WAVE-LIKE LOCOMOTION OF A SNAKE ROBOT: MATHEMATICAL MODELLING AND SIMULATION

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ABSTRACT

As a high redundancy system with high adaptability and stability, rescue robot is widely employed in collapsed buildings for rescue and search work. The work presented here introduces the wave-like locomotion of the snake robot using the modified serpenoid curve for the optimal motion. A systemic complete description of kinematics and dynamics of a snake robot are presented. VREP (virtual Robotic Experimental Platform) is used for simulation to validate the mathematical models and obtained joint torques of the snake robot during the wave-like locomotion.

KEYWORDS: Serpenoid Curve, Biomimetic, Snake Robot & Wave-like Locomotion

1. INTRODUCTION

It has been said that, when you have a conversation in 2050. You will be having a conversation with someone that’s a combination of artificial and biological intelligence. There is a substantial movement, towards the robotics and artificial intelligence as evolutionary, in large part because it is such a sociological leap. A short few years ago, Asimo (Honda) appeared like the only horse, in the contest to build a bipedal robot, one with the walking ability and all-terrain adaptability like a two-legged human[1]. Today, the field of robotics is rife with competitors and several efforts have been made, to design biologically inspired robots, such as Petman[2], Cheetah[3], Uncle sam[4], WSL[5], TBCP-11[6] and etc. Snake robots are very common bio mimetic robots, which often have a high number of degrees of freedom, and they are capable of locomoting without using legs or wheels.

Snake robots provide effective assistance, in areas such as rescue missions, fire fighting and maintenance, due to their ability to move through constricted spaces. These robots are capable of bend and adapt, to the various environments. Their area of cross-section is smaller than their length, which facilitates them to enter into the small orifices and tubes, and get to the various inaccessible to other kinds of robots. The most well-known gaits, used by snakes are rectilinear, serpentine, concertina and side winding locomotion [7].

Depending on the environment, basic gaits of snakes may combine and create various new locomotion modes. The non-snake-like gaits typically do not exist in nature but are employable in locomotion of snake robot. The wave-like locomotion, spinning, flapping gaits are few examples of such gaits. Serpentine locomotion is the frequently employed form of snake locomotion, which is used by real snakes. Hirose et al.[7]described that the main property of snakes in mimicking serpentine locomotion is the variation in coefficients of friction for the normal and tangential directions with respect to the snake body. Thus, the snake robot typically requires wheels to realize friction. This requirement confines adaptability of the snake robot to the environment. Hirose formulated
the serpenoid curve and showed that a snake-like mechanism is capable of generating a net forward thrust by applying the torque along the length of its body. Chen et al.\[8\] demonstrated a model for snake wave like locomotion and presented the kinematics and dynamics analysis. They were considered the effects of the friction coefficient and initial winding angle on the joint torques during the wave-like locomotion. Wave-like locomotion can divide into the four different types of phases. The phases are developed based on the total number of joints contacting the ground and the friction forces resultant on contacting joints. In another literature, snake robot has been fabricated without wheels. This robot is capable of adapting to different environments at the cost of increased power usage. They attained total equations of motions for a snake robot travelling with horizontal wave-like locomotion[9]. Me et al.\[10\] also derived the formulation of the snake robot kinematics and dynamics to analyse the creeping locomotion. They established a comparative study between the creeping locomotion and sinus-lifting locomotion to investigate the motion efficiency. Me et al.\[11\], employed Euler and Newton to derive the equations of dynamics, for n-link snake robot creeping, on a slope. They implemented both symmetrical and unsymmetrical body shapes. They have concluded that the robot performance increases with the unsymmetrical body. Kalyani et al.\[12\], presented a framework for kinematics and dynamics of the snake robot, to analyse the wave-like locomotion, along a symmetrical and unsymmetrical curve of the body. They studied the effects of coefficient of friction, unsymmetrical factor and winding angle on the joint torques. They also described novel kinematics modelling technique for wave-like locomotion. Webots software was used to validate their described technique.

In over the last thirty years, an intense research has been carried out, to develop the snake robots with many degrees of freedom[7]. Various types of limbless motion were attained in snake robots[7], [13], [14]. Two significant planer models, travelling wave like locomotion (Vertical locomotion) and serpentine locomotion (horizontal locomotion) are often implemented in snake robot[15]–[17]. Wave-like locomotion formed in a vertical plane, orthogonal to the supporting plane has great potential for adaptability, to the surrounding environment, without consideration of friction conditions. On the contrary, the serpentine locomotion developed in the supporting plane, and its driving forces generates from the various friction coefficients of the body in the normal and tangential directions, with respect to the supporting plane[18].

