APPLICATION OF FINITE ELEMENT HOMOGENIZATION TECHNIQUE FOR EFFICIENT MODELING OF ADDITIVELY MANUFACTURED METAMATERIALS

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ABSTRACT

The current research is devoted to study of the feasibility of the homogenization method to such objects as additively manufactured metamaterials. The article presents the approaches, analysis details and results of finite element homogenization based on a set of metamaterials produced with metal additive manufacturing using the powder-based materials such as aluminium alloy AlSi10Mg and 17-4PH steel. The numerical study was performed using highly adequate orthotropic models of the materials, and the results from homogenized models were compared both with the results of the full models of the metamaterials and with the experimental data. Results of the performed research show clearly that the method of finite element homogenization is fully applicable for such structures as metamaterials. There is a good correlation between the full models, natural experiments and homogenized models in the considered range of strain and stresses. The formulated methodology can be well applied for developing new and optimizing existing mechanical parts and assembly components for such advanced industries as aviation, automotive, space and others.

KEYWORDS: Additive Manufacturing; 3D Printing; Metal Powders; Selective Laser Melting; Electron Beam Melting; Computer-Aided Engineering; Structural Analysis; Structural Simulations; Advanced Manufacturing Technologies; Metamaterials; Lattice Structures; Material Modeling

INTRODUCTION

Topic Review

During the latest ten years additive technologies, as a group of manufacturing methods with a wide range of capabilities and limited number of disadvantages, have made possible to produce unique metal parts and components, which could not be manufactured earlier by using conventional technologies. Metamaterials or lattice materials are one of such types of structures, since their physical and mechanical properties can be effectively controlled by adjusting the shape of the basic cell.

Despite the huge amount of advantages, the modern metamaterials have a significant disadvantage for mathematical modeling and computational analysis. It is revealed that the finite element simulation of their mechanical behavior is extremely difficult due to the small cross-section of the rods and small dimensions of the basic cell in comparison with the size of the parts and components made of metamaterials. The mentioned feature
makes the processes of the mesh development and post processing very time consuming, and hence the solution requires a lot of computational effort.

The presented article describes the study of the feasibility of the homogenization method – the well-known theoretical approach successfully used for modeling composite materials and other complex structures worldwide, to additively manufactured metamaterials for estimating effective anisotropic properties and performing efficient modeling of these advanced materials.

**Definition of a Metamaterial**

In the beginning of the description of the process and results of the study presented it is important to define the term “metamaterial”, since it may have several meanings.

Most frequently, a “metamaterial” is a composite material, the behavior of which is mostly affected by the internal structure usually possessing the special periodic arrangement, but not by the composite components properties itself. It is assumed here that the periodic structure is artificially designed in such a way that it provides some unique mechanical, electromagnetic or acoustic properties of the material on the macroscale, which are not available for conventional homogeneous materials and cannot be reached directly due to technological issues [1-4].

The understanding of a metamaterial mentioned above is quite close to the definition, which used in this study, however the definition shall be clarified slightly. It is presumed in current paper that a metamaterial is the material that has the following specific properties:

- It possesses three-dimensional periodic lattice structure;
- The characteristic size of the basic cell is low in comparison with the overall dimensions of the part formed by the cells;
- The global behavior of the metamaterial strongly depends on the basic cell parameters;
- It is produced with additive manufacturing technologies using metal powder materials.

The features above can be used to formulate the full definition of a metamaterial that is used in the study: a metamaterial is a material that consists of a large amount of basic cells with the size, which is much lower than the size of the part made of the metamaterial. The behavior of the metamaterials is a function of a combination of the shape, type and dimensions of the basic cell. One important thing to mention is that the behavior of the metamaterials still strongly connected with the properties of the metal material from which it was produced.

**Numerical Modeling of Metamaterials**

The metamaterial properties, which are listed above lead to an important conclusion that such complicated object might be considered in the various ways depending on the modeling level.

