DIGITAL PARAMETERS EFFECT OF LHM MODELING

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ABSTRACT

The metamaterials electromagnetic one are composites artificial made up of a matrix dielectric or magnetic and inclusions dielectric magnetic or metal. These inclusions can be laid out in the matrix either in a random way or in an ordered way; their size is generally small in front of the wavelength.

Research on materials with negative refractive index (NRI) and in particular on the left-handed material (LHM) or metamaterial ones developed very quickly and they have leads to many applications for example in, the radars, the lines of transmission, one of the applications most promising is the perfect lens.

In this work, we applied the dispersive FDTD method for study different effects limiting the resolution of sub-wavelength of LHM slab. To check the validity of our results we compared the variation of transmission coefficient obtained numerically with that calculated analytically.

KEYWORDS: Metamaterial, LHM, FDTD, NRI, Perfect Lens

INTRODUCTION

The current research projects correspond to the studies on the artificial composites hyper frequency, which open new prospects in the domain for reflexion and transmission for an electromagnetic wave.

The metamaterials generally are artificial structures; they consist of a dielectric or magnetic matrix and inclusions dielectric, magnetic or electromagnetic metal.

The concept of metamaterial was mentioned for the first time in the domain of optics in 1968, when the Russian physicist Victor Veselago [1] introduced the notion of Left Handed Metamaterial (LHM), to study theoretically the propagation in such material characterized by its electric permittivity and magnetic permeability simultaneously negative.

In this case the electric field ‘E’, magnetic field ‘H’, wave vector ‘K’ and the vector Poyting ‘S’ are anti parallel, and the group velocity and phase velocity are opposite, and hence a negative refractive index (NRI), the LHMs have special properties such as reversal of the Doppler effect and Cerenkov effect.

In 2001 [2], it was shown that an electromagnetic wave can be propagated in a medium of negative optical index.

An incidental wave crossing the surface of separation of vacuum and such a medium is refracted according to a negative angle, what had never been observed with before with a traditional medium.

The method of Finite Difference Time Domain (FDTD) [3], is known being that a powerful digital technique in electromagnetism, it at proven summer that it is popular among the researchers because of his simplicity in the execution.

In this work, we applied this method because it implies the direct numerical solution of the Maxwell’s equations, which are known as the base of traditional electromagnetism.
PRESENTATION OF CALCULATION METHOD (DISPERSE FDTD)

Characteristics of the Transmission in a LHM

For the study characteristics of transmission in a LHM, the model of Drude is applied for the constant dielectric \( \varepsilon(\omega) \) and the magnetic permeability \( \mu(\omega) \) with the same form of dispersion [4].

\[
\varepsilon(\omega) = \varepsilon_0 \left( 1 - \frac{\omega_{pe}^2}{\omega^2 - j\omega \gamma_e} \right)
\]

(1)

\[
\mu(\omega) = \mu_0 \left( 1 - \frac{\omega_{pm}^2}{\omega^2 - j\omega \gamma_m} \right)
\]

(2)

Where \( \omega_{pe} \) and \( \omega_{pm} \) represent the electric and magnetic frequency plasma, \( \gamma_e \) and \( \gamma_m \) represent the electric and magnetic frequency of collision.

According to the Amp and Faraday's laws [5].

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}
\]

(3)

\[
\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}
\]

(4)

As the relations :

\[
\vec{D} = \varepsilon \vec{E}
\]

(5)

\[
\vec{B} = \mu \vec{H}
\]

(6)

Where \( \varepsilon \) and \( \mu \) are expressed by the equations (1) and (2), the equations (3) and (4) can be discretized according to the procedure, which leads to the conventional FDTD [3].

\[
\vec{B}_{n+1} = \vec{B}_n - \Delta t \nabla \times \vec{E}_{n+1/2}
\]

(7)

\[
\vec{D}_{n+1} = \vec{D}_n + \Delta t \nabla \times \vec{H}_{n+1/2}
\]

(8)

Where \( \Delta t \): Step temporal, \( n \): Iteration count.

