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NON LINEAR SYSTEM IDENTIFICATION USING ROBUST COST FUNCTIONS AND GENETIC ALGORITHM

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ABSTRACT

The paper demonstrates the use of alternative cost functions for identification of non linear systems which are not affected by outliers. Conventionally error based cost function is used for identification which does not provide satisfactory weight update when the training signal is corrupted with random outliers thus leading to incorrect estimation of parameters of system. Using these alternative cost functions identification performance is satisfactory even with outliers.

Keywords: Direct Modeling, robust cost function, Genetic Algorithm

1. INTRODUCTION

For identification of an unknown plant zero mean input is applied to adaptive system as well as the plant, with the objective to adjust weights of adaptive filter causing its output to match that of plant such that the adaptive system becomes a model of system. Practically, most of the plants which needs to be modeled is non linear in nature. Identification of an unknown system becomes a difficult task with the presence of outliers. In past a lot of research activities have been done towards presenting accurate model of estimation of system parameters when random outliers are present in the training samples. In identifying the parameters of nonlinear systems a lot of work has been done earlier using conventional algorithms like least mean square (LMS) and recursive least square (RLS) [1] as well as population based bio inspired evolutionary computing algorithms like (GA) [2], bacterial foraging optimization (BFO) [3], particle swarm optimization (PSO) [4] and differential evolution (DE) [5] as optimization tools using mean square error (MSE) as cost function. With outliers in the training sample and with MSE as the cost function the convergence performance is not satisfactory either with derivative based algorithms or derivative free bio inspired algorithms. Therefore in such conditions when the system to be identified is non linear along with random presence of strong outliers there is a need of articulating a better model for identification. It has been seen from some earlier work done in recent past that in such cases the minimizations of robust cost functions (RCF) provides improved convergence performance and these alternative cost functions are not effected by outliers. Recently Hsieh, Lin and Jeng,2008 Wang, Lee, Liu and Wang,1997 and Tsai and Yu,2000 have successfully used these alternative cost functions which are robust to outliers to propose new learning rules for approximating non linear functions. In the present work we have applied these alternative cost functions for identification of non linear systems by using derivative free Genetic Algorithm (GA). The result of identification model developed is compared with results obtained from conventionally used mean square error function. The weight updating of the model is done through derivative free genetic algorithm (GA). The present work is organized into five sections. In section II development of adaptive model for system identification using alternative cost functions is presented. Section III outlines the GA based identification model. Section IV discusses about the results obtained from the simulation study and Section V lists out the conclusion of present work.

2. DEVELOPMENT OF ADAPTIVE MODEL FOR IDENTIFICATION USING ALTERNATIVE COST FUNCTIONS

Basic model for nonlinear system identification where FIR filter is used to model linear plant is listed out in Fig. 1. The output of plant at any k^{th} instant is given as

$$y(k) = \sum_{i=0}^{N-1} w(i)x(k-i) \tag{1}$$

where, $w(i)$ are the plant coefficients, N is the length of the FIR system and $x(k)$ are the input samples . Non linear distortion is provided by the NL block which is listed in Section IV and its output is expressed according to eqn (2).

$$z(k) = \Omega(y(k)) \tag{2}$$

where $\Omega(.)$ is the nonlinear function generated by the "NL" block. Uniformly generated white signal is given as input to both the system as well as model. The system output $z(k)$ is added with additive white Gaussian noise $n(k)$. The effect of outlier is studied by randomly choosing 10% to 50% of the total training samples and changing its amplitude to ±5. The desired signal $d(k)$ is compared with the estimated output of the model, thereby producing error vector. The objective of the identification model is to minimize this error recursively, iteration after iteration such that $\hat{y}(k)$ approaches the desired plant output when same input $x(k)$ is applied to both the plant and the model.

The error vector is then used to calculate the alternative cost functions described by equation (5),(7) and (8), using GA based algorithm and the results are compared with conventional MSE using GA.

Three alternative cost functions which are robust to outliers are defined in literature by Hsieh, Lin and Jeng, 2008; Wang, Lee, Liu and Wang, 1997; Tsai and Yu, 2000. These robust cost functions (RCFs) are used in the development of robust adaptive identification model. The derivative free population based GA algorithm is then used to minimize these alternative norms of the error obtained from the model iteratively.