In case of meso-level modeling the metamaterials should be considered as complex structures, because their geometry does not allow treating them as conventional materials. So the meso-level modeling should be done with the proper level of detail to make it possible to describe each of the beams of the structure of the metamaterial.
Since the geometry of most metamaterials is composed of a large amount of basic cells, these materials have a periodic internal lattice structure, like conventional metallic materials, which are formed by crystalline lattices. This allows making the assumption that in case of macro-level modeling of some parts or assemblies, which are made of metamaterials or contain some regions made of it, the local effects taking place on the meso-level can be neglected.

The approach mentioned above allows performing finite-element analysis for metamaterial parts using the material continuum models, which are based on the mechanical properties that are equivalent to the macro-level properties of the corresponding metamaterial. And the equivalent properties can be evaluated as the result of the homogenization technique performed for the representative volume element (RVE) of the metamaterial, which is the basic cell forming the material as a rule.

Such type of simplified modeling of metamaterial reduces the computational effort significantly and makes possible to design, analyze and optimize efficient parts and assemblies, which were impossible to perform earlier.

**Review of the Research of Metamaterials**

This section presents a short review of the current studies related to metamaterials and the approaches of efficient modeling of such type of materials.

One of the possible approaches of homogenization is discussed in [5-6]. The researchers studied the behavior of heterogeneous materials in case of a series of load cases. Mechanical properties of the equivalent material were mathematically evaluated.

Article [7] describes the process of preparation of the geometrical model, numerical simulations and natural experiments for two types of metamaterials, which are printed with Ti6Al4V by selective laser melting process. The aim of the work is to develop the algorithm of estimation of the beam diameter for the basic cell subjected to some specific loading conditions. The numerical modeling was carried out in SIMULIA Abaqus with the implicit dynamic solver. The study was performed in assumption that the material is isotropic, and it is loaded with uniaxial tension. The computational results were compared with the experimental data and used for developing the algorithm of estimation of the beam diameter.

A detailed review of the best practices related to modeling of metamaterials is presented in [8]. Many of the described researches are devoted to investigation of the equivalent mechanical properties for additively manufactured metallic metamaterials based on the experimental and numerical studies.

One of analytical approaches of performing the homogenization is made in study [9]. The analytical results are compared with the results of numerical simulations for the case of uniaxial tension of a metamaterial with the basic cell shaped like a body centered cubic crystalline lattice.

Another analytical method of estimation of the equivalent properties of metamaterials is considered in [10]. The procedure is quite easy-to-use and simple, but these advantages lead to a very limited scope of applicability. The analytical results are compared with the results of finite-element simulations. The work also describes a series of criteria that allow detecting the critical loading conditions for a chosen metamaterial.
The authors of [11] present the results of numerical modeling of a polymer metamaterial printed with polyamide PA2200. The considered material had a basic cell with the geometry of the crystalline lattice of a diamond. The results of the performed computations were compared with the experimental data.

As a specific type of metamaterials, the porous composite structures saturated by the air or liquid can be also considered [12]. Study of the dynamic characteristics of cellular and lattice structures is essential for the effective design of the mechanism parts [13].

It is important to mention that the review above includes only the most relevant articles, which show the general direction of the studies that are devoted to metamaterials. However, it allows making the conclusion regarding the high relevance of the topic of current study and confirms that the method of efficient modeling of general metamaterials does not exist still.

THE AIM OF STUDY AND PROPOSED METAMATERIALS

The current study explains the application of the homogenization technique using four metamaterials: two basic cells made of two materials. Since the behavior of metamaterials is strongly affected both by the type of the basic cell and by the material properties, each pair of cell and material form a unique metamaterial.

The fragments of the analyzed metamaterials are shown in Fig. 1 [14]. All the materials are manufactured with selective laser melting process [15]. One of the pairs is made of AlSi10Mg aluminium alloy, another pair is made of 17-4PH steel.

![Figure 1: Fragments of the Analyzed Metamaterials. (a) AlSi10Mg, Cell of Type 1; (b) AlSi10Mg, Cell of Type 2; (c) 17-4PH, Cell of Type 1; (d) 17-4PH, Cell of Type 2.](image)

The metamaterial fragments shown in Fig. 1 consist of 3x3x3 basic cells. The basic cells are shown in Fig. 2. The “metal-air” ratio for the cell of Type 1 is 16:84, for the cell of Type 2 – 19:81 that gives the volumetric porosity equal to 0.84 and 0.81 correspondingly.