Moreover, the auxiliary equation must be taken into account and it can be discretized in the following way:

\[
(\omega^2 - j\omega \gamma_e)\vec{D} = \varepsilon_0 \left( \omega^2 - j\omega \gamma_e - \omega_{pe}^2 \right)\vec{E}
\]

(9)

By using the opposite transform of Fourier, the equation (9) can be rewritten in the time domain in the form:

\[
\left( \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t} \gamma_e \right)\vec{D} = \varepsilon_0 \left( \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t} \gamma_e + \omega_{pe}^2 \right)\vec{E}
\]

(10)

Consequently the equation of \( \vec{E} \) is:
Digital Parameters Effect of LHM Modeling

\[
\tilde{E}^{n+1} = \left[ \frac{1}{\varepsilon_0 (\Delta t)^2} + \frac{\gamma_e}{2 \varepsilon_0 \Delta t} \right] \tilde{D}^{n+1} - \frac{2}{\varepsilon_0 (\Delta t)^2} \tilde{D}^n + \left[ \frac{2}{(\Delta t)^2} - \frac{\omega_{pe}^2}{2} \right] \tilde{E}^n - \left[ \frac{1}{(\Delta t)^2} - \frac{\gamma_e}{2 \Delta t} + \frac{\omega_{pe}^2}{4} \right] \tilde{E}^{n-1}
\]

(11)

Effect of the Numerical Parameters

To model the dielectric conventional, we assume that the results are rather precise how the effect of the numerical parameters of material can be ignored, if the size of cells of the FDTD is smaller than \( \lambda/10 \) [3].

However, the discretization has a disparity between the numerical and analytical permittivity/permeability, by modeling the LHM particularly when the evanescent waves are implied, spatial resolution of the FDTD a significant impact on the accuracy of simulation results.

In the case of the plane waves when: \( \tilde{E}^{n+1} = \tilde{E}^n e^{j \omega \Delta t} ; \tilde{D}^{n+1} = \tilde{D}^n e^{j \omega \Delta t} \)

We have: \( e^{j \omega \Delta t} + e^{-j \omega \Delta t} = 2 \cos(\omega \Delta t) ; e^{j \omega \Delta t} - e^{-j \omega \Delta t} = 2j \sin(\omega \Delta t) \)

The equation (11) can be rewritten as follows:

\[
\tilde{E}^n \left[ \frac{\omega_p^2}{2} + \frac{2}{(\Delta t)^2} \right] \cos(\omega \Delta t) + \tilde{E}^n \left[ \frac{\omega_p^2}{2} - \frac{2}{(\Delta t)^2} \right] + \tilde{E}^n \frac{\gamma_e}{\Delta t} j \sin(\omega \Delta t)
\]

\[
= \tilde{D}^n \frac{2}{\varepsilon_0 (\Delta t)^2} \cos(\omega \Delta t) - \tilde{D}^n \frac{2}{\varepsilon_0 (\Delta t)^2} + \tilde{D}^n \frac{\gamma_e}{\varepsilon_0 \Delta t} j \sin(\omega \Delta t)
\]

(12)

After a simple calculation the numerical permittivity ( \( \tilde{\varepsilon} = \frac{\tilde{D}^n}{\tilde{E}^n} \) ) can be obtained:

\[
\tilde{\varepsilon} = \varepsilon_0 \left[ 1 - \frac{\omega_p^2 (\Delta t)^2 \cos^2 \frac{\omega \Delta t}{2}}{2 \sin \frac{\omega \Delta t}{2} \left( \frac{\omega \Delta t}{2} - j \gamma \Delta t \cos \frac{\omega \Delta t}{2} \right)} \right]
\]

(13)

CALCULATION RESULTS

For the analysis of different spatial resolutions of the FDTD concerning the numerical permittivity, the parameters (\( \omega_p \) and \( \gamma \)) are chosen so that the real part of the analytical permittivity is equal “-1”, the corresponding numerical permittivity is calculated starting from the equation (13), the comparison is shown in Fig1.
We can note that the analytical permittivity is always have an equal value 
“−1”.

For $\Delta x > \lambda / 60$, a considerable difference is observed between the numerical and analytical permittivity, what
leads to the amplification of the coefficient of transmission analyzed in the following section.

