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INVENTORY MODEL FOR DETERIORATING ITEMS IN PRESENCE OF TRADE CREDIT PERIOD, TIME DEPENDENT DEMAND RATE AND DETERIORATION USING DISCOUNTED CASH FLOW APPROACH

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ABSTRACT

In this paper, we are using discounted cash flow approach for deteriorating items in the presence of trade credit period. Demand rate is time dependent and follows the power demand pattern. Shortages are not allowed and deterioration follows the Weibull distribution rate. Mathematical models are derived for three different cases : Instantaneous cash flows, credit only on units in stock, fixed credit period. Inflation and time value of money is also considered. Our approach is to find the optimal value of all future cash flows for three cases. Numerical examples are also given to illustrate the theoretical results. Sensitivity analysis of various parameters are also discussed.

1. INTRODUCTION

Ghare & Schrader (1963) were the first who studied inventory models of deteriorating items. The assumption that the goods in inventory always preserve their physical characteristics is not true in general. There are some items, which are subject to the risks of breakage, deterioration, evaporation, obsolescence and so forth. Managing inventory for deteriorating items is of great concern to the retailers, wholesalers, even to the production managers, who are in the business of perishable items, the items that can deteriorate or lose values under normal conditions such as meat, fish, sea food, poultry, diary products, fruits and vegetables, some special type of medicines, radioactive substances etc., often transportation of which also needs special care. In most of the cases, these items should be used within a short period of time after delivery, as it may not be possible to preserve them in the same manner after delivery.

It is generally assumed that the buyer must pay for the items as soon as he receives them from the supplier, but in reality supplier will allow a certain fixed period called credit period, for settling the amount the retailer owes to him for the items supplied. The credit period reduces the buyer’s cost of holding stock because it reduce the amount of capital invested in stock for the duration of the permissible period. Chung (1989) used the discounted cash flows (DCF) approach for studying the optimal inventory policy in the presence of the trade credit, which permits an explicit recognition of the exact timing of cash flows associated with the inventory system.

A DCF approach permits a proper recognition of the financial implication of the opportunity cost and out of pocket costs in inventory system. Aggarwal and Jaggi (1994) analyzed the credit financing in economic ordering policies of deteriorating items in the presence of trade credit using a DCF approach. Liao et al. (2000) presented a model with deteriorating items under inflation, when delay in payments is permissible. Chang (2004) presented an EOQ model with deteriorating items under inflation when the supplier provides a permissible delay of payments for a large order that is greater than or equal to the pre-determined quantity. Shah and Shah (1998) presented a probabilistic inventory model with cost in case delay in payments is permissible. Chang et al. (2001)

developed a finite time horizon inventory model with both deterioration and monetary time value when payment periods are offered. Huang and Chung (2003) discussed replenishment and payment policies to minimize the total cost of cash discount and payment delays. Huang (2003) considered an EOQ model in which supplier offers a credit period to retailer and retailer offers a credit period to the customers.

Demand also depends on the retailers sales efforts. This situation was discussed by Taylor (2002). He proved that coordination cannot be achieved with linear rebates and returns or target rebates alone. He provided a properly designed target rebate and returns contracts to get coordination. Krishanan et al. (2004) analysed the coordination of contracts for decentralized supply chains with the retailer promotional efforts. They found that a buy back policy cannot be utilized to coordinate the channels and they provided three contracts to achieve channel coordination. Liang et al. (2005) discussed an inventory model with non-instantaneous receipt under trade credit in which the supplier provides not only a permissible delay but also a cash discount to the retailer and obtained the optimal order cycle and orders receipt period so that the total relevant cost per unit time is minimized. Tripathi (2011) presented the economic ordering policies of time dependent deteriorating items in presence of trade credit using discounted cash flow approach. He found the optimal present values of all future cash flows for three cases; instantaneous cash-flows, credit only on units in stock and fixed credit period. In this paper, we are going to extend his idea by taking time-dependent deterioration and time dependent demand rate. Effects of inflation and time value of money is also considered.

2. ASSUMPTIONS

1. Deterioration of items starts after a definite time.
2. Deterioration rate varies with time and follows a two parameter Weibull distribution.