![Figure 2: (a) Basic Cell of Type 1; (b) Basic Cell of Type 2.](image)
The homogenization is performed for the representative volume elements consisting of 1×1×1 basic cells. Surely there are no limitations or restrictions, which do not allow carrying out the homogenization for the fragments consisting of 2×2×2 cells, because such fragments could be considered as the representative volume elements of larger size, but the larger are the dimensions of the representative volume element, the higher is the computational effort that is required for performing the homogenization.

**HOMOGENIZATION TECHNIQUE DESCRIPTION**

**General Approach of Homogenization**

As it was mentioned earlier, homogenization is a technique of evaluation of equivalent global properties of a complex material with a periodic structure. The evaluation is done through natural experiments or virtual testing by numerical modeling. The object of testing is the representative volume element or the basic cell of the metamaterial. The results of the tests for the cell can be extrapolated to the entire material or part due to the periodic structure of the metamaterial. In general, the approach is applicable to the materials with random internal structure, because it is also periodic.

Since the study mostly focuses on the additively manufactured metamaterials with a relatively low size of the basic cell, it is more reasonable to perform the homogenization based on the virtual testing or finite-element modeling of the representative volume element. Such decision is caused by the fact that the natural testing is too complex from the technical point of view and evidently very expensive, as it requires manufacturing of large amount of specimens and special experimental gear.

However, finite-element simulation based on the highly-adequate continuum models of additive materials and detailed modeling of the geometry of the basic cell can provide all the results that are required for performing the homogenization and obtaining the equivalent properties of the metamaterial. In such case the homogenization is actually done numerically, and this may lead to some doubts regarding the resulting accuracy. So in the current study the fully numerical homogenization is followed by the validation with natural testing in order to confirm that the approach provides the sufficient accuracy.

**Theoretical Aspects of Homogenization**

Before the theoretical aspects description of the homogenization technique as applied to metamaterials, it is important to mention that all definitions, expressions and formulas are valid for the representative volume elements (RVE), which differ slightly from the basic cells of metamaterials themselves. The reason for the difference is the “air”, which fills the void space inside the basic cell. Despite the fact that the gas has negligible mechanical properties, it has a significant volume (81 – 84 % in our research), which shall be taken into account in the homogenization procedure calculations.

In order to avoid doubling, all the process of homogenization in the article is explained for the metamaterial of Type 1 made of AlSi10Mg. The geometrical model that is used for performing the homogenization is illustrated in Fig. 3.
In current research the target mathematical model for the equivalent homogeneous material is the orthotropic elastic model [16]. Such decision was made, because most of the parts and assemblies that are used in technical devices operate in the elastic condition without reaching the nonlinear and plastic regions of the stress-strain curve.

Figure 3: Geometrical Model that Performing the Homogenization.

The governing equations written for the averaged stresses and strains of the heterogeneous representative volume element in case of considering it as a homogeneous orthotropic material appear as follows [5]:

\[
E_x^*(\varepsilon_{xx}) = \langle \sigma_{xx} \rangle - \nu_{xy}^* \langle \sigma_{yy} \rangle - \nu_{xz}^* \langle \sigma_{zz} \rangle,
\]
\[
E_y^*(\varepsilon_{yy}) = -\nu_{yx}^* \langle \sigma_{xx} \rangle + \langle \sigma_{yy} \rangle - \nu_{yz}^* \langle \sigma_{zz} \rangle,
\]
\[
E_z^*(\varepsilon_{zz}) = -\nu_{zx}^* \langle \sigma_{xx} \rangle - \nu_{zy}^* \langle \sigma_{yy} \rangle + \langle \sigma_{zz} \rangle,
\]
\[
G_{xy}^*(\gamma_{xy}) = \langle \sigma_{xy} \rangle,
\]
\[
G_{yz}^*(\gamma_{yz}) = \langle \sigma_{yz} \rangle,
\]
\[
G_{zx}^*(\gamma_{zx}) = \langle \sigma_{zx} \rangle.
\]

