NUMERICAL INVESTIGATION ON WING PERFORMANCE USING SINGULARITY METHOD

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ABSTRACT
The numerical method is represented to predict wing performance at low speed. In this paper, the wing is simulated with airfoil NACA 0012, which is divided to a large number of panels, each panel approximated with linear strength vortex to model the flow pattern around an airfoil. The pressure coefficient distribution around the airfoil is numerically obtained for the same speed and different angles of attack which results to calculate the lift and drag coefficients. This paper is focused on the verification of lift and drag coefficients, where very good agreements have been obtained with several well-known classical methods.

KEYWORDS: Linear Strength Vortex, Lift Coefficient, Drag Coefficient & Pressure Coefficient

INTRODUCTION
A cross sectional shape of wing is called airfoil whose aerodynamic characteristics influence the aircraft flying characteristics. Aerodynamic parameters of a wing of an infinite span are defined as airfoil characteristics, where the flow is purely two-dimensional (2D). Lift is generated due to the pressure difference between upper and lower surface of wing. As soon as we know airfoils aerodynamic characteristics that will be used in wing design.

Characteristics of airfoil are usually determined analytically, numerically and in wind tunnels experimentally or at least confirmed by wind tunnels, if computational methods had been used in airfoil design and analysis.

In reference [1], the airfoil NACA 0012 was approximated by sufficient number of panels with linear vortex strength to simulate real flow around airfoil and to calculate pressure coefficient i.e. lift and drag coefficients. Yao et al. [2] studied the aerodynamic performance of wind turbine airfoils and compared the numerical results with experimental data. Mragank et al. [3] evaluated lift and drag coefficients by simulation fluid flow around airfoil using FLUENT software package of ANSYS. Ankan Dash [4], numerically analyzed flow field around airfoil NACA 0012 at different angles of attack and compared the results with available experimental data. In reference [5] CFD study of airfoils by k-omega shear stress transport (SST) model to predict its lift and drag characteristics. Hess [6] is found the solution of flow field by simulating the surface by a number of panels, and solving algebraic set of linear equations to calculate the unknown strengths of the singularities. Devinant et al. [7] test data for different angle of attack. The experimental results show that the coefficient of lift increases with increasing angle of attack. Taha et al. [13] calculate the pressure distribution coefficient of airfoil numerically using linear vortex strength by dividing the airfoil to a sufficient number of panels.
FUNDAMENTAL ASSUMPTIONS

The following assumptions are adopted in this paper:

- The flow about a two-dimensional airfoil is inviscid and irrotational.
- The airfoil is represented by a sufficiently large number of linear vortex panels (Fig. 1).
- The air flow is subsonic.
- Three-dimensional effects are negligible

![Figure 1: Schematics of Airfoil Panelling.](image)

Governing Equations

The first assumption replaces Navier-Stokes equations by potential flow equation:

$$\beta^2 \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$$  \hspace{1cm} (1)

Correction parameter for compressibility is defined as:

$$\beta = \sqrt{1 - M^2}$$  \hspace{1cm} (2)

Where M is Mach number for free stream, and the velocity potential $\Phi$ for small disturbance perturbation has been defined as follows:

$$\Phi = U_\infty x + V_\infty y + \Phi$$  \hspace{1cm} (3)

Transforming $\Phi$ and $(\hat{x}, \hat{y})$ by equation:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \hat{x} \\ \beta \hat{y} \end{pmatrix}$$  \hspace{1cm} (4)

We obtain $\Delta \Phi = 0$, where Laplace operator is:

$$\Delta = \frac{\partial^2}{\partial \hat{x}^2} + \frac{\partial^2}{\partial \hat{y}^2}$$  \hspace{1cm} (5)

This equation is solved for free-stream conditions, using the superposition principle of singular solutions, since Laplace equation is linear. Any combination of singular solutions is also the solution of the Laplace's equation. Here our
task is to select arbitrary constants for singularity solutions that, besides satisfying boundary conditions, also satisfy Laplace's equation.

**Boundary Conditions**

- The normal component of the velocity at any point of the solid surface of airfoil must be equal to zero (i.e. on the control points of the panels) satisfy the following condition:

\[ V_n \cdot \hat{n} = 0 \]  \hspace{1cm} (6)

A control point is indicated by subscript \( i \) whose coordinates are, \( x_{ci}, y_{ci} \).

