**International Journal of Mathematics and Computer Applications Research (IJMCAR)**
**ISSN 2249-6955**
**Vol.2, Issue 1 Mar 2012 1-15**


# DETERIORATING INVENTORY MODEL WITH LINEAR DEMAND AND VARIABLE DETERIORATION TAKING INTO ACCOUNT THE TIME-VALUE OF MONEY

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## ABSTRACT

Most works on inventory models do not consider deterioration, time-value of money and price-dependent demand factor. A deteriorating inventory model taking into account the time-value of money is developed for linear demand. Shortages are completely backordered and variable rate of deterioration is taken. The objective of this study is to enable the retailer to develop a policy which will ensure the largest net profit. Numerical examples are also given to support the theoretical results.

## 1. INTRODUCTION

Ghare and Schrader (1963) were the first authors to analyse deteriorating inventory models. They developed an EOQ model by assuming a constant rate of deterioration, Covert and Philip (1973) have developed an EOQ model for items whose deterioration patterns follow the Weibull distribution. Misra (1979) simultaneously considered both the inflation and time-value of money for internal as well as external inflation rate and analyzed the influence of interest rate and inflation rate on replenishment strategy. Chandra and Bahner (1985) extended the result in Misra to allow for shortages. Abad (1988) considered pricing and lot-size decisions with incremental quantity discounts for a non-deteriorating product. Sarker and Pan (1994) assumed a finite replenishment model and studied the effects of inflation and time-value of money on order quantity when shortages are allowed. Hariga (1995) extended the study to analyze the effects of inflation and time-value of money on an inventory model with time-dependent demand rate and shortages. Wee (1997) developed a replenishment policy for items with a price-dependent demand and a varying rate of deterioration. Teng et al. (1997) investigated all possible policies with linearly increasing demand and mathematically identified the least expensive policy among them. Padmanbhan and Vrat (1995) developed an inventory model in which the backlogging rate depends upon the total number of customers in the waiting line. Ouyang et al. (2003) extended their model by incorporating the inflation and time-value of money for a finite planning horizon. Zhou (2003) used the same backlogging rate to develop a deterministic replenishment model with multiple warehouses. Deterioration starts as soon as the items are received into inventory. In this paper, we are going to develop an deteriorating inventory model with linear demand and variable deterioration taking into account the time value of money.

## 2. ASSUMPTIONS

(1) The rate of deterioration is variable i.e. $\theta(t) = \theta t, \quad 0 < \theta << 1$

(2) Deterioration occurs as soon as the items are received into inventory.

(3) Shortages are completely back-ordered.

(4) The system operates for a prescribed period of planning horizon.

(5) Demand rate is a decreasing linear function of the selling price.

(6) Replenishment rate is finite.

(7) There is no replacement or repair of deteriorating items under consideration.

(8) The replenishment rate is instantaneous, the order quantity and replenishment cycle is same for each period.

## 3. NOTATIONS

(1) T is replenishment cycle.

(2) H is planning horizon.

(3) N is number of replenishments during the planning horizon, N=H/T.

(4) ‘s’ is per unit selling price of the item.

(5) d(s) is demand rate, $d(s) = a - bs$, where $s < \frac{b}{a}$ and $a - bs > 0$ for positive demand.

(6) $I(t_1)$ is inventory level at any time $t_1$, $0 \le t_1 \le T_1$

(7) $I(t_2)$ is inventory level at any time $t_2$, $0 \le t_2 \le T - T_1$

(8) C is per unit cost of the item.

(9) r is interest rate.

(10) q is 2nd , 3rd ……….Nth replenishment lot size (units).

(11) $I_0$ is maximum inventory level.

(12) $C_1$ is cost per replenishment when $t = 0$

(13) $C_2$ is per unit holding cost per unit time.

(14) $C_3$ is per unit shortage cost per unit time.

(15) $T_1$ is time of positive inventory.