Here \( E_x^*, E_y^* \) and \( E_z^* \) are the equivalent Young moduli, \( \nu_{xy}^*, \nu_{yx}^*, \nu_{xz}^*, \nu_{yz}^*, \nu_{zx}^*, \nu_{zy}^* \) – equivalent Poisson ratios, \( G_{xy}^*, G_{yz}^*, G_{zx}^* \) – equivalent shear moduli. What is important, three of six Poisson are independent, and the other three ratios are dependent according to the following formulas:

\[
E_x^* \nu_{xy}^* = E_y^* \nu_{yx}^*,
\]
\[
E_y^* \nu_{yz}^* = E_z^* \nu_{zy}^*,
\]
\[
E_z^* \nu_{zx}^* = E_x^* \nu_{xz}^*.
\]

**Derivation of Elastic and Shear Moduli**

It is possible to conclude from the previous section expressions that the independent quantities \( E_x^*, E_y^*, E_z^*, \nu_{xy}^*, \nu_{yz}^*, \nu_{zx}^* \)
\( G_{xy}^*, G_{yz}^*, G_{zx}^* \) can be derived with the defining equations based on a number of specific natural or virtual experiments [5].

So in case of performing a series of uniaxial tension experiments, where there is only one equivalent nonzero stress and strain component, it is possible to reformulate the defining equations for the tension along X, Y and Z axes respectively in a simplified form:

\[
E_x^*(\varepsilon_{xx}) = \langle \sigma_{xx} \rangle,
\]
\[
E_y^*(\varepsilon_{yy}) = \langle \sigma_{yy} \rangle,
\]
\[
E_z^*(\varepsilon_{zz}) = \langle \sigma_{zz} \rangle.
\]

Then the formulas for evaluation of the elastic moduli of the equivalent homogeneous material can be derived quite easily:
Due to the specific behavior of metamaterials as periodic structures, the numerical uniaxial tension experiments shall be carried out with the boundary conditions, which differ from the traditional ones. Thus, to perform the uniaxial tension test along X axis it is required to comply with the boundary conditions listed below [5]:

\[ x = 0: u_x = 0, \]
\[ x = L: u_x = f(t), \]
\[ y = 0: u_y = 0, \]
\[ y = L: u_y = u^*_y = \text{const}, \]
\[ z = 0: u_z = 0, \]
\[ z = L: u_z = u_z^* = \text{const}. \]

In current case symbol L means the length of the side of the representative volume element, which equals 2.5 mm. Similar boundary conditions are applicable for the uniaxial tension tests along Y and Z axes.

In the same manner in case of performing a series of uniaxial shear tests, where the nonzero components of stress and strain tensors are the shear components corresponding with XY, YZ or XZ planes, it is possible to reformulate the defining equations as follows:

\[ G_{xy} \langle \gamma_{xy} \rangle = \langle \sigma_{xy} \rangle, \]
\[ G_{yz} \langle \gamma_{yz} \rangle = \langle \sigma_{yz} \rangle, \]
\[ G_{xz} \langle \gamma_{xz} \rangle = \langle \sigma_{xz} \rangle. \]

Then the shear moduli can be derived as

\[ G_{xy}^* = \frac{\langle \sigma_{xy} \rangle}{\langle \gamma_{xy} \rangle}, \]
\[ G_{yz}^* = \frac{\langle \sigma_{yz} \rangle}{\langle \gamma_{yz} \rangle}, \]
\[ G_{xz}^* = \frac{\langle \sigma_{xz} \rangle}{\langle \gamma_{xz} \rangle}. \]

Here again due to the behavior of metamaterials, the numerical experiments shall be carried out with the specific boundary conditions. The conditions for the uniaxial shear in XY plane are listed below [5]:

\[ x = 0: u_x = 0, u_y = 0, u_z = 0 \]
\[ x = L: u_x = 1 - \cos(t), u_y = \sin(t) \]
\[ y = 0: u_y = \frac{x}{L} \sin(t), \]
\[ y = L: u_y = \frac{x}{L} \sin(t) , \]
\[ z = 0: u_z = 0, \]
\[ z = L: u_z = u_z^* = \text{const}. \]

The averaged values of stresses and strains mentioned in the formulas are evaluated as it is explained below:
Derivation of Poisson Ratios

The Poisson ratios of the homogeneous material with the properties, which are equivalent to the heterogeneous metamaterial, can be calculated according to the definition as a ratio between the lateral contraction strain and the axial elongation strain of the specimen:

\[ \nu = \frac{d}{d'} \frac{l}{l'} \]

The measurements that are required to perform in order to evaluate the Poisson ratio are explained in Fig. 4.