3. Replenishment is instantaneous.

4. Lead time is zero.

5. Shortages are not allowed.

6. There is no repair or replacement of deteriorating items during the period under consideration.

7. Inflation and time value of money is considered.

8. The demand rate is time-dependent and follows the power demand pattern, $D(t) = D_0 t^{m-1}$, where

$m \in (0, \infty)$ is the pattern index and $D_0 = dm\left(\frac{1}{T}\right)^m$ where d is the demand size during the fixed cycle time T, $D_0 > 0$.

3. NOTATIONS

1. 'C' is unit cost of the item.

2. 'Q' is the order quantity.

3. 'D(t)' is the demand rate at time 't.

4. 'i' is Inventory holding cost fraction.

5. 'iC' is the out-of-pocket inventory carrying cost per unit time.

6. 'R' is constant representing the difference between the discount rate and inflation rate.

7. 'H' is the ordering cost per unit.

8. I(t) is the inventory level at time t.

9. T_1 is the optimal cycle time for case I.

10. T_2 is the optimal cycle time for case II.

11. T_3 is the optimal cycle time for case III.

12. Z_1 (T) is the present value of all future cash-flows for case I.

13. Z_2 (T) is the present value of all future cash-flows for case II.

14. Z_3 (T) is the present value of all future cash-flows for case III.

15. Z_1 (T_1) is the optimal value of all future cash-flows for case I.

16. Z_2 (T_2) is the optimal value of all future cash-flows for case II.

17. Z_3 (T_3) is the optimal value of all future cash-flows for case III.

18. T is the inventory cycle time.

4. MATHEMATICAL FORMULATION

The level of inventory I(t) at time 't' is depleted due to both market demand and deterioration. The differential equation describing the inventory system over (0, T) is given by

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -D_0 t^{m-1}, \ \ 0 \le t \le T \qquad \ldots(1)$$

with the boundary condition, $I(T) = 0$

The solution of equation (1) is given by

$$I(t)e^{\alpha t^{\beta}} = -\int D_0 t^{m-1} e^{\alpha t^{\beta}} dt + C_1$$

$$= -D_0 \int t^{m-1}\left(1 + \alpha t^{\beta}\right) dt + C_1$$

[Assuming a very small value of α $(0 \le \alpha < 1)$, the approximate solution is obtained by neglecting the second and higher order terms of α]

$$= -D_0 \left[\frac{t^m}{m} + \frac{\alpha t^{m+\beta}}{m+\beta}\right] + C_1$$

Now $I(T)=0$

$$\Rightarrow C_1 = D_0\left[\frac{T^m}{m}+\frac{\alpha T^{m+\beta}}{m+\beta}\right]$$

$$\therefore I(t) = D_0\left[\frac{T^m}{m}+\frac{\alpha T^{m+\beta}}{m+\beta}-\left(\frac{t^m}{m}+\frac{\alpha t^{m+\beta}}{m+\beta}\right)\right]e^{-\alpha t^{\beta}} \qquad \ldots(2)$$

Order quantity, $Q = I(0) = D_0\left(\frac{T^m}{m}+\frac{\alpha T^{m+\beta}}{m+\beta}\right)$ …(3)

The number of deteriorating units during one cycle,

$$D(T) = Q - \int_0^T D(t)\,dt$$

$$= Q - \int_0^T D_0 t^{m-1} dt$$

$$= Q - D_0\left[\frac{T^m}{m}\right]$$

$$= D_0\left(\frac{T^m}{m}+\frac{\alpha T^{m+\beta}}{m+\beta}\right) - D_0\left(\frac{T^m}{m}\right)$$

$$= \frac{D_0\alpha}{m+\beta}T^{m+\beta} \qquad \ldots(4)$$

Using DCF approach, we have three cases on the trade credit terms.

Case I : Instantaneous cash-flows : In this case, we present the DCF approach to the inventory model of time-dependent deteriorating items under instantaneous inventory holding cost.

