STUDY OF THE PERFORMANCE OF THE LINEAR AND NON-LINEAR NARROW BAND RECEIVERS FOR 2X2 MIMO SYSTEMS WITH STBC MULTIPLEXING AND ALAMOTI CODING

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ABSTRACT

A detailed analysis of the performance of 2X2 Multiple Input Multiple Output (MIMO) antenna system with STBC multiplexing and Alamouti coding and BPSK modulation for high data rate transmission has been carried out assuming flat fading Rayleigh channel. Combining different linear and non-linear detection techniques, Bit Error Rate (BER) performance have been studied in comparison to the optimally equalized 1X2 antenna Maximal Ratio Combining (MRC) techniques of receive diversity. The combination of linear techniques such as Zero Force (ZF) equalization methods and non-linear techniques such as Successive Interference Cancellation (SIC) shows that for BER ~10^{-3} there is an improvement of SNR by ~2.2 dB compared to ZF case. The combination of ZF, SIC and non-linear techniques such as optimal ordering showed further improvement in the SNR ~2dB. The combination of linear techniques such as Minimum Mean Square Equalization (MMSE) methods with SIC and optimal ordering indicates that for BER by 10^{-3} significant improvement in SNR ~ 5 dB over ZF case. A detailed comparison of different MIMO linear and non-linear detector systems with 1x2 transmit diversity system indicates that the maximum BER performance can be achieved in case of MIMO-Maximum likelihood (ML)
equalization detection where the SNR values closely match with that of transmit diversity system. The results are presented and discussed in the paper.

INTRODUCTION

Basic narrowband MIMO detectors

Narrowband MIMO detectors are used to reduce complexity in fast Rayleigh fading channels. MIMO techniques offer the promise of high spectral efficiency and robustness to fading. Key to their success is the MIMO detector at the receiver, whose job it is to recover the symbols that are transmitted simultaneously from multiple antennas. They are categorized into linear and non-linear receivers. Linear receivers are ZF receiver which implements matrix (pseudo)-inverse (ignores noise enhancement problems) and MMSE receiver optimizes the noise and offers a compromise between residual interference between input signals and noise enhancement [2]. Non-linear receivers are ML and SIC. ML is exhaustive optimum detection receiver uses Complexity exponential in QAM. This paper coverage ranges from simple linear detectors based on the zero-forcing and MMSE criteria to the optimal maximum-likelihood detector [1]. The successive-cancellation or decision-feedback detectors are described. The performance-complexity tradeoff for a variety of detection strategies are quantified.

MIMO detection methods are derived with an assumption that the noise power of all receiving paths is equal. This assumption is not valid in practical systems performing signal level to the dynamic range of analog to digital converters (ADC). If the independent scheme with the different gain for each receiving path to minimize the quantization noise is used, the output noise of is not white in the spatial domain, i.e. the noise variances of receiving paths are not identical. Therefore, this spatially colored noise effects should be taken into consideration for the optimal detection [4].
This briefly proposes a new signal detection method called optimized ordered successive interference cancellation (OSIC) with signals the method for spatially multiplexed MIMO systems. By using the OSIC algorithm and the ML metric, the proposed method achieves near-ML performance without requiring a large number of signals.

**IMPLEMENTATION**

A MIMO detector for detecting receive symbols, which correspond to symbols transmitted through transmit antennas from receive signals, when the transmit data transmitted by the terminal group are received through receive antennas.

A terminal identifier for identifying the receive symbols detected by the MIMO detector as symbols which correspond to respective terminals in the terminal group; a symbol de-mapper for de-mapping the receive symbols identified by the terminal identifier to binary data which correspond to a modulation method used by the terminal Group and a reverse data processor for performing de-interleaving, decoding of error correction codes, and descrambling on the binary data de-mapped by the symbol de-mapper, and detecting receive data of the respective terminals. The symbol de-mapper and the reverse data processor are provided as the same number as that of the terminals in the terminal group. The terminal of the terminal group comprises: a data processor for performing scrambling, error correction encoding, and interleaving on the transmit data, and processing them as binary data; a symbol mapper for mapping the binary data transmitted by the data processor according to a desired modulation method; and a parallel converter for paralleling the symbols mapped by the symbol mapper to the respective antennas, and providing them by consideration of the number of the transmit antennas.
A spatial multiplexing detection method using a MIMO technology comprises:

