ANALYSIS OF MHD EFFECT ON RAYLEIGH STEP SLIDER BEARING LUBRICATED WITH COUPLE-STRESS FLUIDS

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ABSTRACT: The magneto-hydrodynamic Rayleigh step slider bearing lubricated with couple-stress fluids is analyzed under transverse magnetic field. The expression for non-dimensional pressure, non-dimensional load carrying capacity, frictional force and the co-efficient of friction is derived and results are discussed for various non-dimensional parameters. It is observed that the effect of Hartmann number and couple-stress parameter enhances the load carrying capacity, frictional force and the co-efficient of friction.

KEYWORDS: Couple Stress, Magneto Hydrodynamics & Rayleigh Step Bearing

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INTRODUCTION

The analysis of interface of conducting fluids with electromagnetic occurrences is called Magnetohydrodynamic (MHD). MHD lubrication is significantly than hydrodynamic lubrication, because most MHD bearing models have been developed from the perspective of mathematical analysis simplicity rather than practicality. The unexpected variation in the viscosity of lubricants with temperature, made the use of an electrically conducting fluid more popular. The first author to represent a study of MHD infinite inclined plane slider bearing in the existence of uniform transverse magnetic field was Snyder [1] and found that a significant increase in load capacity was possible with liquid metal lubricants in the presence of magnetic field. Shukla [2] examined optimum load of MHD slider bearing using film thickness and conductivity functions as bounded control variables and concluded that if the conductivity of bearing surface is of step-type in nature, it is more desirable for a uniformly distributed magnetic field. Hughes [3] analysed the MHD finite step slider bearing in the presence of magnetic field applied both tangentially and transversely to the fluid film. For the transverse field it is found that only a slight increase in pressurization can be affected on open circuit conditions and that short circuit conditions are unfavorable. Anwar [4] analysed the MHD characteristic of inclined slider bearing with arbitrary magnetic field. It is found that a non-uniform applied magnetic field gives higher load capacity as compared with uniform magnetic fields.

In 1918, Lord Rayleigh discovered a film geometry where a step is split the lubricant film into two zones viz; h₁ at the entry zone and h₂ at the exit zone and result holds good when the viscosity of lubricant is considered as function of pressure. Many researchers have been studied the characteristics of the Rayleigh step bearing lubricated with non-Newtonian fluids to name few are Jianming and Gaobing [5], and Naduvinamani and Siddangouda [6] have studied the effect of couple stress on the characteristics of Rayleigh step bearings.

The slider bearings have practical applications in machine design and in other kinds of machine elements in
which rectilinear sliding motions occur. The thrust area of the step bearings is in the clutch plate, gear box, thrust bearings and journal bearings etc. Awareness of the features of bearing is important when taking into consideration specific operational circumstances. Several analyses on lubrication performance of slider bearings have been concluded in the last few years. Later to upgrade the lubricating performance the increased utilization of Newtonian lubricant which has been blended with long chain polymers has been observed. Using additives stabilizes the flow properties and minimizes lubricant sensitivity to change shear rate. A variety of micro-continuum theories have been carried out, but Stokes [7] theory is the simplest micro-continuum theory which allows the existence of Couple stresses and body couples. The load carrying capability, force of friction increases, and frictional coefficient decreases for a slider bearing greased with couple-stress fluid was concluded by Ramanaiah and Sarkar [8]. A relative study is made between parabolic and inclined slider lubricated with couplestress fluid by Moobalaji and John [9] and found that parabolic slider has more superior load carrying capability than inclined slider. The rough slider bearing with couple stress was analyzed by Naduvianamani et al [10]. Buzurke et al [11] examined the porous step slider with couple stress fluids and many authors [12-14] discussed the effect of couple stress lubricants on the characteristics of bearing systems. Combined effect of Magnetic field and couple stress lubricant was discussed by several researchers such as Kashinath and Hanumagowda [15] presented the composite slider bearing with MHD couple stress, and noticed due to magnetic field there is a increase in pressure, load carrying, frictional force and coefficient of friction. Hanumagowda et al [16-21] studied Land Tapered slider bearing and plane slider bearing, and Naduvianamani et al [22] discussed the effect of MHD couple stress on exponential slider bearings and noted that the increase in load carrying capacity. The aim of the current paper is to conduct a study on MHD Rayleigh step slider bearing lubricated with Couple-stress Fluids.