Hence at the beginning of each cycle, there will be cash out flows of ordering cost and purchasing cost. Since, the inventory carrying cost is proportional to the value of the inventory, the out-of-pocket inventory carrying cost per unit time at ‘t’ is iCI(t). Hence, the present value of cash flow for the first order cycle $z_1(T)$ is

$$z_1(T) = H + CQ + iC\int_0^T I(t)e^{-Rt}dt$$

$$= H + CD_0\left(\frac{T^m}{m} + \frac{\alpha T^{m+\beta}}{m+\beta}\right) + iC\int_0^T D_0\left[\frac{T^m}{m} + \frac{\alpha T^{m+\beta}}{m+\beta} - \frac{t^m}{m} - \frac{\alpha t^{m+\beta}}{m+\beta} - \frac{\alpha T^{m+\beta}}{m} + \frac{\alpha T^{\beta} t^m}{m}\right]e^{-Rt}dt$$

$$e^{-Rt} \approx 1 - Rt$$

$$= H + CD_0\left(\frac{T^m}{m} + \frac{\alpha T^{m+\beta}}{m+\beta}\right) + iCD_0\left[\frac{T^m t}{m} + \frac{\alpha T^{m+\beta}.t}{m+\beta} - \frac{t^{m+1}}{m(m+1)} - \frac{\alpha t^{m+\beta+1}}{(m+\beta)(m+\beta+1)}\right.$$

$$\left. - \frac{\alpha T^{m+\beta}.t}{m} + \frac{\alpha T^{\beta} t^{m+1}}{m(m+1)} - \frac{RT^m t^2}{2m} + \frac{Rt^{m+2}}{m(m+2)}\right]_0^T$$

$$= H + CD_0\left(\frac{T^m}{m} + \frac{\alpha T^{m+\beta}}{m+\beta}\right) + iCD_0\left(\frac{T^{m+1}}{m+1} - \frac{\alpha\beta T^{m+\beta+1}}{(m+1)(m+\beta+1)} - \frac{RT^{m+2}}{2(m+2)}\right)$$

….(5)

Hence the present value of all future cash flows is

$$Z_1(T) = \sum_{n=0}^{\infty} z_1(T)e^{-nRT} = \frac{z_1(T)}{1-e^{-RT}}$$

Since $e^{-RT} \approx 1 - RT$ (approx.)

$\therefore\ 1 - e^{-RT} \approx 1 - (1 - RT) = RT$

$\therefore$ Equation (5) becomes

i.e.

$$Z_1(T)=\frac{1}{RT}\left[H+CD_0\left(\frac{T^m}{m}+\frac{\alpha T^{m+\beta}}{m+\beta}\right)+iCD_0\left(\frac{T^{m+1}}{m+1}-\frac{\alpha\beta T^{m+\beta+1}}{(m+1)(m+\beta+1)}-\frac{RT^{m+2}}{2(m+2)}\right)\right]$$

$$Z_1(T)=\frac{1}{R}\left[\frac{H}{T}+CD_0\left(\frac{T^{m-1}}{m}+\frac{\alpha T^{m+\beta-1}}{m+\beta}\right)+iCD_0\left(\frac{T^m}{m+1}-\frac{\alpha\beta T^{m+\beta}}{(m+1)(m+\beta+1)}-\frac{RT^{m+1}}{2(m+2)}\right)\right] \quad \ldots(6)$$

The optimal value of T can be found by solving $\frac{\partial Z_1(T)}{\partial T}=0$

Diff. equation (6) partially w.r.t. 'T', and equating to zero, we get

$$\frac{\partial Z_1(T)}{\partial T}=\frac{1}{R}\left[\frac{-H}{T^2}+CD_0\left(\frac{m-1}{m}T^{m-2}+\frac{\alpha(m+\beta-1)}{m+\beta}T^{m+\beta-2}\right)\right.$$

$$\left.+iCD_0\left(\frac{mT^{m-1}}{m+1}-\frac{\alpha\beta(m+\beta)T^{m+\beta-1}}{(m+1)(m+\beta+1)}-\frac{R(m+1)T^m}{2(m+2)}\right)\right]=0 \qquad \ldots(7)$$

and

$$\frac{\partial^2 Z_1(T)}{\partial T^2}=\frac{1}{R}\left[\frac{2H}{T^3}+CD_0\left(\frac{(m-1)(m-2)T^{m-3}}{m}+\frac{\alpha(m+\beta-1)(m+\beta-2)T^{m+\beta-3}}{m+\beta}\right)\right.$$

$$\left.+iCD_0\left(\frac{m(m-1)T^{m-2}}{m+1}-\frac{\alpha\beta(m+\beta)(m+\beta-1)T^{m+\beta-2}}{(m+1)(m+\beta+1)}-\frac{R(m+1)mT^{m-1}}{2(m+2)}\right)\right]>0$$

Thus optimum value of T can be found from equation (7). Let it be 'T_1'. Then, the optimum value of order quantity $Q_1 = D_0\left(\frac{T_1^m}{m} + \frac{\alpha T_1^{m+\beta}}{m+\beta}\right)$ and minimum cost $Z_1(T_1)$ can be found from equation (6).