## 4. MODEL DEVELOPMENT

At $t = 0$, initial replenishment $I_0$ is made. During the period $T_1$, the inventory level decreases due to demand and deterioration till it becomes zero at $t = T_1$. During the time interval $t_2 = T - T_1$ shortages occur and are accumulated until at $t = T$ before they are backordered. The inventory system at any time t can be represented by the following differential equations:

$$\frac{dI(t_1)}{dt_1} + \theta t\, I(t_1) = -d(s);\ 0 \le t_1 \le T_1 \qquad \text{.....(1)}$$

$$\frac{dI(t_2)}{dt_2} = -d(s);\ 0 \le t_2 \le T - T_1 \qquad \text{.....(2)}$$

and the boundary conditions are

$$I(0) = I_0,\ \ I(T_1) = 0$$

Solution of equation (1) can be given as

$$\left[ I(t_1) e^{\frac{\theta t^2}{2}} \right]_0^{t_1} = -d(s) \int_0^{t_1} e^{\frac{\theta t^2}{2}} dt, \quad 0 \le t_1 \le T_1$$

$$\Rightarrow \ I(t_1) e^{\frac{\theta t_1^2}{2}} - I_0 = -d(s) \int_0^{t_1} e^{\frac{\theta t^2}{2}} dt$$

$$\Rightarrow I(t_1) = \frac{I_0 - d(s) \int_0^{t_1} e^{\frac{\theta t^2}{2}} dt}{e^{\frac{\theta t_1^2}{2}}}, \quad 0 \le t_1 \le T_1 \qquad \text{..... (3)}$$

Where $I_0 = d(s)\int_0^{T_1} e^{\frac{\theta t^2}{2}} dt = d(s)\int_0^{T_1} \sum_{n=0}^{\infty} \frac{\theta^n t^{2n}}{2^n \lfloor n} dt$

$$d(s)\sum_{n=0}^{\infty}\left[\frac{\theta^n t^{2n+1}}{2^n \lfloor n (2n+1)}\right]_0^{T_1} = d(s)\sum_{n=0}^{\infty} \frac{\theta^n T_1^{2n+1}}{2^n \lfloor n (2n+1)} \qquad \text{….. (4)}$$

Since $0 < \theta << 1$, $\therefore \; I_0 \approx d(s)\left[T_1 + \frac{\theta T_1^3}{6}\right]$ ….(5)

and solution of equation (2) is given by

$$I(t_2) = -d(s)t_2, \quad 0 \le t_2 \le T - T_1 \qquad \text{…..(6)}$$

The total cost in this model includes the replenishment cost, material cost, holding cost and shortage cost. The time-value of money with compounding interest rate is taken. The objective is to maximize the total profit.

## 4.1 Present-Value Sales Revenue

$$R = s\int_0^{T_1} d(s)e^{-rt_1}dt_1 + se^{-rT}\int_0^{T-T_1} d(s)dt_2$$

$$= s\,d(s)\left\{\left[\frac{e^{-rt_1}}{-r}\right]_0^{T_1} + e^{-rT}(T - T_1)\right\}$$

$$= s\,d(s)\left[\frac{1 - e^{-rT_1}}{r} + e^{-rT}(T - T_1)\right]$$

$$\approx s\,d(s)\left[\frac{1 - 1 + rT_1 - \frac{r^2 T_1^2}{2}}{r} + (1 - rT)(T - T_1)\right]$$

$$R \approx s\,d(s)\left[T - \frac{rT_1^2}{2} - rT^2 + rTT_1\right] \qquad \text{….. (7)}$$

## 4.2 Present-Value Ordering cost

Since replenishment in each cycle is done at the start of each cycle; the present value replenishment cost is $C_0 = C_1$. …..(8)

## 4.3 Present-Value Inventory Cost

Inventory occurs during period $T_1$.

$\therefore$ Present-Value Inventory cost during the period is

$$C_H = C_2 \int_0^{T_1} I(t_1) e^{-rt_1} dt_1$$

$$= C_2 \int_0^{T_1} \left\{ d(s) \frac{\int_0^{T_1} e^{\frac{\theta t^2}{2}} dt - \int_0^{t_1} e^{\frac{\theta t^2}{2}} dt}{e^{\frac{\theta t_1^2}{2}}} \right\} e^{-rt_1} dt_1$$

$$= C_2 \int_0^{T_1} \left\{ d(s) \left[ \sum_{n=0}^{\infty} \frac{\theta^n \left(T_1^{2n+1} - t_1^{2n+1}\right)}{2^n \lfloor n (2n+1)} \right] \left[ \sum_{n=0}^{\infty} \frac{(-rt_1)^n}{\lfloor n} \right] \left[ \sum_{n=0}^{\infty} \frac{\left(-\frac{\theta t_1^2}{2}\right)^n}{\lfloor n} \right] \right\} dt_1$$