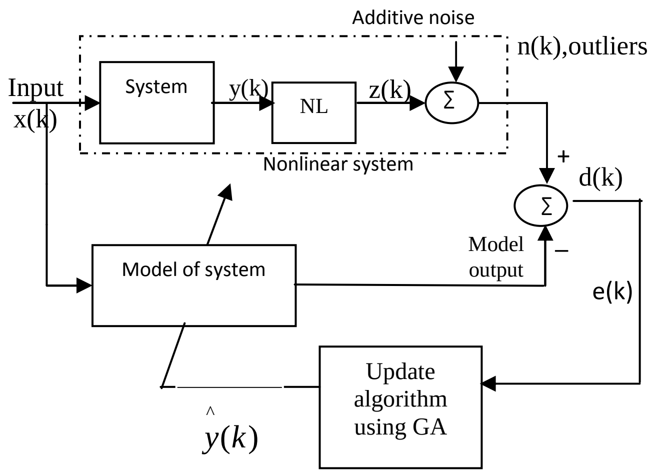

Fig 1: The adaptive model for identification with outliers

Alternative cost functions which are robust against outliers are defined mathematically as follows:

(a) Wilcoxon Norm : (McKean, 2004; Hsieh, Lin and Jeng, 2008) (RCF1)

A score function in the range 0 to 1 should satisfy following condition

$$\int_0^1 \Omega^2(u)du < \infty \tag{3}$$

It has the characteristics

$$\int_0^1 \Omega(u)du = 0 \text{ and } \int_0^1 \Omega^2(u)du = 1 \tag{4}$$

The score associated with this score function is given by

$$a_\Omega(i) = \Omega\left(\frac{i}{m+1}\right), \quad i \in m \tag{5}$$

where m is a fixed positive integer.

Looking at equation (4) it can be observed that $a_\Omega(1) \le a_\Omega(2) \le .... \le a_\Omega(m)$. The Wilcoxon norm (McKean, 2004; Hsieh, Lin and Jeng, 2008) is a pseudo-norm on $\Re^{'}$ and is defined as

$$C_1 = \sum_{i=1}^{l} a(R(v_i))v_i = \sum_{i=1}^{l} a(i)v_i, \quad v = [v_1, v_2, ..., v_l]^T \in \Re^{'} \tag{6}$$

where $R(v_i)$ denotes the rank of v_i among $v_1, v_2, ..., v_l$ and $v_{(1)} \le v_{(2)} \le ....v_{(m)}$ are the ordered values of $v_1, v_2, ..., v_m$. The score is given by $a(i) = \Omega[i/(m+1)]$. In statistics, different types of score functions have been dealt but the commonly used one is given by $\Omega(u) = \sqrt{12}(u - 0.5)$.

(b) Robust Cost Function-2 (Tsai and Yu, 2000) (RCF2) It is defined as

$$C_2 = \sigma(1 - \exp(-e^2 / 2\sigma)) \tag{7}$$

where e^2 = mean square error and σ is a constant which is adjusted during training phase.

(c) Robust Cost Function – 3 (Wang, Lee, Liu and Wang, 1997) (RCF3)

The third cost function is defined as

$$C_3 = \log(1 + \frac{e^2}{2}) \tag{8}$$

where e^2 is mean square error.

In the model presented in Fig 1 the weights of the model is updated iteration after iteration by using GA by minimizing the alternative cost functions of the errors defined in (5), (7) and (8). GA is used as the update algorithm.

3. GA BASED NONLINEAR SYSTEM IDENTIFICATION

In the present investigation we have used genetic algorithm (GA) for updating the weights of the model with the objective of minimizing the squared error. Then the performance is compared by taking alternative cost functions. In this section steps pertaining to identification of non linear system using GA is outlined.

Step 1: Initialization of Parameter for Genetic algorithm

N= Initial population, $\mathbf{P_m}$= Mutation probability,

S= Selection rate.

