{h}{d} \leq 2.5 \right) \).

**PROPOSED EMPIRICAL EQUATION**

On the basis of shear data base of the experimental test results [Rafeeqi et al (2011)], an empirical equation is developed for predicting the shear capacity of reinforced concrete deep \( \left( \frac{h}{d} \leq 1 \right) \) beams. The proposed equation is

\[
\tau_{\text{prf}} = \left[ \frac{\tau_{\text{c}}} {\sigma_{\text{c}}} \right]^{0.33} \rho^{0.1} \quad (1)
\]

Eq. 1 is applicable to 151 reinforced concrete deep \( \left( \frac{h}{d} \leq 1 \right) \) test beams in the shear database [Rafeeqi et al (2011)]. The proposed empirical equation contains the basic influencing parameters i.e. concrete compressive strength \( f_{\text{c}} \), shear
span to depth ratio $\frac{s}{d}$ and ratio of longitudinal reinforcement $f_r$. The average Margin of safety $\left(\frac{V_{exp}}{V_{pre}}\right)_{avg}$ for proposed empirical is 2.26 with a COR of 0.760 (Table 2). The proposed equation predicts the shear capacity of deep beams more accurately and safely as it has relatively high COR and relatively low average Margin of safety $\left(\frac{V_{exp}}{V_{pre}}\right)_{avg}$ as compared to Euro code EC2 (2002) equation and also the applicability of the proposed equation is over a larger number of beams (151) as opposed to 114 beams for equation used in Euro codeEC2 [2].

**Table 2**: Summary of results showing the Average Margin of Safety $\left(\frac{V_{exp}}{V_{pre}}\right)_{avg}$ with coefficient of correlation (COR) for NSC and HSC reinforced deep and short beams using proposed empirical equation

<table>
<thead>
<tr>
<th>Proposed Empirical Equation</th>
<th>No. of Deep Beams ($\frac{s}{d} &lt; 1$) used for Evaluation</th>
<th>Average Margin of Safety $\left(\frac{V_{exp}}{V_{pre}}\right)_{avg}$</th>
<th>Coefficient of Correlation (COR)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>151</td>
<td>2.26</td>
<td>0.760</td>
</tr>
</tbody>
</table>

**SUMMARY AND CONCLUSIONS**

From the evaluation study of Euro code EC2 to predict the shear capacity of reinforced concrete deep and short beams, the following conclusions can be drawn:

1) For NSC as well as HSC deep beams, the predictive accuracy of Euro code is overly conservative.
2) Euro code EC2 [2] equation is be more accurate in predicting the shear capacity of reinforced concrete short \( \left( \frac{d}{h} \leq 2.5 \right) \) beams as compared to reinforced concrete deep \( \left( \frac{d}{h} \leq 1 \right) \) beams., however it may not be safe for predicting the shear capacity of HSC short beams \( \left( \frac{d}{h} \leq 2.5 \right) \).

3) For reinforced concrete deep \( \left( \frac{d}{h} \leq 1 \right) \) beams, an Equation is proposed, that predicts the shear capacity more accurately and safely as compared to Euro code EC2 equation and applicability of the proposed equation is over a larger number (151) of test beams as compared to 114 beams for Euro code EC2 equation.

REFERENCES

1. ACI Committee 318, Building Code Requirement for Reinforced Concrete (ACI 318-08), American Concrete Institute, Detroit.