$$\approx C_2 \int_0^{T_1} d(s) \left\{ \left(T_1 - t_1 + \frac{\theta}{6}\left(T_1^3 - t_1^3\right)\right)(1 - rt_1)\left(1 - \frac{\theta t_1^2}{2}\right) \right\} dt_1$$

$$\approx C_2 d(s) \int_0^{T_1} \left\{ \left(T_1 - t_1 + \frac{\theta}{6}\left(T_1^3 - t_1^3\right)\right) - rT_1 t_1 + rt_1^2 - \frac{\theta}{2} t_1^2 T_1 + \frac{\theta}{2} t_1^3 \right\} dt_1$$

$$\approx C_2 d(s)\left[\frac{T_1^2}{2}-\frac{rT_1^3}{6}+\frac{\theta T_1^4}{12}\right] \quad \text{.....(9)}$$

## 4.4 Present-Value Shortage Cost

All shortage during $(T-T_1)$ will be completely backordered at T, the present value shortage cost for the period is

$$C_s = C_3 \int_0^{T-T_1} \left[-I(t_2)\right] e^{-r(T_1+t_2)}\, dt_2$$

$$= C_3 \int_0^{T-T_1} d(s)\, t_2 e^{-rT_1} e^{-rt_2}\, dt_2$$

$$= C_3\, d(s) \int_0^{T-T_1} t_2 e^{-r(T_1+t_2)}\, dt_2$$

$$= C_3\, d(s)\left[\frac{t_2 e^{-r(T_1+t_2)}}{-r}-\frac{e^{-r(T_1+t_2)}}{r^2}\right]_0^{T-T_1}$$

$$= \frac{C_3\, d(s)}{r^2}\left[e^{-rT_1}+e^{-rT}\left(-rT+rT_1-1\right)\right]$$

$$\approx \frac{C_3\, d(s)}{r^2}\left[\left(1-rT_1+\frac{r^2T_1^2}{2}-\frac{r^3T_1^3}{6}\right)+\left(1-rT-\frac{r^2T^2}{2}-\frac{r^3T^3}{6}\right)\left(-rT+rT_1-1\right)\right]$$

$$\approx \frac{C_3\, d(s)}{6}\left[3T^2-6TT_1-2rT^3+3rT^2T_1+3T_1^2-rT_1^3\right] \quad \text{.....(10)}$$

## 4.5 Present-Value Item Cost

Replenishment is done at t = 0 and t = T the replenished items are consumed by demand as well as by deterioration during $T_1$.The present-value item cost, $C_p$ includes item cost and deterioration cost i.e.

$$C_p = CI_0 + Ce^{-rT}\int_0^{T-T_1} d(s)\,dt_2$$

$$= C\,d(s)\left[T_1 + \frac{\theta T_1^3}{6}\right] + C(1 - rT)\,d(s)(T - T_1)$$

[By equation (4)]

$$= C\,d(s)\left[T - rT^2 + rTT_1 + \frac{\theta T_1^3}{6}\right] \quad \text{.....(11)}$$

The first cycle present-value net profit is

$$P = R - C_0 - C_H - C_s - C_P$$

$$P = s\,d(s)\left[T - \frac{rT_1^2}{2} - rT^2 + rTT_1\right] - C_1 - C_2\,d(s)\left[\frac{T_1^2}{2} - \frac{rT_1^3}{6} + \frac{\theta T_1^4}{12}\right]$$

$$-\frac{C_3 d(s)}{6}\left[3T^2 - 6TT_1 - 2rT^3 + 3rT^2T_1 + 3T_1^2 - rT_1^3\right] - C\,d(s)\left[T - rT^2 + rTT_1 + \frac{\theta T_1^3}{6}\right]$$

.....(12)

There are N cycles during the planning horizon. Since inventory starts and ends at zero; an extra replenishment at t=H is required to satisfy backorders of last cycle in the planning horizon. Hence, total number of replenishment = N + 1 times the first replenishment lot size $= I_0$ and the $2^{nd}$, $3^{rd}$ and $N^{th}$ replenishment lost size

$$q = I_0 + \int_0^{T-T_1} d(s)\,dt_2 \qquad \text{.....(13)}$$

and (N+1)th replenishment lot size $= \int_0^{T-T_1} d(s)\,dt_2$ .....(14)