Step 2: ***n*** uniformly distributed random signals which are applied to the actual nonlinear system and to the adaptive model simultaneously over the interval [-1, 1] are generated. In the present investigation we have taken three standard 3 tap FIR models as plant, defined in section IV.

Step 3: Plant's output is corrupted with random outliers, which gives desired output. On the other hand the estimated output is obtained by applying same input to adaptive model.

Step 4: ***n*** number of errors are produced by comparing each samples of the desired output with the corresponding estimated output.

Step 5: The mean square error (MSE) for each i^{th} weight vector is calculated by using the following fitness function:

$$MSE(i) = \sum_{i=1}^{n} e^2(n,i)/n \qquad (9)$$

Step 6: Random population of ***n*** chromosomes which are assumed to be suitable solutions are then generated and fitness of each chromosome in the population is evaluated.

Step 8: To perform the crossover operation two parent chromosomes are randomly selected from the initial population according to their fitness (the better fitness, the bigger chance to be selected).

Step 9: With a crossover probability, the parents are crossed to form new offspring (children). In case where no crossover is performed, offspring is the exact replica of the parents.

Step 10: With a mutation probability new offspring at each position in chromosome are mutated and new offspring are then placed in the new population. The crossover and the mutation operators prevent the model to be trapped in local minima.

Step 11: To quantify the learning behavior of the adaptive model after every iteration minimum of MSE (MMSE) is calculated.

Step 12: The learning process continues until the cost function decreases to the possible minimum values, which is then assumed to be the end of the training. The resulting weight vector represents the final weights of the identification model.

In the present approach, the error is produced in same manner as in the case of normal identification problem i.e the steps 1 to 4 are common with normal identification using MSE as cost function. The only exception in training is as follows:

Assuming that the error vector of i th bacterium at j th generation due to application of n input samples to the model be represented as follows:-

$$[e_{1,i}(j), e_{2,i}(j), \ldots\ldots\ldots\ldots, e_{n,i}(j)]^T . \qquad (10)$$

The obtained errors are then arranged in an increasing manner out of which the rank $R\{e_{n,i}(j)\}$ of each n^{th} error term is found out. The score associated with each rank of the error term is evaluated as

$$a(l) = \sqrt{12}\left(\frac{l}{N+1} - 0.5\right) \qquad (11)$$

where $a(l)$ $(1 \le l \le n)$ denotes the rank associated with each error term. At j^{th} generation of each i^{th} bacterium the Wilcoxon norm is then calculated as

$$C_i(j) = \sum_{l=1}^{n} a(l)\, e_{l,i}(j) \qquad (12)$$

Similarly, the other two RCFs described in using (7) and (8) are computed from the error vector obtained in equation (10). The learning phase continues till the robust cost functions decreases to minimum values which is treated as end to the training phase.

4. SIMULATION STUDY

To evaluate the present model of identification rigorous simulation study is carried out by taking three linear systems which are described as follows:

$$System1 : H_1(z) = 0.209 + 0.995z^{-1} + 0.209z^{-2}$$
$$System2 : H_2(z) = 0.304 + 0.903z^{-1} + 0.304z^{-2}$$
$$System3 : H_3(z) = 0.341 + 0.876z^{-1} + 0.341z^{-2}$$

Three nonlinearities described below are then added to these linear systems.

NL1 $y_n(k) = \tanh(y(k))$

NL2 $y_n(k) = y(k) + 0.2y^2(k) - 0.1y^3(k)$

NL3 $y_n(k) = y(k) + 0.2y^3(k)$

where $y(k)$ is the output of the linear system and $y_n(k)$ is the overall output of the system after adding nonlinearities, noise and outliers.