- The Kutta condition must be satisfied at the trailing edge to ensure velocity at the trailing edge is finite

**General Linear Vortex Distribution**

At some arbitrary point \((x, y)\) velocity induced along the segment by vorticity with linear strength variation (see Fig. 2) is calculated by applying the superposition principle [11]. By this principle, contribution of all vortices \( \gamma(x_0)dx_0 \) along vortex segment placed between \( x_1 \) and \( x_2 \) are added to obtain:

\[ u = \frac{1}{2\pi} \int_{x_1}^{x_2} \gamma(x_0) \frac{x_0 y}{(x - x_0)^2 + y^2} dx_0 \]  \hspace{1cm} (7)

\[ v = \frac{1}{2\pi} \int_{x_1}^{x_2} \gamma(x_0) \frac{x_0 (x - x_0)}{(x - x_0)^2 + y^2} dx_0 \]  \hspace{1cm} (8)

In local coordinate system these velocity components are expressed. For arbitrary position of the vortex segment, transformation the coordinates of the arbitrary point \((x, y)\) and the coordinates of the end points of the vortex to the coordinate system fixed to segment. By vorticity strengths \( \gamma_1 \) and \( \gamma_2 \) at segment’s end points vorticity distribution \( \gamma(x_0) \) is determined. From boundary conditions (see Eq. (6)) their magnitude is determined.

\[ \gamma(x_0) = \gamma_1 + \Delta(x_0 - x_1) \]  \hspace{1cm} (9)
Where
\[ \Delta = \frac{\gamma_2 - \gamma_1}{x_2 - x_1} \]

Integration of expressions for \( u \) and \( v \) gives induced velocity in the segment-fixed coordinate system by vortex segment at any point \((x, y)\).

\[
\begin{align*}
\frac{\gamma_2}{2\pi} (\theta_2 - \theta_1) + \frac{\Delta}{2\pi} \left[ y \ln \frac{r_2}{r_1} - x (\theta_2 - \theta_1) \right] \\
\frac{-\gamma_1}{2\pi} i n \frac{r_1}{r_2} - \frac{\Delta}{2\pi} \left[ x \ln \frac{r_2}{r_1} + (x_2 - x_1) + y (\theta_2 - \theta_1) \right]
\end{align*}
\]

Angles \( \theta_1 \) and \( \theta_2 \), as well as \( r_1 \) and \( r_2 \), are shown in Fig. 2.

Numerical Solution of the Flow about the Airfoil

In the Eqs. (10) and (11) the subscripts 1 and 2 refer to first and last point of a panel, globally defined by points numerated as \( j \) and \( j+1 \) respectively. In this work the airfoil NACA 0012 is given with \((N+1)\) pair of \((x, y)\) coordinates ordered counterclockwise, starting from the trailing edge of the airfoil. The airfoil shape is approximated by \( N \) panels connecting these \((N+1)\) point coordinates of the airfoil. In expressions for induced velocities, \( \gamma_1 \) and \( \gamma_2 \) are local parameters, unique for each panel. These coefficients are used to model vortex strength variation over the panel. If the strength of \( \gamma \) at the beginning of each panel is set equal to the strength of the vortex at the end point of the previous panel, the continuous vortex distribution is obtained.

All vortices \((\gamma_j, \gamma_{j+1}, \ldots)\) at the end points of the panels should determine by the numerical procedure, see Fig. 3. If the airfoil shape is approximated by \( N \) distributed vortex panels, then the number of unknown parameters is equal to the number of points which define vortex segments, i.e. \( N+1 \), one greater than number of panels. To apply Eqs. (10) and (11), subscripts 1 and 2 should be replaced by \( j \) and \( j + 1 \) respectively.
In local coordinate system the induced velocity components of the panel at \(i^{th}\) control point is expressed in terms of the panel-edge vorticity strengths \(\gamma_j\) and \(\gamma_{j+1}\). This way, Eqs. (10) and (11) become:

\[
\begin{align*}
    u_{ij} &= \frac{\gamma_j}{2\pi} \left( \theta_{j+1} - \theta_j \right) + \frac{\gamma_{j+1} - \gamma_j}{2\pi(x_{j+1} - x_j)} \left[ y_i \ln \frac{r_j}{r_{j+1}} - x_i (\theta_{j+1} - \theta_j) \right] \\
    v_{ij} &= -\frac{\gamma_j}{2\pi} \ln \frac{r_j}{r_{j+1}} - \frac{\gamma_{j+1} - \gamma_j}{2\pi(x_{j+1} - x_j)} \left[ x_i \ln \frac{r_j}{r_{j+1}} + (x_{j+1} - x_j) + y_i (\theta_{j+1} - \theta_j) \right]
\end{align*}
\]  

(12)  

(13)

Transformation of control point coordinates with respect to vorticity segment \(J\) to segment fixed coordinate system is seen by Eqs. (14) and (15):

\[
\begin{align*}
    X_{ij} &= (x_{ci}) \cos \alpha_j + (y_{ci}) \sin \alpha_j \\
    Y_{ij} &= (y_{ci}) \cos \alpha_j - (x_{ci}) \sin \alpha_j
\end{align*}
\]

(14)  

(15)

Where \((x_{ci}, y_{ci})\) are global coordinates of the control point, while \((X_{ij}, Y_{ij})\) are local coordinates of the control point, as viewed from the local coordinate system fixed to segment \(J\) (see Fig. 3). Starting points local coordinates are \((0, 0)\), while the end points local coordinates of the segment are given as shown in Eqs. (16) and (17).

\[
\begin{align*}
    X_{j+1} &= (x_{j+1} - x_j) \cos \alpha_j + (y_{j+1} - y_j) \sin \alpha_j \\
    Y_{j+1} &= (y_{j+1} - y_j) \cos \alpha_j + (x_{j+1} - x_j) \sin \alpha_j
\end{align*}
\]

(16)  

(17)

The segment slope with respect to global \(x\)-axis is \(\alpha_j\).