![Figure 4: Illustration of the Measurements for Evaluation of Poisson Ratio.](image)

MATHEMATICAL MODELS OF ALSI10MG AND 17-4PH

The virtual finite-element testing of the representative volume elements shall be based on the highly-adequate continuum models of the materials, which are used for 3D printing of the lattice metamaterials [17].

Since the additive materials may have noticeable anisotropy due to the difference between the mechanical properties within the plane of the printer platform and in the build direction, in current research the basic solid materials are described with the orthotropic elastic-plastic mathematical models with the plasticity according to Hill’s theory. The mechanical parameters that are used to define the models are listed in Table 1 and Table 2, the parameters of the Hill’s plasticity criteria are listed in Table 3. The plastic curves are shown in Fig. 5.
Application of Finite Element Homogenization Technique for Efficient Modeling of Additively Manufactured Metamaterials

Table 1: Mechanical Properties of AlSi10Mg

<table>
<thead>
<tr>
<th>Material Property</th>
<th>X Axis or XY Plane</th>
<th>Y Axis or YZ Plane</th>
<th>Z Axis or XZ Plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus (GPa)</td>
<td>82</td>
<td>77</td>
<td>77</td>
</tr>
<tr>
<td>Shear modulus (GPa)</td>
<td>23</td>
<td>24</td>
<td>25</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.350</td>
<td>0.348</td>
<td>0.346</td>
</tr>
</tbody>
</table>

Table 2: Mechanical Properties of 17-4PH

<table>
<thead>
<tr>
<th>Material Property</th>
<th>X Axis or XY Plane</th>
<th>Y Axis or YZ Plane</th>
<th>Z Axis or XZ Plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus (GPa)</td>
<td>186</td>
<td>178</td>
<td>193</td>
</tr>
<tr>
<td>Shear modulus (GPa)</td>
<td>68</td>
<td>62</td>
<td>61</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.300</td>
<td>0.300</td>
<td>0.300</td>
</tr>
</tbody>
</table>

Table 3: Parameters of Hill’s Plasticity Models for AlSi10Mg and 17-4PH

<table>
<thead>
<tr>
<th>Parameter Type</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>L</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlSi10Mg</td>
<td>0.517</td>
<td>0.559</td>
<td>0.424</td>
<td>1.444</td>
<td>1.293</td>
<td>1.384</td>
</tr>
<tr>
<td>17-4PH</td>
<td>0.507</td>
<td>0.515</td>
<td>0.478</td>
<td>0.952</td>
<td>0.940</td>
<td>1.063</td>
</tr>
</tbody>
</table>

Figure 5: Plastic Curves for AlSi10Mg and 17-4PH.

HOMOGENIZATION PROCEDURE

Modeling Process Description

The homogenization procedure for metamaterials in current research is developed and verified using SIMULIA Abaqus finite-element software. The loading of the representative volume elements is performed in the static mode assuming that the displacements are small. Each of the analyses described below runs for about thirty minutes.

For the sake of brevity the procedure of homogenization in this section is described for the metamaterial of Type 1 made of AlSi10Mg.

All the virtual tests are done using the same finite-element model of the basic cell of Type 1 shown in Fig. 6 and having the following characteristics:

- Element type – C3D4 (four-node linear tetrahedra);
- Number of elements – 202 148;
- Number of nodes – 41 850.
The simulations for the cell of Type 2 are done with a similar model. The typical fields of displacement vector magnitude for the cases of uniaxial tension and uniaxial shear are shown in Fig. 7 and Fig. 8.
The equivalent values of elastic and shear moduli can be calculated using the linear fragments of the stress-strain diagrams that are obtained from the virtual tension and shear tests.