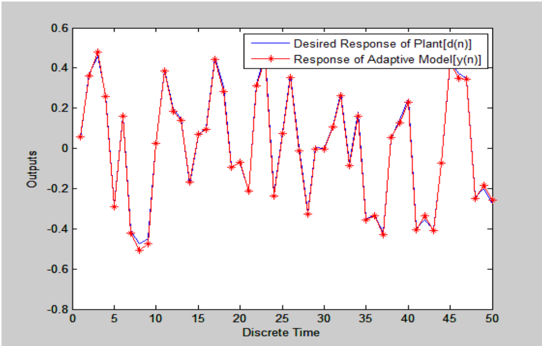

Fig 2(a) Comparison of responses of example-1 using NL1 with 50% outliers within the range -5 to +5 using RCF1

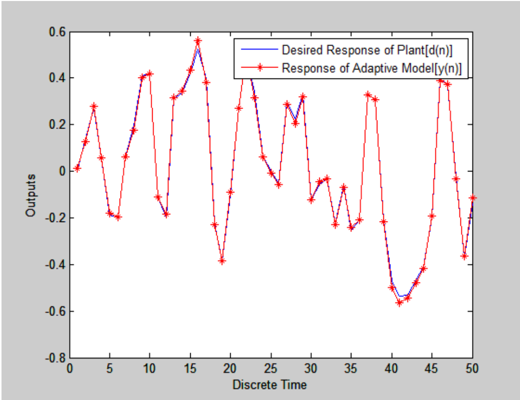

Fig 2(b) Comparison of responses of example-2 using NL1 with 50% outliers within the range -5 to +5 using RCF1

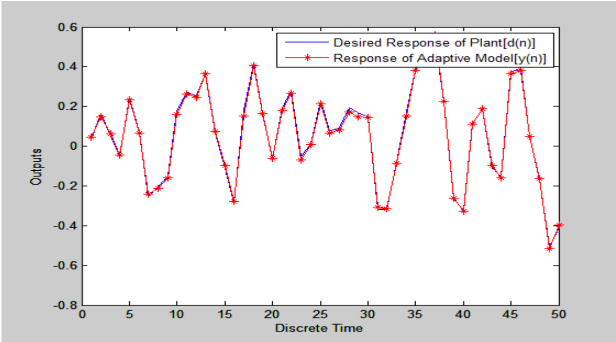

Fig 2(c) Comparison of responses of example-3 using NL1 with 50% outliers within the range -5 to +5 using RCF1

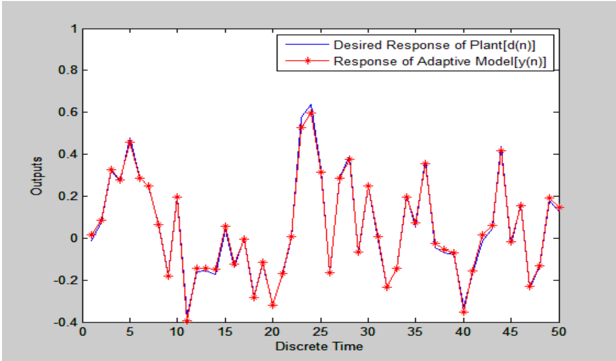

Fig 2(d) Comparison of responses of example-1 using NL2 with 50% outliers within the range -5 to +5 using RCF1

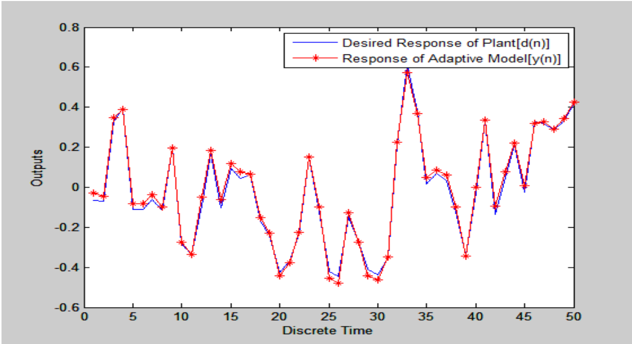

Fig 2(e) Comparison of responses of example-2 using NL2 with 50% outliers within the range -5 to +5 using RCF1

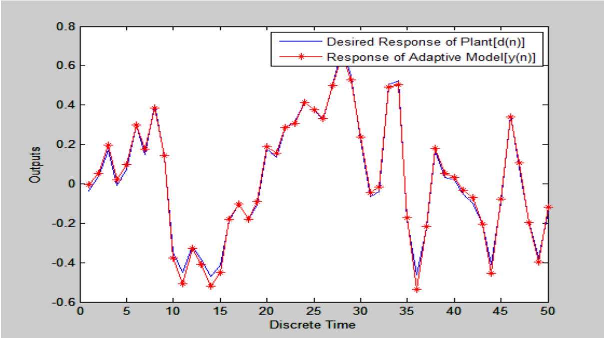

Fig 2(f) Comparison of responses of example-3 using NL2 with50% outliers within the range -5 to +5 using RCF1

