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ON SOME NEW POLYNOMIAL AND ITS EIGEN VALUES

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ABSTRACT

The path layer matrix for a connected graph is a matrix $P=[p_{ij}]$ where $p_{ij} =$ number of vertices having a path of length j from v_i .In this paper we define a new polynomial for a connected graph with respect to its path layer matrix and obtain eigen values of characteristic polynomial corresponding to it for some standard graphs.

KEY WORDS: Path layer matrix of a graph, Eigen values, Energy.

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1. INTRODUCTION

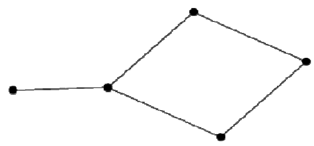
The path layer matrix for a connected graph is a matrix $P=[p_{ij}]$

where $p_{ij} =$ number of vertices having a path of length j from v_i It is generalization of the degree sequence of graph ,known as path degree sequence of graph. [1]..This matrix is also known as atomic path code of a molecular graph [2].Related invariants have found interesting applications in Organic Chemistry to characterize branching in molecules, To establish similarity among molecular graphs and in drug design.[2]-[5].

Let *G* be a simple connected undirected graph on *n* vertices and *m* edges. Let *P* be the path layer matrix of *G*. The characteristic polynomial of $\Phi[PP^T(G:$

$\lambda)] = \det[\lambda I - PP^T(G)]$, where I is an identity matrix. The roots of equation $\Phi[PP^T(G:\lambda)] = 0$, can be called path energy of a graph G is defined as $E_p(G) = \sum_{i=1}^{n} |\lambda_i|$.

Ex:



H

For graph H, the path layer matrix is $P(H) = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 3 & 1 & 2 & 0 \\ 2 & 2 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 2 & 1 & 3 & 0 \end{bmatrix}$

$P_M(H) = PP^T(H)$ is the matrix, $\begin{bmatrix} 10 & 9 & 10 & 10 & 7 \\ 7 & 14 & 12 & 12 & 13 \\ 10 & 12 & 13 & 13 & 12 \\ 10 & 12 & 13 & 13 & 12 \\ 7 & 13 & 12 & 12 & 14 \end{bmatrix}$

The characteristic polynomial corresponding to P_M may be called as path polynomial denoted by Pa (G :λ). The path polynomial for H as in figure is,

$$Pa(H:\lambda) = \lambda^5 - 64\lambda^4 + 407\lambda^3 - 492\lambda^2 + 148\lambda$$

The path eigen values i.e, roots of Pa(H:λ) are 0, .4705,1 5.5179,57.0117

Lemma 1 [6]: If a and b are scalars then,

$$\begin{vmatrix} a & b & b & \cdots & b & b \\ b & a & b & \cdots & b & b \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ b & b & b & \cdots & a & b \\ b & b & b & \cdots & b & a \end{vmatrix} = (a-b)^{n-1}\{a+(n-1)b\}$$

Path polynomials of Some Standard Graphs

Complete Graph

Let K_n be a complete graph of order n. Then the path layer matrix of K_n is given by

$$P(K_n)=\begin{bmatrix} n-1 & n-1 & \cdots & \cdots & n-1 \\ n-1 & n-1 & \cdots & \cdots & n-1 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n-1 & n-1 & \cdots & \cdots & n-1 \end{bmatrix}$$

$$PP^T(K_n)=\begin{bmatrix} (n-1)^3 & (n-1)^3 & \cdots & \cdots & (n-1)^3 \\ (n-1)^3 & (n-1)^3 & \cdots & \cdots & (n-1)^3 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (n-1)^3 & (n-1)^3 & \cdots & \cdots & (n-1)^3 \end{bmatrix}$$

Hence the path polynomial of K_n is ,

Pa (K_n:λ)=

$$\begin{vmatrix} \lambda-(n-1)^3 & -(n-1)^3 & \cdots & \cdots & -(n-1)^3 \\ -(n-1)^3 & \lambda-(n-1)^3 & \cdots & \cdots & -(n-1)^3 \\ \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -(n-1)^3 & -(n-1)^3 & \cdots & \cdots & \lambda-(n-1)^3 \end{vmatrix} = 0$$

$$= [\lambda + n(n-1)^3] (\lambda)^{n-1} = 0$$

Thus path eigen values are $\lambda = 0$ 'n-1' times & $-n(n-1)^3$ then path energy (sum of absolute path eigen values) is $n(n-1)^3$.

Star ($K_{1,n}$)

Let $K_{1,n}$ be a star graph of order n+1. Then the path layer matrix of $K_{1,n}$ is given by

$$P(K_{1,n}) = \begin{bmatrix} n & 0 & 0 & \cdots & 0 \\ 1 & n-1 & 0 & \cdots & 0 \\ 1 & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & n-1 & 0 & \cdots & 0 \end{bmatrix}$$

$$PP^T (K_{1,n}) = \begin{bmatrix} n^2 & n & \cdots & \cdots & n \\ n & 1+(n-1)^2 & \cdots & \cdots & 1+(n-1)^2 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n & 1+(n-1)^2 & \cdots & \cdots & 1+(n-1)^2 \end{bmatrix}$$

Hence the path polynomial of K_n is ,

$$Pa(K_n:\lambda)=\begin{vmatrix} \lambda-n^2 & -n & \cdots & \cdots & -n \\ -n & \lambda-(n^2-2n+2) & \cdots & \cdots & -(n^2-2n+2) \\ \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -n & -(n^2-2n+2) & \cdots & \cdots & \lambda-(n^2-2n+2) \end{vmatrix}=0$$