**THEORETICAL FORMULATION**

The physical configuration of the problem is shown in Figure1 in which lower surface have pure tangential sliding motion relative to the other surface and separate by the lengths $L_1$ and $L_2$. The film thickness in the entry region is $h_1$ and in exit region is $h_2$. The uniform magnetic field $B_0$ is applied perpendicular to the two plates. The lubricant in the film region is incompressible. Stokes[7] couplestress fluid, body forces and body couples are assumed to be absent. The basic equations governing the hydro-magnetic flow of the lubricant in the fluid region are

$$
\begin{align*}
\mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial v}{\partial y} - \sigma B_0 u = \frac{\partial p}{\partial x} + \sigma E_x B_0 \\
\frac{\partial p}{\partial y} = 0 \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\
\int_{y=0}^{h} (E_x + B_0 u) dy = 0
\end{align*}
$$

From figure1 the boundary of the upper surface is given by

$$
\begin{align*}
h = \begin{cases} 
  h_1 \text{ for } 0 \leq x \leq L_1 \\
  h_2 \text{ for } L_1 \leq x \leq L
\end{cases}
\end{align*}
$$

where $L_1 + L_2 = L$
Boundary conditions are

At the upper surface $y = h$

$$u = 0 \frac{\partial^2 u}{\partial y^2} = 0 \quad v = 0 \quad (5)$$

At the lower surface $y = 0$

$$u = U \frac{\partial^2 u}{\partial y^2} = 0 \quad v = 0 \quad (6)$$

Solution of equation (1) using condition (4) and the boundary conditions (5) and (6) is

$$u = -\frac{h^2}{2\mu l M_0} \frac{\partial p}{\partial x} \left\{ \frac{B^2}{A} \tanh \left( \frac{A h}{2l} \right) - \frac{A^2}{B} \tanh \left( \frac{B h}{2l} \right) \right\} - \frac{U}{2(A^2 - B^2)} \left\{ B^2 \xi_1 - A^2 \xi_4 \right\} \quad (7)$$

where

$$\xi_1 = \frac{\sinh \left( \frac{A h}{l} \right) - \sinh \left( \frac{A y}{l} \right) + \sinh \left( \frac{A (h - y)}{l} \right)}{\sinh \left( \frac{A h}{l} \right)}, \quad \xi_2 = \frac{\sinh \left( \frac{B h}{l} \right) - \sinh \left( \frac{B y}{l} \right) + \sinh \left( \frac{B (h - y)}{l} \right)}{\sinh \left( \frac{B h}{l} \right)},$$

$$\xi_3 = \frac{\sinh \left( \frac{A h}{l} \right) - \sinh \left( \frac{A y}{l} \right) - \sinh \left( \frac{A (h - y)}{l} \right)}{\sinh \left( \frac{A h}{l} \right)}, \quad \xi_4 = \frac{\sinh \left( \frac{B h}{l} \right) - \sinh \left( \frac{B y}{l} \right) - \sinh \left( \frac{B (h - y)}{l} \right)}{\sinh \left( \frac{B h}{l} \right)}.$$

Integration of the continuity equation (3) over the film thickness with boundary conditions (5) and (6) results in the modified Reynolds equation in the form
Using the non-dimensional quantities, the non-dimensional MHD Reynolds-type equation is

\[
\frac{\partial}{\partial x}\left[ \frac{6h_0^2h}{\mu M_0^2} \left( \frac{A^2 - B^2}{B \tanh \left( \frac{Bh}{2l} \right)} - \frac{2l}{h} \frac{\partial p}{\partial x} \right) \right] = 6\mu \frac{dh}{dx}
\]

where \( f(H, l', M_0) = \frac{12H^2}{l'M_0^2} \left( \frac{A^2 - B^2}{B^2 \tanh \left( \frac{B'H}{l'} \right)} - \frac{B^2}{A^2} \tanh \left( \frac{A'H}{l'} \right) \right) \)

\( x' = \frac{x}{L}, \quad P = \frac{ph_0^2}{6\mu UL}, \quad l' = \frac{2l}{h}, \quad H = \frac{h}{h_2}, \quad L_1' = \frac{L_1}{L}, \quad L_2' = \frac{L_2}{L} \)

\[ A^* = \left[ 1 + \left( 1 - \left( l'^2M_0^2 / \mu \right) \right)^{1/2} \right]^{1/2} \]

\[ B^* = \left[ 1 - \left( 1 - \left( l'^2M_0^2 / \mu \right) \right)^{1/2} \right]^{1/2} \]

The relevant boundary conditions for pressure are

\[ P = 0 \text{ at } x' = 0, 1 \]  

(9)

Integrating both sides of equation (8) twice w.r.t. \( x' \) and using the boundary conditions (9) we get

\[ p_i' = \frac{(h_i' - 1)L_2'x'}{L_1'f^+(l', l', M_0) + L_2'f^+(h_i', l', M_0)} \quad 0 \leq x' \leq L_1' \]