The purpose of solving the issues like what error rate can be tolerated, what is the ultimate measure of performance (e.g., frame-error-rate, worst-case complexity, or average complexity), and what computational platform is used. Additionally, the bits may be part of a larger code word and different vectors in that code word may either see the same H (slow fading) or many different realizations of H (fast fading). This complicates the picture, because notions that are important in slow fading (such as spatial diversity) are less important in fast fading, where diversity is provided anyway by time variations [8].

The multiple input multiple output code division multiple access systems can get good performance or high capacity, but the computational complexity of detection is usually high. When the SNR is high enough, it simply outputs the zero forcing detection results, which leads to a faster detecting process.

### MIMO Receiver Design

The Fig. 1 shows the probable MIMO transmission schemes with 2 transmit antennas and 2 receive antennas.

![Fig. 1 2 Transmit 2 Receive (2×2) MIMO channel](image)
1. MIMO with ZF SIC equalizer

An attempt is made to improve the bit error rate performance by trying out Successive Interference Cancellation (SIC). We will assume that the channel is a flat fading Rayleigh multipath channel and the modulation is BPSK. To do the SIC, the receiver needs to perform the following:

Using the ZF equalization approach described above, the receiver can obtain an estimate of the two transmitted symbols $x_1$, $x_2$, i.e.

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = (HH^H)^{-1}H^Hy.$$ 

Take one of the estimated symbols (for example $\hat{x}_2$) and subtract its effect from the received vector $y_1$ and $y_2$, i.e.

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{1,2} \hat{x}_2 \\ y_2 - h_{2,2} \hat{x}_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} x_1 + n_1 \\ h_{2,1} x_1 + n_2 \end{bmatrix}$$

Expressing in matrix notation,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & n_1 \\ h_{2,1} & n_2 \end{bmatrix} \begin{bmatrix} x_1 \\ n_1 \end{bmatrix}$$

$$r = h x_1 + n$$

The equalized symbol is,

$$\hat{x}_1 = \frac{r_h}{r_h^H r_h} r.$$ 

This forms the explanation for ZF Equalizer with Successive Interference Cancellation (ZF-SIC) approach.
The BER performance for different SNR values for a 1X2 Transmit diversity scheme and combined ZF and SIC detectors for a MIMO system is shown in figure 2. It can be seen from the figure the BER values decreases with the SNR in all the three cases. The figure indicates that for BER of $10^{-3}$, the transmit diversity system shows the minimum SNR ~ 11 dB. For the same BER performance the MIMO -SIC detectors indicate SNR ~25 dB and MIMO- ZF detectors SNR ~ 22.5 dB. The figure indicate that there is a marked difference in SNR > 11 dB between the transmit diversity and MIMO receiver diversity. The SIC detectors indicate SNR improvement ~2.2 dB over that of ZF equalization detectors.

MIMO with ZF-SIC and optimal ordering

In the variant of ZF-SIC optimal ordering, assume that the channel is a flat fading Rayleigh multipath channel and the modulation is BPSK. Using the ZF equalization, the receiver can obtain an estimate of the two transmitted symbols $x_1$, $x_2$, i.e.
\[
\begin{bmatrix}
\mathbf{y}_1 \\
\mathbf{y}_2
\end{bmatrix} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \begin{bmatrix}
\mathbf{y}_1 \\
\mathbf{y}_2
\end{bmatrix}.
\]

**Successive Interference Cancellation with optimal ordering**

In classical SIC, the receiver arbitrarily takes one of the estimated symbols, and subtract it effect from the received symbol \(y_1\) and \(y_2\). However, we can have more intelligence in choosing whether we should subtract the effect of \(\mathbf{y}_1\) first or \(\mathbf{y}_2\) first. To make that decision, let us find out the transmit symbol (after multiplication with the channel) which came at higher power at the receiver. The received power at the both the antennas corresponding to the transmitted symbol \(x_1\) is,

\[P_{x_1} = |h_{1,1}|^2 + |h_{2,1}|^2\]

The received power at the both the antennas corresponding to the transmitted symbol \(x_2\) is, \[P_{x_2} = |h_{1,2}|^2 + |h_{2,2}|^2\].