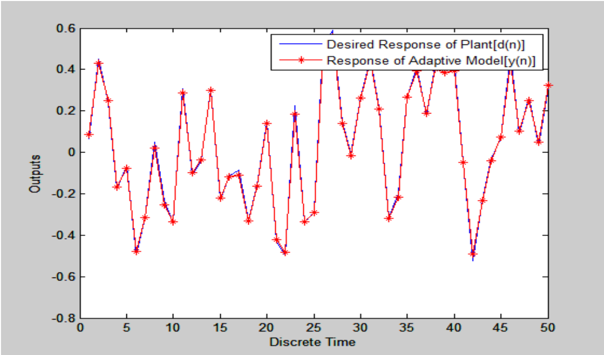

Fig 2(g) Comparison of responses of example-1 using NL3 with 50% outliers within the range -5 to +5 using RCF1

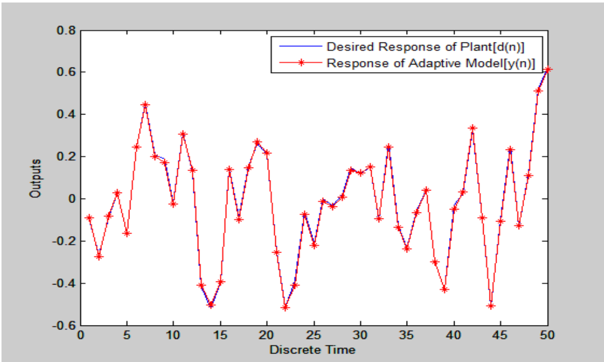

Fig 2(h) Comparison of responses of example-2 using NL3 with 50% outliers within the range -5 to +5 using RCF1

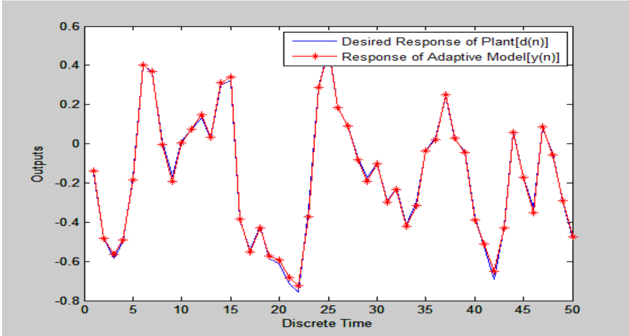

Fig 2(i) Comparison of responses of example-3 using NL3 with 50% outliers within the range -5 to +5 using RCF1

Sum of squared error is used as quantitative measure for performance evolution of the identification model. Various parameters used in study are as follows: Number of Input samples=500, population size=16, number of iterations=100, mutation rate=0.15, σ for RCF2=0.4. Simulation is carried out for different values of outliers. In the results presented from fig 2(a) to 2(i) only the comparisons of responses of RCF1 is shown for various plants along with all three nonlinearities discussed. The others results are presented in form of tables listed from 1(a)-1(h).It is evident from these plots and tables that robust cost functions perform better even in presence of outliers as high as 50% , where the MSE exhibits poor identification performance. Moreover of all the robust cost functions discussed the wilcoxon norm based cost function which is rank based cost function provides best response matching in presence of outliers.