Case II : Credit only on the items in stock

In this case, payment is connected to the subsequent use of items. Here, there exist a credit period M. During this period, the customers make payment to the supplier immediately after the use of the items and the remaining balance is paid by the customer on the last day of the credit period. Here we have two cases depending on the value of 'T' and credit period 'M'.

Sub case I : If $T \le M$, then the present value of cash flows for the first cycle is

$$z_2(T) = H + C\int_0^T D_0 t^{m-1}.e^{-Rt}dt + CD(T)e^{-RM} + iC\int_0^T I(t)e^{-Rt}dt$$

$$= H + CD_0\int_0^T t^{m-1}(1-Rt)dt + \frac{CD_0\alpha}{m+\beta}T^{m+\beta}e^{-RM} + iCD_0\left(\frac{T^{m+1}}{m+1} - \frac{\alpha\beta T^{m+\beta+1}}{(m+1)(m+\beta+1)} - \frac{RT^{m+2}}{2(m+2)}\right)$$

$$= H + CD_0\left(\frac{T^m}{m} - \frac{RT^{m+1}}{m+1}\right) + \frac{CD_0\alpha}{m+\beta}T^{m+\beta}(1-RM) + iCD_0\left(\frac{T^{m+1}}{m+1} - \frac{\alpha\beta T^{m+\beta+1}}{(m+1)(m+\beta+1)} - \frac{RT^{m+2}}{2(m+2)}\right) \quad \dots(8)$$

Sub Case II : If T>M, then the present value of cash flows for the first cycle is

$$Z_2(T) = H + C\int_0^M D_0 t^{m-1}e^{-Rt}dt + C\left(Q - \int_0^M D_0 t^{m-1}dt\right)e^{-RM} + iC\int_0^T I(t)e^{-Rt}dt$$

$$= H + CD_0\left(\frac{M^m}{m} - \frac{RM^{m+1}}{m+1}\right) + CD_0\left(\frac{T^m}{m} + \frac{\alpha T^{m+\beta}}{m+\beta} - \frac{M^m}{m}\right)(1-RM)$$

$$+iCD_0\left(\frac{T^{m+1}}{m+1}-\frac{\alpha\beta T^{m+\beta+1}}{(m+1)(m+\beta+1)}-\frac{RT^{m+2}}{2(m+2)}\right)$$

$$=H+CD_0\frac{T^m}{m}(1-RM)+\frac{\alpha T^{m+\beta}}{m+\beta}+CD_0\frac{RM^{m+1}}{m(m+1)}$$

$$+iCD_0\left(\frac{T^{m+1}}{m+1}-\frac{\alpha\beta T^{m+\beta+1}}{(m+1)(m+\beta+1)}-\frac{RT^{m+2}}{2(m+2)}\right) \quad ..(9)$$

The present value of all future cash flows is

$$Z_2(T)=\frac{z_2(T)}{1-e^{-RT}}=\frac{z_2(T)}{RT}$$

$$\therefore\ Z_2(T)=\frac{1}{R}\left[\frac{H}{T}+CD_0\left(\frac{T^{m-1}}{m}-\frac{RT^m}{m+1}\right)+\frac{CD_0\alpha T^{m+\beta-1}}{m+\beta}(1-RM)\right.$$

$$\left.+iCD_0\left(\frac{T^m}{m+1}-\frac{\alpha\beta T^{m+\beta}}{(m+1)(m+\beta+1)}-\frac{RT^{m+1}}{2(m+2)}\right)\right] \quad \text{for } T\le M \qquad \ldots(10)$$

and

$$Z_2(T)=\frac{1}{R}\left[\frac{1}{T}\left(H+\frac{CD_0\,RM^{m+1}}{m(m+1)}\right)+CD_0\left(\frac{T^{m-1}}{m}(1-RM)+\frac{\alpha T^{m+\beta-1}}{m+\beta}\right)\right.$$

$$\left.+iCD_0\left(\frac{T^m}{m+1}-\frac{\alpha\beta T^{m+\beta}}{(m+1)(m+\beta+1)}-\frac{RT^{m+1}}{2(m+2)}\right)\right] \quad \text{for } T>M \qquad \ldots(11)$$