If \(P_{x_1} > P_{x_2}\), then the receiver decides to remove the effect of \(\mathbf{y}_1\) from the received vector \(y_1\) and \(y_2\) and then re-estimate \(\mathbf{y}_2\).

\[
\begin{align*}
\begin{bmatrix}
\mathbf{r}_1 \\
\mathbf{r}_2
\end{bmatrix} &= \begin{bmatrix}
\mathbf{y}_1 - h_{1,2} x_1^* \\
\mathbf{y}_2 - h_{1,2} x_1^*
\end{bmatrix} = \begin{bmatrix}
h_{1,2} x_1 + n_1 \\
h_{2,2} x_1 + n_2
\end{bmatrix}
\end{align*}
\]

Expressing in matrix notation,

\[
\begin{align*}
\begin{bmatrix}
\mathbf{r}_1 \\
\mathbf{r}_2
\end{bmatrix} &= \begin{bmatrix}
\mathbf{y}_1 - h_{1,2} x_2^* \\
\mathbf{y}_2 - h_{2,2} x_2^*
\end{bmatrix} = \begin{bmatrix}
h_{1,1} x_1 + n_1 \\
h_{2,1} x_1 + n_2
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
\mathbf{r}_1 \\
\mathbf{r}_2
\end{bmatrix} &= \begin{bmatrix}
h_{1,1} \\
h_{2,1}
\end{bmatrix} \begin{bmatrix}
x_1 \\
n_1
\end{bmatrix} + \begin{bmatrix}
n_2
\end{bmatrix}
\end{align*}
\]
$$r = h x_2 + n$$

Optimal way of combining the information from multiple copies of the received symbols in receive diversity case is to apply MRC. The equalized symbol is,

$$\mathbf{R}_2 = \frac{\mathbf{H}^H \mathbf{y}_2}{\mathbf{H}^H \mathbf{H}}.$$

Else if $\mathbf{P}x_1 <= \mathbf{P}x_2$ the receiver decides to subtract effect of $\mathbf{R}_2$ from the received vector $y_1$ and $y_2$, and then re-estimate $\mathbf{R}_1$:

$$\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} y_1 - h_{1,2} x_2^\wedge \\ y_2 - h_{1,2} x_2^\wedge \end{pmatrix} = \begin{pmatrix} h_{1,1} x_1 + n_1 \\ h_{2,1} x_1 + n_2 \end{pmatrix}$$

Expressing in matrix notation,

$$\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} h_{1,1} & x_1 + n_1 \\ h_{2,1} & n_2 \end{pmatrix}$$

$$r = h x_1 + n$$

Optimal way of combining the information from multiple copies of the received symbols in receive diversity case is to apply MRC. The equalized symbol is,

$$\mathbf{R}_1 = \frac{\mathbf{H}^H \mathbf{y}_1}{\mathbf{H}^H \mathbf{H}}.$$

Doing SIC with optimal ordering ensures that the reliability of the symbol which is decoded first is guaranteed to have a lower error probability than the other symbol. This results in lowering the chances of incorrect decisions resulting in erroneous interference cancellation. Hence gives lower error rate than simple successive interference cancellation.
The simulation is done using MATLAB script which performs the following: Generate random binary sequence of +1’s and -1’s, grouping them into pair of two symbols and send two symbols in one time slot, multiply the symbols with the channel and then add white Gaussian noise, equalize the received symbols with ZF criterion, finding the power of received symbol from both the spatial dimensions, taking the symbol having higher power, subtract from the received symbol, performing MRC for equalizing the new received symbol and hard decision decoding and count the bit errors and lastly by repeating for multiple values of $E_b/N_0$ and plot the simulation and theoretical results.