The time-value of money affects all the replenishment periods and hence must be considered separately. The total net present-value profit for the planning horizon is $P_1(s, T_1, N) = P\left(1 + e^{-rT} + e^{-2rT} + ............ + e^{-(N-1)rT}\right) - C_1 e^{-rH}$

$$= \sum_{n=0}^{N-1} P e^{-r\,nT} - C_1 e^{-rH}$$

$$= P\left(\frac{1 - e^{-rNT}}{1 - e^{-rT}}\right) - C_1 e^{-rH}, \text{ where } T = \frac{H}{N}$$

$$= P\left(\frac{1 - e^{-rH}}{1 - e^{-rT}}\right) - C_1 e^{-rH}$$

i.e.

$$P_1(s, T_1, N) = \left(\frac{1 - e^{-rH}}{1 - e^{-rT}}\right)\left\{sd(s)\left[T - \frac{rT_1^2}{2} - rT^2 + rTT_1\right] - C_1 - C_2 d(s)\left[\frac{T_1^2}{2} - \frac{rT_1^3}{6} + \frac{\theta T_1^4}{12}\right]\right.$$

$$\left. - \frac{C_3 d(s)}{6}\left[3T^2 - 6TT_1 - 2rT^3 + 3rT^2T_1 + 3T_1^2 - rT_1^3\right] - Cd(s)\left[T - rT^2 + rTT_1 + \frac{\theta T_1^3}{6}\right]\right\} - C_1 e^{-rH}$$

...(15)

Now $$\frac{\partial P_1}{\partial T_1} = \left(\frac{1 - e^{-rH}}{1 - e^{-rT}}\right)\left\{sd(s)\left[-rT_1 + rT\right] - C_2 d(s)\left[T_1 - \frac{rT_1^2}{2} + \frac{\theta T_1^3}{3}\right]\right.$$

$$\left. - \frac{C_3 d(s)}{6}\left[-6T + 3rT^2 + 6T_1 - 3rT_1^2\right] - Cd(s)\left[rT + \frac{\theta T_1^2}{2}\right]\right\} \qquad \text{.....(16)}$$

$$\frac{\partial P_1}{\partial s}=\left(\frac{1-e^{-rH}}{1-e^{-rT}}\right)\left\{\left(sd'(s)+d(s)\right)\left(T-\frac{rT_1^2}{2}-rT^2+rTT_1\right)-d'(s)\left[C_2\left(\frac{T_1^2}{2}-\frac{rT_1^3}{6}+\frac{\theta T_1^4}{12}\right)\right.\right.$$

$$\left.\left.+\frac{C_3}{6}\left(3T^2-6TT_1-2rT^3+3rT^2T_1+3T_1^2-rT_1^3\right)-C\left(T-rT^2+rTT_1+\frac{\theta T_1^3}{6}\right)\right]\right\}$$

.....(17)

For extreme conditions $\frac{\partial P_1}{\partial T_1}=0,\frac{\partial P_1}{\partial s}=0$

$\therefore$ By equation (16)

$$s\left(-rT_1+rT\right)-C_2\left(T_1-\frac{rT_1^2}{2}+\frac{\theta T_1^3}{3}\right)-\frac{C_3}{6}\left(-6T+3rT^2+6T_1-3rT_1^2\right)-C\left(rT+\frac{\theta T_1^2}{2}\right)=0$$

.....(18)

$$\frac{\partial^2 P_1}{\partial T_1^2}=\left(\frac{1-e^{-rH}}{1-e^{-rT}}\right)\left\{sd(s)(-r)-C_2d(s)\left(1-rT_1+\theta T_1^2\right)-\frac{C_3d(s)}{6}\left(6-6rT_1\right)-C\,d(s)\theta T_1\right\}$$

$$=\frac{\left[1-\left(1-rH\right)\right]}{\left[1-\left(1-rT\right)\right]}\left(-d(s)\right)\left\{rs+C_2\left(1-rT_1+\theta T_1^2\right)+C_3\left(1-rT_1\right)+C\,\theta T_1\right\}$$

$$=-\frac{Hd(s)}{T}\left\{rs+C_2\left(1-rT_1+\theta T_1^2\right)+C_3\left(1-rT_1\right)+C\theta T_1\right\}$$