The necessary condition for $Z_2(T)$ to be minimum is $\frac{\partial Z_2(T)}{\partial T}=0$

$$\frac{\partial Z_2(T)}{\partial T}=\frac{1}{R}\left[\frac{-H}{T^2}+CD_0\left(\frac{(m-1)T^{m-2}}{m}-\frac{mRT^{m-1}}{m+1}\right)+\frac{CD_0\alpha(m+\beta-1)T^{m+\beta-2}}{m+\beta}(1-RM)\right.$$

$$+iCD_0\left(\frac{mT^{m-1}}{m+1}-\frac{\alpha\beta(m+\beta)T^{m+\beta-1}}{(m+1)(m+\beta+1)}-\frac{R(m+1)T^m}{2(m+2)}\right)\Bigg]=0 \text{ for } T\le M$$

…(12)

and

$$\frac{\partial Z_2(T)}{\partial T}=\frac{1}{R}\Bigg[\frac{-1}{T^2}\left(H+\frac{CD_0RM^{m+1}}{m(m+1)}\right)+CD_0\left(\frac{(m-1)T^{m-2}(1-RM)}{m}+\frac{\alpha(m+\beta-1)T^{m+\beta-2}}{m+\beta}\right)$$

$$+iCD_0\left(\frac{mT^{m-1}}{m+1}-\frac{\alpha\beta(m+\beta)T^{m+\beta-1}}{(m+1)(m+\beta+1)}-\frac{R(m+1)T^m}{2(m+2)}\right)\Bigg]=0 \text{ for } T>M$$

…(13)

$$\frac{\partial^2 Z_2(T)}{\partial T^2}=\frac{1}{R}\Bigg[\frac{2H}{T^3}+CD_0\left(\frac{(m-1)(m-2)T^{m-3}}{m}-\frac{m(m-1)RT^{m-2}}{m+1}\right)$$

$$+\frac{CD_0\alpha(m+\beta-1)(m+\beta-2)}{m+\beta}(1-RM)T^{m+\beta-3}$$

$$+iCD_0\left(\frac{m(m-1)T^{m-2}}{m+1}-\frac{\alpha\beta(m+\beta)(m+\beta-1)T^{m+\beta-2}}{(m+1)(m+\beta+1)}-\frac{Rm(m+1)T^{m-1}}{2(m+2)}\right)\Bigg]>0$$

$$\text{for } T\le M$$

and

$$\frac{\partial^2 Z_2(T)}{\partial T^2}=\frac{1}{R}\Bigg[\frac{2}{T^3}\left(H+\frac{CD_0RM^{m+1}}{m(m+1)}\right)+CD_0\left(\frac{(m-1)(m-2)T^{m-3}}{m}(1-RM)\right.$$

$$\left.+\frac{\alpha(m+\beta-1)(m+\beta-2)T^{m+\beta-3}}{m+\beta}\right)$$

$$+iCD_0\left(\frac{m(m-1)T^{m-2}}{m+1}-\frac{\alpha\beta(m+\beta)(m+\beta-1)T^{m+\beta-2}}{(m+1)(m+\beta+1)}-\frac{Rm(m+1)T^{m-1}}{2(m+2)}\right)\Bigg]>0$$

for T> M

and the optimum value of $T = T_2$ can be found from equation (12) and (13).

Hence if the payments to the supplier is done immediately after the use of materials and if the credit period (M) is longer than cycle length (T), then only out of pocket cost and the discounted cost of deterioration are relevant in finding the optimal cycle length. When $T \le M$, then there would be no opportunity cost in the expression of total cash flows because in this case, the firm finances the inventory investment with the trade credit offered by its supplier.