Performing $R_i - R_2$ i= 3,4,…….n+1 we get

$$Pa(K_{1,n}:\lambda)=\begin{vmatrix} \lambda-n^2 & -n & \cdots & \cdots & -n \\ -n & \lambda-(n^2-2n+2) & \cdots & \cdots & -(n^2-2n+2) \\ 0 & -\lambda & \lambda & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & -\lambda & 0 & \cdots & \lambda \end{vmatrix}=0$$

Performing $C_2+\sum_{3}^{n+1} C_i$

$Pa(K_{1,n}:\lambda)=$

$$\begin{vmatrix} \lambda-n^2 & -n^2 & \cdots & \cdots & -n \\ -n & \lambda-n(n^2-2n+2) & \cdots & \cdots & -(n^2-2n+2) \\ 0 & 0 & \lambda & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda \end{vmatrix}=0$$

Performing $R_2 + \frac{n}{\lambda - n^2} R_1$

$Pa(K_{1,n}:\lambda)=$

$$\begin{vmatrix} \lambda-n^2 & -n^2 & \cdots & \cdots & -n \\ 0 & \lambda-n(n^2-2n+2)-\frac{n^3}{\lambda-n^2} & \cdots & \cdots & -(n^2-2n+2) \\ 0 & 0 & \lambda & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda \end{vmatrix} = 0$$

Which is in the diagonal form .On direct expansion we have

$$Pa(K_{1,n}:\lambda) = (\lambda - n^2)[\lambda - n(n^2 - 2n + 2) - \frac{n^3}{\lambda - n^2}]\lambda^{n-1} = 0$$

$$= \lambda^{n-1}[\lambda^2 - n\,(n^2 - n + 2)\,\lambda + n^3\,(n-1)^2] = 0$$

Thus path eigen values are $\lambda = 0$ 'n–1' times & $\frac{(n^3 - n^2 + 2n) \pm n\sqrt{n^4 - 6n^3 + 13n^2 - 8n + 4}}{2}$ giving path energy (sum of absolute path eigen values) as, $n^3 - n^2 + 2n$

Cycle C_n

Let c_n be a cycle of order n. Then the path layer matrix of C_n upon n.

Case1 : If n is even say n =2k, then

$$P(C2k) = \begin{bmatrix} 2 & 2 & \cdots & 2 & 1 & 2 & \cdots & 2 \\ 2 & 2 & \cdots & 2 & 1 & 2 & \cdots & 2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 2 & 2 & \cdots & 2 & 1 & 2 & \cdots & 2 \\ 2 & 2 & \cdots & 2 & 1 & 2 & \cdots & 2 \end{bmatrix}$$

In the first k-1 columns there are all 2's ,followed by a column of all 1's then again all 1 s in the following k-1 columns.

$$PP^T(C_{2k}) = \begin{bmatrix} 4k-3 & 4k-3 & \cdots & \cdots & 4k-3 \\ 4k-3 & 4k-3 & \cdots & \cdots & 4k-3 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 4k-3 & 4k-3 & \cdots & \cdots & 4k-3 \end{bmatrix}$$

Hence the path polynomial of C2k is,

Pa(C2k:λ)=

$$\begin{vmatrix} \lambda-(4k-3) & -(4k-3) & \cdots & \cdots & -(4k-3) \\ -(4k-3) & \lambda-(4k-3) & \cdots & \cdots & -(4k-3) \\ \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -(4k-3) & -(4k-3) & \cdots & \cdots & \lambda-(4k-3) \end{vmatrix}$$

The determinant is in the form as per lemma 1

Hence, Pa(C2k:λ) =λ2k-1[λ-(4k-3)-(2k-1)(4k-3)]

= λ2k-1[λ-2k (4k-3)]

Thus path eigen values are $\lambda = 0$ '2k-1' times & 2k(4k-3) then path energy (sum of absolute path eigen values) is 2k(4k-3).

Case 2 : Odd cycle C2k+1

$$P(C2k+1) = \begin{bmatrix} 2 & 2 & \cdots & 2 & 1 \\ 2 & 2 & \cdots & 2 & 1 \\ \vdots & \vdots & \ddots & 2 & 1 \\ \vdots & \vdots & \vdots & 2 & 1 \\ 2 & 2 & \cdots & 2 & 1 \end{bmatrix}$$

In the first 2k-1 columns there are all 2's ,followed by a column of all 1's

$$PP^T(C_{2k+1}) = \begin{bmatrix} 8k-3 & 8k-3 & \cdots & \cdots & 8k-3 \\ 8k-3 & 8k-3 & \cdots & \cdots & 8k-3 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 8k-3 & 8k-3 & \cdots & \cdots & 8k-3 \end{bmatrix}$$

Hence the path polynomial of C2k+1 is,

Pa(C2k+1:λ)=

$$\begin{vmatrix} \lambda-(8k-3) & -(8k-3) & \cdots & \cdots & -(8k-3) \\ -(8k-3) & \lambda-(8k-3) & \cdots & \cdots & -(8k-3) \\ \vdots & \vdots & \ddots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -(8k-3) & -(8k-3) & \cdots & \cdots & \lambda-(8k-3) \end{vmatrix}$$

The determinant is in the form as per lemma 2.3.1

Hence, $Pa(C_{2k+1}:\lambda) = \lambda^{2k}[\lambda-(8k-3)-2k(8k-3)]$

$$= \lambda^{2k}[\lambda-(2k+1)(8k-3)]$$

Thus path eigen values are $\lambda = 0$ '2k' times & (2k+1) (8k– 3) then path energy (sum of absolute path eigen values) is (2k+1) (8k– 3).

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