The distances between the end points and control point of the segment are:

\[R_{ij}, R_{i,j+1}\]

Angles between the lines connecting end of the segment with control point and segments are given as:

\[\theta_{ij}, \theta_{i,j+1}\]

A contribution to the induced velocity at control points is necessary to separate into parts influenced only by end segment vorticities. Eqs. (12) and (13) can be divided into a portion of velocity influenced by \(\gamma_{j+1}\) and a portion of velocity influenced by \(\gamma_j\). The superscripts \(\gamma_{j+1}\) and \(\gamma_j\) represent the contribution of the ending and the contribution of the beginning vorticity strength.

\[
\begin{align*}
    u_{ij}' &= \frac{\gamma_j}{2\pi} \left( \theta_{j+1} - \theta_j \right) + \frac{Y_{ij}}{X_{j+1}} \ln \frac{R_j}{R_{j+1}} - \frac{X_{ij}}{X_{j+1}} (\theta_{j+1} - \theta_j) \\
    u_{ij}'^{j+1} &= \frac{\gamma_{j+1}}{2\pi} \left( \frac{X_{ij}}{X_{j+1}} (\theta_{j+1} - \theta_j) - \frac{Y_{ij}}{X_{j+1}} \ln \frac{R_j}{R_{j+1}} \right) \\
    v_{ij}' &= \frac{\gamma_j}{2\pi} \ln \frac{R_j}{R_{j+1}} \left( X_{ij} - 1 \right) - \frac{Y_{ij}}{X_{j+1}} (\theta_{j+1} - \theta_j)
\end{align*}
\]

(18)  

(19)  

(20)
There represent the induced velocity components influenced by strengths of vorticity at the beginning and at the end of each segment. The calculations of the Eqs. (18), (19), (20) and (21) are based on the assumption that $\gamma_j = 1$ and $\gamma_{j+1} = 0$. At any point in the flow field in local (segment fixed) coordinate system the induced velocity is:

$$u_{ij} = u_i^j + u_i^{j+1}$$  \hspace{1cm} (22)$$

$$v_{ij} = v_i^j + v_i^{j+1}$$  \hspace{1cm} (23)$$

The Eqs. (18), (19), (20) and (21) can be arranged to separate vorticity strengths $\gamma_j$ and $\gamma_{j+1}$.

Into airfoil coordinate system, induced velocity components, calculated in segment fixed coordinate system, have to be transformed back, and summed up to determine induced velocity at a control point $(x_c, y_c)$ by the vorticity segment $J$:

$$u_{ij} = \sum u_{ij} \cos \alpha_j - \sum v_{ij} \sin \alpha_j$$  \hspace{1cm} (24)$$

$$v_{ij} = \sum u_{ij} \sin \alpha_j + \sum v_{ij} \cos \alpha_j$$  \hspace{1cm} (25)$$

In global coordinate system components of induced velocity at $(x, y)$ point are obtained in the flow field by transforming local induced velocity components, due to vortex segment between points $j$ and $j+1$, according to:

$$u_{ij} = \sum (f_{ij} \cdot \gamma_j + g_{ij} \cdot \gamma_{j+1}) \cos \alpha_j - \sum (w_{ij} \cdot \gamma_j + z_{ij} \cdot \gamma_{j+1}) \sin \alpha_j$$  \hspace{1cm} (26)$$

$$v_{ij} = \sum (f_{ij} \cdot \gamma_j + g_{ij} \cdot \gamma_{j+1}) \sin \alpha_j + \sum (w_{ij} \cdot \gamma_j + z_{ij} \cdot \gamma_{j+1}) \cos \alpha_j$$  \hspace{1cm} (27)$$

After rearrangement Eq. (26) can be written in the form:

$$u_{ij} = a_{ij} \cdot \gamma_j + b_{ij} \cdot \gamma_{j+1}$$  \hspace{1cm} (28)$$

For Eq. (27) similarly:

$$v_{ij} = k_{ij} \cdot \gamma_j + s_{ij} \cdot \gamma_{j+1}$$  \hspace{1cm} (29)$$