Case III: Fixed Credit Period : I n this case credit period is fixed and hence, the customer pays the full purchase amount on the last day of the credit period. The present value of cash-flows for one cycle, $z_3(T)$ is

$$z_3(T) = H + CQe^{-RM} + iC\int_0^T I(t)e^{-Rt}dt$$

$$= H + CD_0\left(\frac{T^m}{m}+\frac{\alpha T^{m+\beta}}{m+\beta}\right)e^{-RM} + iCD_0\left(\frac{T^{m+1}}{m+1}-\frac{\alpha\beta T^{m+\beta+1}}{(m+1)(m+\beta+1)}-\frac{RT^{m+2}}{2(m+2)}\right)$$

$$= H + CD_0\left(\frac{T^m}{m}+\frac{\alpha T^{m+\beta}}{m+\beta}\right)(1-RM) + iCD_0\left(\frac{T^{m+1}}{m+1}-\frac{\alpha\beta T^{m+\beta+1}}{(m+1)(m+\beta+1)}-\frac{RT^{m+2}}{2(m+2)}\right) \quad \ldots(14)$$

The present value of all future cash-flows is

$$z_3(T) = \frac{z_3(T)}{1-e^{-RT}} = \frac{z_3(T)}{1-(1-RT)} = \frac{1}{RT} z_3(T)$$

$$\left[\because e^{-RT} \approx 1 - RT \text{ approx.}\right]$$

$\therefore$ By equation (14)

$$Z_3(T) = \frac{1}{R}\left[\frac{H}{T} + CD_0\left(\frac{T^{m-1}}{m} + \frac{\alpha T^{m+\beta-1}}{(m+\beta)}\right)(1-RM) + iCD_0\left(\frac{T^m}{m+1} - \frac{\alpha\beta T^{m+\beta}}{(m+1)(m+\beta+1)} - \frac{RT^{m+1}}{2(m+2)}\right)\right] \quad \ldots(15)$$

For optimum value of T,

$$\frac{\partial Z_3(T)}{\partial T} = 0$$

$$\frac{\partial Z_3(T)}{\partial T} = \frac{1}{R}\left[\frac{-H}{T^2} + CD_0\left(\frac{(m-1)T^{m-2}}{m} + \frac{\alpha(m+\beta-1)T^{m+\beta-2}}{(m+\beta)}\right)(1-RM)\right.$$

$$\left. + iCD_0\left(\frac{mT^{m-1}}{m+1} - \frac{\alpha\beta(m+\beta)T^{m+\beta-1}}{(m+1)(m+\beta+1)} - \frac{R(m+1)T^m}{2(m+2)}\right)\right] = 0 \quad \ldots(16)$$

and

$$\frac{\partial^2 Z_3(T)}{\partial T^2} = \frac{1}{R}\left[\frac{2H}{T^3} + CD_0\left(\frac{(m-1)(m-2)T^{m-3}}{m} + \frac{\alpha(m+\beta-1)(m+\beta-2)T^{m+\beta-3}}{(m+\beta)}\right)(1-RM)\right.$$

$$\left. + iCD_0\left(\frac{m(m-1)T^{m-2}}{m+1} - \frac{\alpha\beta(m+\beta)(m+\beta-1)T^{m+\beta-2}}{(m+1)(m+\beta+1)} - \frac{Rm(m+1)T^{m-1}}{2(m+2)}\right)\right] > 0 \quad \ldots(17)$$

Hence the optimum value of $T = T_3$ can be obtained from equation (16) and the corresponding optimal order quantity is $Q = Q_3 = D_0\left(\frac{T_3^m}{m} + \frac{\alpha T_3^{m+\beta}}{m+\beta}\right)$.

The corresponding optimal present value of all future cash-flows $Z_3(T) = Z_3(T_3)$ is obtained from equation (15). Equation (16) contains trade credit, the correct opportunity cost and the cost of deterioration, which are the discounted cost of deterioration. This results that effective capital cost should be less than that of the instantaneous payments.

5. NUMERICAL EXAMPLES

Case I :

Example 1.

Let d = 50 units, m = 2, $\alpha = 0.001$, $\beta = 1$, i = 0.20, C = 20, H = 250,

T = 50 days = 0.1369 year, M = 100 days = 0.27399 years,

$$D_0 = dm\left(\frac{1}{T}\right)^m = 5335.72$$

Sensitivity Analysis on 'R'

R	$Z_1(T)$
0.15	61858.90
0.18	51547.86
0.21	44107.83
0.24	38593.34
0.27	34304.43
0.30	30873.30

i.e. there is decrease in present value of cash-flows with increase in ‘R’.