Table 1(a): Comparison of Sum of squared error for system-1 and NL1

Outlier %	RCF_1 NMSE	RCF_2 NMSE	RCF_3 NMSE	MSE-CF NMSE
10	0.0017	0.0021	0.0155	0.0762
20	0.0022	0.0029	0.0084	0.0661
30	0.0018	0.0030	0.0149	0.1562
40	0.0027	0.0032	0.0340	0.3426
50	0.0025	0.0065	0.0277	0.1178

Table 1(b) Comparison of Sum of squared error for system-1 and NL2

outlier%	RCF_1 NMSE	RCF_2 NMSE	RCF_3 NMSE	MSE-CF NMSE
10	0.0043	0.0082	0.0174	0.0851
20	0.0046	0.0083	0.0122	0.0196
30	0.0053	0.0131	0.0320	0.0650
40	0.0039	0.0073	0.0271	0.0638
50	0.0049	0.0135	0.0939	0.1599

Table 1(c): Comparison of Sum of squared error for system-1 and NL3

Outlie%	RCF_1 NMSE	RCF_2 NMSE	RCF_3 NMSE	MSE-CF NMSE
10	0.0017	0.0020	0.0034	0.0060
20	0.0018	0.0028	0.0071	0.0254
30	0.0029	0.0031	0.0111	0.1035
40	0.0025	0.0047	0.0051	0.1926
50	0.0011	0.0018	0.0095	0.0783

Table 1(d): Comparison of Sum of squared error for system-2 and NL1

outlier%	RCF_1 NMSE	RCF_2 NMSE	RCF_3 NMSE	MSE-CF NMSE
10	0.0024	0.0028	0.0049	0.0367
20	0.0023	0.0026	0.0082	0.0223
30	0.0026	0.0045	0.0046	0.1820
40	0.0023	0.0055	0.0163	0.4808
50	0.0019	0.0050	0.0362	0.4151

Table 1(e): Comparison of Sum of squared error for system-2 and NL2

outlier%	RCF_1 NMSE	RCF_2 NMSE	RCF_3 NMSE	MSE-CF NMSE
10	0.0058	0.0084	0.0184	0.0478
20	0.0051	0.0088	0.0093	0.0166
30	0.0085	0.0091	0.0143	0.0232
40	0.0051	0.0104	0.0161	0.0412
50	0.0050	0.0056	0.0076	0.3431

Table 1(f): Comparison of Sum of squared error for system-2 and NL3

outlier%	RCF_1 NMSE	RCF_2 NMSE	RCF_3 NMSE	MSE-CF NMSE
10	0.0010	0.0014	0.0036	0.0957
20	0.0013	0.0016	0.0081	0.0504
30	0.0014	0.0027	0.0077	0.0551
40	0.0015	0.0031	0.0180	0.1082
50	0.0011	0.0075	0.0084	0.0664

Table 1(g): Comparison of Sum of squared error for system-3 and NL1

outlier%	RCF_1 NMSE	RCF_2 NMSE	RCF_3 NMSE	MSE-CF NMSE
10	0.0023	0.0026	0.0059	0.0650
20	0.0019	0.0046	0.0038	0.0542
30	0.0017	0.0027	0.0088	0.0311
40	0.0021	0.0040	0.0051	0.01618
50	0.0059	0.0073	0.1106	0.4361

Table 1(h): Comparison of Sum of squared error for system-3 and NL2

outlier%	RCF_1 NMSE	RCF_2 NMSE	RCF_3 NMSE	MSE-CF NMSE
10	0.0053	0.0081	0.0072	0.0483
20	0.0036	0.0079	0.0187	0.0955
30	0.0069	0.0132	0.0098	0.1232
40	0.0063	0.0134	0.0202	0.1109
50	0.0081	0.0108	0.0323	0.1361

Table 1(k): Comparison of Sum of squared error for system-3 and NL3

outlier%	RCF_1 NMSE	RCF_2 NMSE	RCF_3 NMSE	MSE-CF NMSE
10	0.0016	0.0020	0.0036	0.0182
20	0.0014	0.0016	0.0029	0.0363
30	0.0020	0.0043	0.0044	0.0197
40	0.0011	0.0055	0.0099	0.1093
50	0.0132	0.0086	0.0678	0.0765

5. CONCLUSION

The paper has introduced a scheme for nonlinear system identification using GA for updating the weights of the adaptive system for outliers present in the training sample itself. Rigorous simulation study has been carried out and the study demonstrates that accurate and robust models can created by using GA and robust cost functions in comparison to conventional squared error based norm. Of all the robust cost functions the Wilcoxon based norm outperforms the other cost functions in terms of performance matching.

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