The averaged stresses and strains that are required to perform the evaluation of the equivalent moduli are calculated in SIMULIA Abaqus based on the results of each of the virtual tests using a special script. The script is developed in Python and does the direct weighted integration of the components of stress and strain tensors within the volume of the analyzed basic cell. The result of this processing is a set of values: stresses and strains which were volume averaged and equivalent moduli. All these values are listed in Table 4. The results obtained well correlate with those known from the papers [18].

<table>
<thead>
<tr>
<th>Type of the Virtual Test</th>
<th>Average Stress (kPa)</th>
<th>Average Strain (%)</th>
<th>Equivalent Elastic Modulus (MPa)</th>
<th>Equivalent Shear Modulus (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension along X axis</td>
<td>1895</td>
<td>0.300</td>
<td>632</td>
<td>–</td>
</tr>
<tr>
<td>Tension along Y axis</td>
<td>1905</td>
<td>0.300</td>
<td>635</td>
<td>–</td>
</tr>
<tr>
<td>Tension along Z axis</td>
<td>1889</td>
<td>0.300</td>
<td>630</td>
<td>–</td>
</tr>
<tr>
<td>Shear in XY plane</td>
<td>996</td>
<td>0.060</td>
<td>–</td>
<td>1660</td>
</tr>
<tr>
<td>Shear in YZ plane</td>
<td>1080</td>
<td>0.060</td>
<td>–</td>
<td>1800</td>
</tr>
<tr>
<td>Shear in XZ plane</td>
<td>921</td>
<td>0.060</td>
<td>–</td>
<td>1535</td>
</tr>
</tbody>
</table>

The values of equivalent Poisson ratios calculated according to the definition are listed in Table 5.

<table>
<thead>
<tr>
<th>Type of the Virtual Test</th>
<th>Axial Strain (%)</th>
<th>Lateral Strain (%)</th>
<th>Poisson Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension along X axis</td>
<td>0.200 (Y)</td>
<td>0.092 (X)</td>
<td>0.462</td>
</tr>
<tr>
<td>Tension along Y axis</td>
<td>0.200 (Z)</td>
<td>0.091 (Y)</td>
<td>0.457</td>
</tr>
<tr>
<td>Tension along Z axis</td>
<td>0.200 (Z)</td>
<td>0.092 (X)</td>
<td>0.461</td>
</tr>
</tbody>
</table>

Accuracy of the homogenized model

The aim of this part of the research is to compare the mechanical behavior of the heterogeneous representative volume element with the mechanical behavior of the cube that is made of the equivalent homogenized material.

The comparison is carried out using a simple finite-element model of a homogeneous cube with the side of 2.5 mm, which is subjected to the same series of virtual tests as the heterogeneous basic cell.

The finite element model is shown in Fig. 9. The characteristics of the model are listed below:

- element type – C3D8 (four-node linear hexahedra);
- number of elements – 8 000;
- number of nodes – 9 261.
Figure 9: Finite-Element Model of a Homogeneous Cube for Performing the Accuracy Tests.

The boundary conditions that are used for loading the model are exactly the same as the conditions, which are used for carrying out the homogenization.

The comparison of direct and homogenized modeling of the basic cell in one of the load cases is shown in Fig. 10. The figure shows the results obtained from a uniaxial testing along X axis for the metamaterial of Type 1 made of AlSi10Mg. The curves for other types of metamaterials and loading conditions show the same correlation between the heterogeneous and homogeneous modeling. Thus, the demonstrated results are quite representative.

Figure 10: Comparison of Stress-Strain Curves for Uniaxial Tension along X Axis Performed for the Metamaterial of Type 1 made of AlSi10Mg.

VALIDATION

The aim of the validation is to compare the results of the natural tests, which are performed for relatively large lattice metamaterial specimens, with the results of finite-element modeling that is carried out both with the direct approach and with the approach using the equivalent homogenized model. Proposed specimens consisted of 5x5x5 basic cells. This size led to the assumption that it is large enough to consider lattice structure as continuous material. The comparison has two main goals:

- Confirmation of the fact that the natural experiments within the homogenization technique can be replaced by the finite-element virtual testing considering highly-adequate material models;
Confirming the accuracy of the evaluated mechanical characteristics of the equivalent homogenized material.