![BER plot for BPSK in 2x2 MIMO equalized by ZF-SIC with optimal ordering](image)

Fig. 3: BER plot for BPSK in 2x2 MIMO equalized by ZF-SIC with optimal ordering

The MIMO-ZF equalization with SIC detection is compared with optimal ordering and shown in figure 3 along with 1X2 transmit diversity scheme. It can be seen from the figure the performance of BER decreases with the SNR in all the cases. The figure indicates that for BER of $10^{-3}$ the transmit diversity SNR ~11 dB. For the same BER values the figure shows different SNR for the two cases. The SNR for optimal ordering is ~20 dB and for ZF-SIC the SNR ~24dB.
The figure indicates that there is a marked difference in SNR > 9 dB between the transmit diversity and MIMO receiver diversity. The figure also suggests that between ZF-SIC and optimal ordering there is an improvement ~4 dB performance at BER of 10^{-3}.

**Minimum Mean Square Error (MMSE) equalizer for 2x2 MIMO channel**

The MMSE approach tries to find a coefficient W which minimizes the criterion,

\[
B \{ [W_y - \alpha W_y]^{HF} \}.
\]

Solving,

\[
W = [H^H H + N_0 I]^{-1} H^H.
\]

When comparing to the equation in ZF equalizer, apart from the \(N_0 I\) term both the equations are comparable. When the noise term is zero, the MMSE equalizer reduces to ZF equalizer.

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**Fig. 4** BER plot for 2x2 MIMO with MMSE equalization for BPSK in Rayleigh channel.

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The simulated results for the MIMO - ZF equalizer and MMSE detectors are shown in figure 4 along with 1X2 transmit diversity scheme. The figure indicates that at BER of $10^{-3}$ the transmit diversity SNR ~11 dB. The figure shows that at $10^{-3}$ BER, the MMSE equalizer the SNR ~22 dB and for ZF equalizer the SNR ~ 25 dB. The graphs indicate that there is a marked difference in SNR >11 dB between the transmit diversity and MIMO receiver diversity. The figure clearly indicates ~ 4dB improvement between the ZF and MMSE detectors.

**MIMO with MMSE SIC and optimal ordering**

It is known that SIC with optimal ordering improves the performance with ZF equalization [3]. The concept of SIC is extended to the MMSE equalization extended and simulate the performance.

The MMSE approach tries to find a coefficient $W$ which minimizes the criterion,

$$
\mathcal{E} \left[ |W y_2 - x_1|^2 |W y_2 - x_2|^2 \right].
$$

Solving, $W = \left[ \mathbf{H}^H \mathbf{H} + \mathbf{N}_0 \mathbf{I} \right]^{-1} \mathbf{H}^H \mathbf{y}.

Using the MMSE equalization, the receiver can obtain an estimate of the two transmitted symbols $x_1, x_2$ i.e.,

$$
\begin{bmatrix}
\hat{x}_1 \\
\hat{x}_2
\end{bmatrix} = \left( \mathbf{H}^H \mathbf{H} + \mathbf{N}_0 \mathbf{I} \right)^{-1} \mathbf{H}^H \mathbf{y}.
$$

SIC with optimal ordering is however can have more intelligence in choosing whether we should subtract the effect of $\mathbf{y}_1$ first or $\mathbf{y}_2$ first. To make that decision, find out the transmit symbol which came at higher power at the receiver. The received power at the both the antennas corresponding to the transmitted symbol $x_1$, is, $P_{x_1} = |s_1|^2 + |s_2|^2$. The received power at the both the antenna corresponding to the transmitted symbol $x_2$
is $P_{\text{SNR}} = |\mathbf{v}_1, \mathbf{v}_2|^2 + |\mathbf{v}_3, \mathbf{v}_4|^2$. If $P_{\text{SNR}} > P_{\text{SNR}}$, then the receiver decides to remove the effect of $\mathbf{v}_1$ from the received vector $\mathbf{v}_1$ and $\mathbf{v}_2$, $\mathbf{v}_3$. Else if $P_{\text{SNR}} < P_{\text{SNR}}$, the receiver decides to subtract effect of $\mathbf{v}_2$ from the received vector $\mathbf{v}_1$ and $\mathbf{v}_3$, and then re-estimate $\mathbf{v}_1$. Once the effect of either $\mathbf{v}_1$ or $\mathbf{v}_2$ is removed, the new channel becomes a 1X2 receive antenna case and the symbol on the other spatial dimension can be optimally equalized by MRC.