While replacing $\Delta x < \lambda / 100$ and $\Delta t = \Delta x / \sqrt{2c}$ and the work frequency in the equation (13), we will have:

$$\varepsilon = -0.9993 - 0.001 j .$$

A small difference is observed between the real part of the relative permittivity and “−1”. However, if the size of
the cells is larger for example in the case of $\Delta x = \lambda / 40$ we will have: $\varepsilon = -0.9959 - 0.001 j .$

The difference between the real part of $\varepsilon$ and “−1” is more significant. We observe an amplification of the
coefficient of transmission as, it is illustrated in Fig2. By using the same permittivity in the analytical formulation, we can
obtain the corresponding coefficient of transmission which is also traced in Fig2 for the comparison.

We note:

For the great values of “Kx” there is a small shift between the analytical and numerical results caused by the big
size of the cell, which causes an insufficient taking away of points.

Figure 1: Comparison of the Real Part of the Analytical and Numerical
Permittivity for Different Spatial Resolutions

Figure 2: Coefficient of Transmission of LHM Slab, Using the FDTD Method with the Average of the Permittivity
and without Correction of the Parameters of Material ($\varepsilon_r = \mu_r = -0.9959 - 0.001 j$) with $\Delta x = \lambda / 40$
The advantage of considering the permittivity electric numerical is the correction of the disparity for simulations of the FDTD.

After a simple calculation, the frequency plasma and the frequency of corrected collision are obtained:

\[
\tilde{\omega}_p = \frac{2 \sin \omega \Delta t \left[ -2 (\varepsilon'_r - 1) \sin \frac{\omega \Delta t}{2} - \varepsilon'_r \chi \Delta t \cos \frac{\omega \Delta t}{2} \right]}{(\Delta t)^2 \cos^2 \frac{\omega \Delta t}{2}} \tag{14}
\]

\[
\tilde{\gamma} = \frac{2 \varepsilon'_r \sin \frac{\omega \Delta t}{2}}{(\varepsilon'_r - 1) \Delta t \cos \frac{\omega \Delta t}{2}} \tag{15}
\]

Where \( \varepsilon'_r \) and \( \varepsilon'_r \) are the real and imaginary part of the relative permittivity \( \varepsilon_r \).

In the case of \( \varepsilon_r = -1 - 0.001 j \), we replace \( \varepsilon'_r = -1 \) and \( \varepsilon'_r = -0.001 j \), in equation (14) and (15) we obtain:

\( \tilde{\omega}_p = 1.4157 \omega \) and \( \tilde{\gamma} = 5.0051 \times 10^{-4} \omega \).

After correction of the disparity while replacing \( \tilde{\omega}_p \) and \( \tilde{\gamma} \) by their expressions in the equation (13), we obtain the corrected numerical permittivity \( \tilde{\varepsilon} \).

By using the corrected parameters, the result of the FDTD and its comparison with the analytical solution are shown in Fig3.

We can see that there is no disparity and consequently no amplification of the coefficient of transmission, the shift between the two solutions for \( \Delta x = \lambda/40 \) is caused by the insufficient taking away of points, by using a special step \( \Delta x = \lambda/100 \) the results of simulation show the good agreement with the analytical solution.

![Figure 3: Coefficient of Transmission of LHM Slab, Using the FDTD Method with the Average of the Permittivity and with Correction of the Material Parameters for \( \Delta x = \lambda/40 \), \( \Delta x = \lambda/100 \) and \( \tilde{\varepsilon}_r = -1 - 0.001 j \), the Same Permittivity is Employed Analytically](image)

CONCLUSIONS

We also saw that the resolution sub-wavelength of the LHM slab, is limited by the losses, the thickness of the slab and the disparity of the LHM with its surrounding medium.
The condition of spatial resolution in simulations of the conventional FDTD ($\Delta x < \lambda / 10$) is not sufficient for the LHM, because of the evanescent waves. The comparison also suggests that a spatial resolution less than FDTD ($\Delta x < \lambda / 80$) is necessary for accurate modeling of LHM.

Thus we showed that the discretization in the FDTD present a disparity between numerical and analytical permittivity/permeability, by modeling the LHM particularly when evanescent waves are involved means that, the spatial resolution in the FDTD has a significant impact on the accuracy of simulation results.

REFERENCES