Sensitivity Analysis on ‘d’

If we take R = 0.15, and change the value of d, keeping all other parameters are same

d	D_0	$Z_1(T)$
30	3201.6	47785.97
40	4268.9	59651.16
50	5355.7	61858.90
60	6403.2	83376.82
70	7470.4	95240.42
80	8537.6	107104.02

Hence with increase in demand size, there is an increase in present cash flows.

Sensitivity Analysis on ‘C’

All other parameters are same as example 1, R = 0.15

C	$Z_1(T)$
15	49370.65
20	61858.90
25	74166.22
30	86564.00
35	98961.79
40	111359.57

With increase in C, $Z_1(T)$ increases.

Case II.

Example 2 :

Let d = 50 units, m = 2, $\alpha = 0.001$, $\beta = 1$, i = 0.20, C = 20, H = 250, T = 50 days ,M = 100 days, D_0 = 5335.72

Since $T \leq M$, this is sub case I.

Sensitivity Analysis on 'R'

R	$Z_2(T)$
0.15	61321.76
0.18	51061.99
0.21	43596.06
0.24	38061.79
0.27	33757.34
0.30	30313.93

i.e. with increase in R, present value of future cash flows decreases.

Sensitivity Analysis on 'd'

If all other parameters are same except d, R = 0.15,

d	D_0	$Z_2(T)$
30	3201.6	41554.67
40	4268.9	51348.41
50	5355.7	61321.76
60	6403.2	70932.04
70	7470.4	80724.49
80	8537.6	90516.95

Hence present value of future cash-flows increases with increase in value of d.

Sensitivity Analysis on 'C'

All other parameters are same as example 2, R = 0.15

C	$Z_2(T)$
15	49035.27
20	61321.76
25	73606.92
30	85892.84
35	97892.67
40	110464.68

With increase in 'C', present value of future cash-flows increases

If all other parameters are same as example 2, Let T = 50 days, M = 30 days i.e. T > M, then this is the subcase II.

Sensitivity Analysis on 'R'

R	$Z_2(T)$
0.15	61466.31
0.18	51207.69
0.21	43740.83
0.24	38205.79
0.27	33901.14
0.30	30457.38

With increase in R, present value of future cash flows decreases.

Sensitivity Analysis on 'd'

Let R = 0.15 and all other parameters are same as above and T>M.

d	D_0	$Z_2(T)$
30	3201.6	41641.85
40	4268.9	51464.65
50	5355.7	61466.31
60	6403.2	71106.39
70	7470.4	80927.90
80	8537.6	90749.42

i.e. present value of future cash-flows increases with increase in 'd'.

Sensitivity Analysis on 'C'

All other parameters are same as example R = 0.15

C	$Z_2(T)$
15	49006.05
20	61466.31
25	73558.21
30	85834.39
35	98110.57
40	110386.75

With increase in 'C', present value of future cash-flows increases.

Case III.

Example 3 :

Let d = 50 units, m = 2, $\alpha = 0.001$, $\beta = 1$, i = 0.20, C = 20, H = 250, T = 50 days, M = 75 days

Sensitivity Analysis on 'R'

R	$Z_3(T)$
0.15	60255.01
0.18	49961.87
0.21	42609.28
0.24	37094.20
0.27	32803.87
0.30	29374.04

With increase in R, present value of future cash flows decreases.

Sensitivity Analysis on 'd'

Let R = 0.15 and all other parameters are same as above.

d	D_0	$Z_3(T)$
30	3201.6	40926.52
40	4268.9	50510.84
50	5355.7	60255.01
60	6403.2	69675.73
70	7470.4	79258.80
80	8537.6	88841.87

With increase in demand size, present value of cash-flows increases.

Sensitivity Analysis on 'C'

All other parameters are same, R = 0.15

C	$Z_3(T)$
15	48246.62
20	60255.01
25	72292.98
30	84070.42
35	96339.26
40	108362.39

With increase in C, present value of future cash-flows increases.

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