Due to vorticity strength $\gamma_j$ components of induced velocity are given by the Eqs. (30) and (31):

$$u_i^{j-1} = (a_{ij} + b_{ij+1}) \gamma_j = A_{ij} \cdot \gamma_j$$  \hspace{1cm} (30)$$

$$A_{ij} = \begin{cases} a_{i1} & j = 1 \\ a_{ij} + b_{i,j-1} & j = 2 \ldots N \\ b_{ij} & j = N + 1 \end{cases}$$  \hspace{1cm} (31)$$
Where \( a_{i1} \) and \( b_{i1} \) represent the coefficients of first and last segment.

\[
v_{i,j}^{j-1} = (k_{ij} + s_{i,j-1})\gamma_j = B_{ij} \gamma_j
\]  \hspace{1cm} (32)

\[
B_{ij} = \begin{cases} 
  k_{i1} & j = 1 \\
  k_{ij} + s_{ij} & j = 2 \ldots N \\
  s_{ij} & j = N + 1
\end{cases}
\]  \hspace{1cm} (33)

At a control point \((x_{ci}, y_{ci})\) the total velocity is obtained when all contributions are summed up:

\[
u_i = \sum_{j=1}^{N+1} A_{ij} \gamma_j + U_{\infty}
\]  \hspace{1cm} (34)

\[
v_i = \sum_{j=1}^{N+1} B_{ij} \gamma_j + V_{\infty}
\]  \hspace{1cm} (35)

Require boundary conditions that the normal velocity component to the surface of airfoil at arbitrary control point \(i\) is equal to zero:

\[
\vec{v}_{i} \cdot \hat{n}_i = u_i n_{xi} + v_i n_{yi} = 0
\]  \hspace{1cm} (36)

There are \(N\) segments and thus \(N\) conditions, since the control point is defined in the middle of the segment. From Kutta condition additional necessary condition is obtained:

\[
\gamma_1 + \gamma_{N+1} = 0
\]  \hspace{1cm} (37)

The coefficients of linear vortex strength panels are determined from the solution of equations system.

Now, as the vorticity strengths \(\gamma_j\) are known for all panels, then at each control point the induced velocity can be easily calculated by Eqs. (22) and (23). The pressure, drag and lift coefficients can be calculated.

By the custom written Fortran 90 computer code the calculations are performed.

RESULTS AND DISCUSSION

In the numerical method applied her, the flow is considered incompressible and in viscid flow and the results is compared with experimental data after numerically correction from wind tunnel T-38 (VTI Žarkovo, Belgrade)see reference [8] and Transonic cryogenic tunnel (0.3-m NASA Langley TCT) see reference [9]. The numerical results also compared with analytical method for Abbott, and Doenhoff see reference [10].

Pressure Distribution

The coefficient of pressure distribution is obtained from her applied numerical method for airfoil NACA 0012 with Mach number \(M = 0.3\) and angles of attack \(\alpha = 2^\circ, 4^\circ\) and \(6^\circ\).

The result is compared with pressure distribution coefficient of mentioned two wind tunnel experimental data as shown in Fig. 4, 5, and 6.
It can be seen that the increasing of angles attack, the increasing of pressure coefficient distribution, and leads to increase the velocity on the upper surface of wing and decrease it on the lower surface, therefore this difference between velocities (i.e. pressure difference) lead to create the lift and drag forces. The large difference in pressure coefficient seems in the leading edge of airfoil while very small difference in trailing edge.

The numerical methods applied in this work indicate a good correlation between the results obtained from numerical method and the data from wind tunnels experiment.

![Figure 4: Pressure Coefficient Distribution.](image)

![Figure 5: Pressure Coefficient Distribution.](image)
Lift and Drag Coefficients

The lift and drag coefficients are calculated at Mach number \( M = 0.3 \) and angles of attack \( 2^\circ, 4^\circ \) and \( 6^\circ \), using numerical method with linear vortex panels. Figures 7 and 8 compares lift and drag coefficients curves for inviscid flow obtained by her applied numerical method and analytical method from Abbott and von Doenhoff [10] and experimental coefficients obtained from two wind tunnels (T-38 and NASA) references [8] and [9]. Good agreement is obtained at low angles of attack as a result of fully attached flow.
CONCLUSIONS

The airfoil 0012 has been divided to a large number of panels, each panel simulated by linear vortex strength to calculate the pressure distribution coefficient (i.e. lift coefficient) for Mach number $M = 0.3$ and different angles of attack.

The numerical method applied in this work has been verified by comparing the obtained results with well-known analytical method and two experimental wind tunnel data mentioned before and there was a good compatibility between her applied method and the methods that were compared.

The linear vortex strength panels enhance this numerical scheme and make it an efficient tool and reduce the computational time. This methodology is suitable for calculating wing performance. In the future work, to three-dimension model this method will be extended.

REFERENCES


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