![BER plot for 2×2 MIMO channel with MMSE-SIC equalization with and without optimal ordering](image)

**Fig. 5: BER plot for 2×2 MIMO channel with MMSE-SIC equalization with and without optimal ordering**

**Observations**

The figure 5 shows the BER performance between MMSE equalization with simple SIC case and SIC with optimal ordering along with the 1X2 transmit diversity scheme. The figure indicates that at BER of $10^{-3}$ the transmit diversity SNR ~11 dB. For the same BER, the MMSE equalizer shows SNR ~19 dB for SIC case ~13 dB. The graphs indicate that there is only a small difference in SNR >2dB between the transmit diversity and MIMO receiver diversity. Between the MMSE-SIC and optimal ordering there is a significant
improvement in the SNR ~ 6dB. The performance of the MIMO detector system closely matching with that of the 1X2 transmit diversity scheme.

**MIMO with ML equalization**

From the above receiver structures, we saw that MMSE equalization with optimally ordered SIC gave the best performance. Here we are discussing another receiver structure called **ML decoding** which gives us an even better performance [6]. We will assume that the channel is a flat fading Rayleigh multipath channel and the modulation is BPSK.

**Maximum Likelihood (ML) Receiver**

The ML receiver tries to find \( \mathbf{x} \) which minimizes,

\[
J = \left| \mathbf{y} - \mathbf{H} \mathbf{x} \right|^2
\]

\[
\begin{pmatrix}
  y_1 \\
  y_2
\end{pmatrix}
= \begin{pmatrix}
  h_{1,1} & h_{1,2} \\
  h_{2,1} & h_{2,2}
\end{pmatrix}
\begin{pmatrix}
  x_1^\wedge \\
  x_2^\wedge
\end{pmatrix}
\]

Since the modulation is BPSK, the possible values of \( x_1 \) is +1 or -1. Similarly \( x_2 \) also take values +1 or -1. So, to find the Maximum Likelihood solution, we need to find the minimum from the all four combinations of \( x_1 \) and \( x_2 \). The estimate of the transmit symbol is chosen based on the minimum value from the above four values i.e if the minimum is \( J + 1, + 1 \Rightarrow [1, 1] \), if the minimum is \( J + 1, - 1 \Rightarrow [1, 0] \), if the minimum is \( J - 1, + 1 \Rightarrow [0, 1] \) and if the minimum is \( J - 1, - 1 \Rightarrow [0, 0] \).

The simulation mainly includes finding the minimum among the four possible transmit symbol combinations, based on the minimum chose the estimate of the transmit symbol and repeat for multiple values of \( E_b/N_0 \) and plot the simulation and theoretical results.
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Fig. 6: BER plot 2×2 MIMO Rayleigh channel with ML equalization

The figure 6 shows the BER performance for different SNR for MIMO-ML equalization detection and 1x2 transmit diversity system. The BER decreases with increase with SNR and indicates that at BER ~$10^{-3}$, the SNR values ~11 dB for both the cases and shows that the MIMO detection with ML equalization shows same performance as those of 1x2 transmit diversity system.

Fig. 7 Comparison of BER plot for 2 transmit 2 receive MIMO channel for BPSK modulation for different linear and non-linear detectors
A detailed comparison of the BER performance with SNR for all the MIMO linear and non-linear detector system is shown in figure 7. The figure clearly demonstrates that when compared to 2X1 transmit diversity system which shows the SNR ~11dB at BER $10^{-3}$, the different types of detection techniques show that SNR values decreases from ZF equalizer to ML equalizer techniques. Further the results demonstrate that the highest BER values can be achieved for lowest SNR values using ML equalization detectors.

CONCLUSIONS

From the results and discussions presented above we conclude the following,

1. The combination of linear and non-linear detection techniques can improve the BER performance of MIMO systems.

2. When compared to various MIMO linear and non-linear detection techniques, the maximum BER performance very close to 1x2 transmit diversity can be achieved for MIMO-Maximum Likelihood detection (ML) scheme with receiver diversity.

REFERENCES

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