$<0$

$[\because r<1]$ ....(19)

$$\frac{\partial^2 P_1}{\partial s^2} = \frac{H}{T}\left\{2d'(s)\left(T - \frac{rT_1^2}{2} - rT^2 + rTT_1\right)\right\}$$

$$\left[\begin{array}{l}\because d''(s) = 0 \\ d(s) = a - bs, b > 0 \\ d'(s) = -b\end{array}\right]$$

$$= -\frac{2bH}{T}\left(T - \frac{rT_1^2}{2} - rT^2 + rTT_1\right) < 0$$

$[\because r < 1]$ ……(20)

$$\frac{\partial^2 P_1}{\partial T_1 \partial s} = \frac{H}{T}\left\{(sd'(s) + d(s))(-rT_1 + rT) - C_2 d'(s)\left(T_1 - \frac{rT_1^2}{2} + \frac{\theta T_1^3}{3}\right)\right.$$

$$\left. - \frac{C_3 d'(s)}{6}\left(-6T + 3rT^2 + 6T_1 - 3rT_1^2\right) - Cd'(s)\left(rT + \frac{\theta T_1^2}{2}\right)\right\}$$

$$= \frac{H}{T}\left\{d'(s)\left[s(-rT_1 + rT) - C_2\left(T_1 - \frac{rT_1^2}{2} + \frac{\theta T_1^3}{3}\right) - C_3\left(-6T + 3rT^2 + 6T_1 - 3rT_1^2\right)\right.\right.$$

$$\left.\left. - C\left(rT + \frac{\theta T_1^2}{2}\right)\right] + d(s)(-rT_1 + rT)\right\}$$

$$= \frac{H}{T}\left\{d'(s).0 + d(s)r(T - T_1)\right\}$$

[By equation (18)]

$$= \frac{H}{T}d(s)r(T - T_1)$$ …..(21)

$$\therefore \left(\frac{\partial^2 P_1}{\partial T_1^2}\right)\left(\frac{\partial^2 P_1}{\partial s^2}\right) - \left(\frac{\partial^2 P_1}{\partial T_1 \partial s}\right)^2 > 0$$ …..(22)

Hence values of $T_1$ and s given by equations (16) & (17) will maximize the profit.

## 5. NUMERICAL EXAMPLE

Let the values of parameters of inventory model are:

C1 = 80/order $\theta$ = 0.05

C2 = 0.6/unit/year r = 0.08

C3 = 1.4/unit/year d(s) = 200 – 4s/unit/year

C = 5/unit/year H= 10 years

1. From table 1 ,we conclude that with increase in number of replenishments, the total net present value profit decreases.

2. If r = 0, i.e. when the time- value of money is not considered then keeping the other parameters same ,we observe from table 2, that the net present-value profit is higher than the case when the time-value of money is considered.

**Table 1**

| **No. of Cycles (N)** | **Sales Price (s)** | **Time Interval (year) (T1)** | **Time Interval (year) (T-T1)** | **Time Interval (year) (T)** | **Present Value Profit** |
|---|---|---|---|---|---|
| 7 | 23.577 | 1.017 | 0.412 | 1.429 | 12652.61 |
| 8 | 23.497 | 0.894 | 0.356 | 1.250 | 12643.66 |
| 9 | 23.447 | 0.799 | 0.312 | 1.111 | 12585.83 |

**Table 2**

| **No. of Cycles (N)** | **Sales Price (s)** | **Time Interval (year) (T1)** | **Time Interval (year) (T-T1)** | **Time Interval (year) (T)** | **Present Value Profit** |
|---|---|---|---|---|---|
| 7 | 23.577 | 1.017 | 0.412 | 1.429 | 18666.55 |
| 8 | 23.497 | 0.894 | 0.356 | 1.250 | 18663.76 |
| 9 | 23.447 | 0.799 | 0.312 | 1.111 | 18597.21 |

## 6. CONCLUSIONS

In this paper, an Inventory model with variable deterioration rate, taking into account the time-value of money is considered. This type of model is useful for all items which deteriorates with time but not at a constant rate. Emphasis is on profit-maximization. A numerical example is also given to illustrate the theory. We observe that total net present value profit is higher when time-value of money is not considered.

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