**Natural Testing**

The natural testing was performed by loading the test specimens with tension and compression. Since the metamaterials have complex lattice geometry, it is not possible to mount them in the machine directly, so the specimens were designed in such a way that they have special additional blocks for installing into the machine. The shapes of the specimens are explained in Fig. 11.

![Shapes of the Test Specimens for (a) Tension Testing and for (b) Compression Testing.](image)

The experiments were done with Instron 8850 Axial-Torsion System. The photographs of the deformed specimens are shown in Fig. 12.

![Deformed Specimens of the Metamaterial of Type 1 made of AlSi10Mg.](image)

**Direct Finite-Element Modeling**

The direct modeling of the virtual tension and compression tests is performed with the same finite-element models. The model of Type 1 is shown in Fig. 13.

The characteristics of the finite-element model are as follows:

- Element type – C3D4 (four-node linear tetrahedra);
- Number of elements – 1 156 665;
- Number of nodes – 273 733.

During the simulation the bottom face of the specimen is fully fixed, the top face has a prescribed axial displacement.
One run takes about five hours.

![Finite-Element Model for Direct Modeling of the Experiments.](image1)

**Figure 13: Finite-Element Model for Direct Modeling of the Experiments.**

**Modeling Based on the Equivalent Homogenized Material**

The reduced modeling utilizes a unified finite-element model shown in Fig. 14 both for tension and compression. The characteristics are listed below:

- Element type – C3D8 (four-node linear hexahedra);
- Number of elements – 21,761;
- Number of nodes – 24,552.

One run takes about thirty minutes.

![Finite-Element Model for Modeling of the Experiments with Equivalent Homogenized Material.](image2)

**Figure 14: Finite-Element Model for Modeling of the Experiments with Equivalent Homogenized Material.**

**Results**

The force-deformation curves obtained from the natural testing, direct modeling and modeling based on the equivalent homogenized material for the four analyzed metamaterials are presented in Fig. 15 – Fig. 18.
Figure 15: Force-Deformation Curve for Tension for AlSi10Mg Specimens.

Figure 16: Force-Deformation Curve for Compression for AlSi10Mg Specimens.

Figure 15: Force-Deformation Curve for Tension for 17-4PH Specimens.
Figure 16: Force-Deformation Curve for Compression for 17-4PH Specimens.

The figures show clearly that all the curves have good correlation in the linear range, which is the target range for the study. Obviously, the linear homogenized model is not applicable for the nonlinear range, but the direct model still behaves quite well even at high levels of deformation.

CONCLUSIONS

The paper presents the methodology of homogenization technique as applied to the metamaterials manufactured with 3D printing with metal powders. The method of developing homogenized material models allows evaluating the mechanical properties of the conventional solid material with the behavior that is equivalent to the macro-level behavior of the considered lattice metamaterial.

The article shows the theoretical aspects of the homogenization technique, explains the process of evaluation of the equivalent properties based on the metamaterial made of aluminium alloy. After that a comparison is done to confirm that the mechanical behavior of the homogeneous equivalent material is the same as the global behavior of the real metamaterial. The performed validation makes possible to prove that the results of the numerical modeling have a good correlation with the experimental data. The homogenized model works well in the linear range, and the direct model shows good results even at high levels of deformation.

From the practical point of view the discussed technique allows performing stress analyses for parts and assemblies that contain metamaterials much faster and efficiently: the direct simulation from the validation section runs for five hours, and the homogenized model runs ten times faster.

Results of the study show clearly that the proposed method of finite element homogenization is fully applicable for such structures as metamaterials. It provides a good correlation between the natural experiment results, direct modeling and homogenized models in the considered range of strain and stresses. The formulated methodology can be applied for developing and optimizing the mechanical parts and assembly components for such advanced industries as aviation, automotive, space and others. The obtained technology allows improving the process of design, analysis and optimization of the high-end components and systems.

REFERENCES