(10a)

\[ p_2' = \frac{(h_2' - 1)L_1'(1 - x')}{L_1'f^+(h_i', l', M_0) + L_2'f^+(h_i', l', M_0)} \quad L_1' \leq x' \leq 1 \]

(10b)

The non-dimensional load carrying capacity is

\[ W^* = \frac{Wh_0^2}{6\mu UL} = \frac{1}{L_1'} \int_0^{L_1'} p_1' dx' + \frac{1}{L_2'} \int_{L_1'}^{1} p_2' dx' \]

Using (10a) and (10b) the non dimensional load carrying capacity is obtained in the form

\[ W^* = \frac{(h_i' - 1)L_1'L_2'}{2\left[ L_2'f^+(h_i', l', M_0) + L_1'f^+(l', l', M_0) \right]} \]

(11)

The components of stress tensor required for calculating frictional force is

\[ \tau_{yn} = \mu \frac{\partial u}{\partial y} - \eta \frac{\partial^2 u}{\partial y^2} \]
The frictional force on the bearing surface is

\[
F = \int \left[ -\frac{\mu UL}{2h^* (A^2 - B^2)} \left( A^2 \coth \left( \frac{Bh}{2l} \right) - B^2 \coth \left( \frac{Ah}{2l} \right) \right) - \frac{h}{2} \frac{\partial p}{\partial x} \right] dx
\]

The dimensionless frictional force is

\[
F^* = -\frac{Fh^*}{6\mu UL} = G(h_1^*, l^*, M_0) L_1^* + G(1, l^*, M_0) L_2^*
\]

\[
+ \frac{(h_1^* - 1)^2 L_2^*}{2 \left( f(l^*, M_0) L_1^* + f(h_1^*, l^*, M_0) L_2^* \right)}
\]

where

\[
G(h_1^*, l^*, M_0) = \frac{\tau M_0^2}{24(A^2 - B^2)} \left( A^2 \coth \left( \frac{Bh_1^*}{l^*} \right) - B^2 \coth \left( \frac{Ah_1^*}{l^*} \right) \right)
\]

\[
G(1, l^*, M_0) = \frac{\tau M_0^2}{24(A^2 - B^2)} \left( A^2 \coth \left( \frac{B}{l^*} \right) - B^2 \coth \left( \frac{A}{l^*} \right) \right)
\]

The co-efficient of friction is given by

\[
C = \frac{F^*}{W}
\]

**Limiting case of the Present Study**

As \( M_0 \to 0 \) equations (11), (12) and (13) reduces to the case studied by Ramanaiah and Sarkar[8](case 2.1). The comparison of present analysis with Ramanaiah and Sarkar[8] is given in table 1.

**RESULTS AND DISCUSSIONS**

The effect of MHD on Rayleigh step slider bearing lubricated with couple-stress fluids is studied. The results are discussed for various non-dimensional parameters namely Hartmann number \( M_0 \), couple-stress parameter \( l^* \), bearing length \( L_1^* \) as follows.

**Pressure**

Figures 2 shows the variation of non-dimensional pressure \( P^* \) with \( x^* \) for distinct values of \( M_0 \) and \( l^* \) with fixed values \( L_1^* = 0.7 \), \( h_1^* = 2 \). It is observed that the pressure increases with increasing values of Hartmann number \( M_0 \) and couple-stress parameter \( l^* \) as compared with Non-magnetic case and Newtonian case. The deviation of non-dimensional pressure \( P^*_{\text{max}} \) with \( L_1^* \) as function of \( M_0 \) and \( l^* \) with fixed values \( h_1^* = 2 \) is depicted in Figures 3. It is found that pressure is maximum for \( L_1^* \) lies between 0.7 and 0.8.
Load Carrying Capacity

Figures 4 shows the variation of non-dimensional load carrying capacity $W^*$ with step height $h_1^*$ for various values of $M_0$ and $l^*$ with fixed values $L_1^* = 0.7$. It is observed that the effect of Hartmann number $M_0$ and couple stress parameter $l^*$ increases load carrying capacity. The non-dimensional load increases with $h_1^*$ until a maximum is obtained, and then after decreases with $h_1^*$. In Figure 5 the graph of non-dimensional load carrying capacity $W^*$ with $L_1^*$ for different values of $M_0$ and $l^*$ with fixed values $h_1^* = 2$ is plotted and observed that maximum load carrying capacity is for $L_1^*$ lies in the interval 0.7 and 0.8.

| Table 1: Present analysis is compared with work done by Ramanan and Sarkar [8] by varying MHD and couple stress parameter and values are tabulated for $W^*$, $F^*$, $C$ and with $L_1^* = 0.5$. |

<table>
<thead>
<tr>
<th>Rananan and Sarkar [8] analysis</th>
<th>Present analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l^*$</td>
<td>$M_0 = 0$</td>
</tr>
<tr>
<td>$h_1^* = 2$</td>
<td>$h_1^* = 3$</td>
</tr>
<tr>
<td>$W^*$</td>
<td>0.02778</td>
</tr>
<tr>
<td>$F^*$</td>
<td>0.015278</td>
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Frictional Force

Figure 6 displays the variation of non-dimensional frictional force $F^*$ with step height $h_i^*$ for different values of $M_0$ and $l'$ with fixed values $L_1^*=0.7$. It is observed that frictional force increases with increasing values of $M_0$ and $l'$ as compared with Non-magnetic case and Newtonian case. For the bearing with a Non-magnetic lubricant the frictional force decreases with increasing value of the step height $h_i^*$ and the bearing with a magnetic lubricant the frictional force increases with increasing value of the step height $h_i^*$. Figure 7 shows the variation of non-dimensional frictional force $F^*$ with $L_1^*$ for different values of $M_0$ and $l'$ with fixed values $h_i^*=2$. It is seen that frictional force decreases with an increase of $L_1^*$ for nonmagnetic case and for magnetic case frictional force increases with an increase of $L_1^*$ until a maximum is obtained, and there after decreases with $L_1^*$.

Coefficient of Friction

Figure 8 shows the variation of coefficient of friction $C$ with step height $h_i^*$ for different values of $M_0$ and $l'$ with fixed values $L_1^*=0.7$. It is observed that coefficient of friction increases with an increase in Hartmann number $M_0$ and decreases due to increase in couplestress parameter $l'$. Figure 9 depicts, the deviation of coefficient of friction $C$ with $L_1^*$ for different values of $M_0$ and $l'$ with fixed values $h_i^*=2$. It is noted that the coefficient of friction $C$ decreases with $L_1^*$ until a minimum is obtained, and there after increases with $L_1^*$.

The relative percentage increase in the non-dimensional load carrying capacity $R_{W^*}$, non-dimensional frictional force $R_{F^*}$ and coefficient of friction $R_{C}$ are defined by

$$R_{W^*} = \left( \frac{W_{\text{magnetic}}^* - W_{\text{non-magnetic}}^*}{W_{\text{non-magnetic}}^*} \right) \times 100$$

$$R_{F^*} = \left( \frac{F_{\text{magnetic}}^* - F_{\text{non-magnetic}}^*}{F_{\text{non-magnetic}}^*} \right) \times 100$$

and

$$R_{C} = \left( \frac{C_{\text{magnetic}} - C_{\text{non-magnetic}}}{C_{\text{non-magnetic}}} \right) \times 100$$

The values of $R_{W^*}$, $R_{F^*}$ and $R_{C}$ are listed in Table 2 for various values of $l'$, $M_0$ with $L_1^*=0.7$, $h_i^*=2$. It is clear that an increase of nearly 66.43%, 185% and 71.23% in $W^*$, $F^*$ and $C$ is observed for $l'=0.2$ and $M_0=4$. 

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CONCLUSIONS

Using the continuity equation and the MHD motion equations the characteristics of Rayleigh step slider bearings with an electrically conducting fluid in the presence of a transverse magnetic field are investigated. According to the results and discussion, conclusions can be drawn as follows.

- The Pressure, load carrying capacity, frictional force and coefficient of friction of Rayleigh step slider bearing is significant for larger values of couples stress parameter \( l' \) as compared to Newtonian case.
- The applied magnetic field increases the load carrying capacity, the frictional force and the coefficient of friction as compared with the corresponding non-magnetic case.
- The relative load \( R_W^* \), the relative frictional force \( R_F^* \) and the relative coefficient of friction \( R_C \) increases with increasing values \( M_0 \), an increase of about 66.43\% in \( W^* \), 184.97\% in \( F^* \) and 71.23\% in \( C \) is observed for \( l'=0.2 \) and \( M_0=4 \).

Numerical example of Rayleigh step slider bearing with MHD and couple stress is given in Table 3.
Analysis of Mhd Effect on Rayleigh Step Slider Bearing Lubricated with Couple-Stress Fluids

<table>
<thead>
<tr>
<th>l'</th>
<th>M₀</th>
<th>R₀'</th>
<th>R₀'</th>
<th>R₀'</th>
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<td>6</td>
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<td>386.3871</td>
<td>108.7643</td>
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</table>

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