Typically, in the horizontal locomotion, the width of the motion is decided by the amplitude of the snake body shape. However, in the vertical locomotion, the width of the motion is nearly equal to the snake body section. Thus, vertical locomotion has few advantages over the horizontal locomotion, in conditions of narrow environments. Furthermore, in the vertical locomotion snake robot can regulate the body shape and also, every joint’s torque to deliver a proper driving and supporting forces. On the other hand, in the horizontal locomotion, this can be attained only by controlling the robot body shape. Moreover, it has been described that, 3D locomotion may be attained through the combination of vertical and horizontal locomotion’s [19]. However, wave-like locomotion is often considered as low efficient gaits because of obscure in its dynamics analysis[20]. In this paper, firstly we derived body shape of the snake robot by using modified serpenoid curve. Secondly, we have developed the kinematics of the snake robot and calculated the displacement, velocity and acceleration of the gravity centre of the snake robot body. Thirdly we have developed the dynamics of the snake robot. At end, modelled robot implemented in the V-REP software for the simulation and results are presented.

2. WAVE-LIKE LOCOMOTION OF THE SNAKE ROBOT USING MODIFIED SERPENOID CURVE

Movements of snakes are classified into four types of motions: rectilinear, serpentine, side winding and
concertina. The serpenoid motion was considered as the efficient and economic motion for the snake robots in terms of motor torques, friction forces and energy. However, the conventional serpenoid motion is based on the fixed constraints. Alternating the constraints, to achieving optimal motion leads to the robot parts slip. Using the serpenoid motion is no longer has been considered as the optimal, due to the side slipping. Modified serpenoid motion equations, were developed for the optimal motion even when the discrete or continuous change in the constraints. In this article, we have implemented the modified serpenoid curve as the body shape of the snake robot during wave like locomotion. The modified serpenoid curve is given by the:

The distance covered from the origin to a point on serpenoid curve or arc length is given by the equation:

\[ c = \int_0^t \sqrt{1+ z'}^2 dx = \int_0^t \omega dt \quad \text{where} \quad \omega = \frac{dc}{dt} \]  

(1)

Defining all constraints based on s, the curve will have various characteristics with every point. It is sufficient to define tangential angles to present the curve based on the length of the arc. All points can be then stated by

\[ x = \int \cos \phi(c) ds \quad z = \int \sin \phi(c) ds \]  

(2)

The angles including the variable constraints are represented as

\[ \phi(c) = p(c) \cos \left( \int_0^c q(\sigma) d\sigma + B_0 \right) + \left( \int_0^c r(\sigma) d\sigma + C_0 \right) \]  

(3)

Where p, q and r are constraints of serpenoid; various types of serpenoid curves can be defined by varying them. p, q and r specify the undulation, periods and angular speed, respectively. \( \sigma \) specifies the position on curve and speed of motion can be represented, by the speed of changing in \( \sigma \).

Figure 1 representing three continuous changes in constraints and Figure 2 representing discrete changes, in constraints as follows:

\[ p(c) = \frac{\pi}{2} + \frac{\pi}{6} \sin(2c) \]

\[ q(c) = 5\pi + 2\pi \sin(c) \]

\[ r(c) = \sin \left( \frac{5}{2} \right) \]
Equation (3) is appropriate for a continuous snake body on the other hand, for a real robot snake consisting of solid segments, a discrete approximation is required. For a robot to track such a curve, the joint angles can be considered as control inputs. Thus, an equation must be given to estimate the joint angles, using (3).

3. KINEMATICS OF THE SNAKE ROBOT

Consider a snake robot as shown in Fig., consisting of \( n \) links, connected through \( n-1 \) joints. Each link is facilitated with a servo motor (torque actuator) and is rigid with uniformly distributed mass. Snake robot has base frame O-X-Y and each link has represented with its own local coordinate system \( o_i . x_i , y_i \) on the joint. The robot fixed on supporting plane can mobilize, in the vertical plane. In this instance, the robot system consists of total \( n+2 \) degrees of freedom, among 2 for the position, \( n+2 \) for the shape, and \( d \) for the orientation. If the base frame O-X-Y is fixed in the inertial frame, consider the \( (i+1)_{th} \) link of the robot. The position, velocity and the acceleration of the \( (i+1)_{th} \) joint, relative to that of \( i_{th} \) joint are represented as follows:

\[
x_{i+1} = x_i + d \cos \phi_i \\
y_{i+1} = y_i + d \sin \phi_i
\]

(1)

(2)

Where, \( x_{i+1} \) and \( y_{i+1} \) represents the position of the \( i \)th joint, with X and Y axis, respectively.

\[
x_{i+1} = x_i - d \sin \phi_i \dot{\phi}_i \\
y_{i+1} = y_i + d \cos \phi_i \dot{\phi}_i
\]

(3)

(4)

Where, \( x_{i+1} \) and \( y_{i+1} \) represents the velocity of the \( i \)th joint with X and Y axis, respectively

\[
x_{i+1} = x_i - d \cos \phi_i \dot{\phi}_i^2 - d \sin \phi_i \ddot{\phi}_i \\
y_{i+1} = y_i - d \sin \phi_i \dot{\phi}_i^2 + d \cos \phi_i \ddot{\phi}_i
\]

(8)

(9)
Where, \( x_{i+1} \) and \( y_{i+1} \) represents the acceleration of the \( i_{th} \) joint with X and Y axis, respectively.

Where, \( i = 1, 2, 3, 4, 5, \ldots, n \)

Likewise, the position, velocity and acceleration of the gravity centres of the \( i \) link can be derived, as follows:

\[
x_{gi} = x_i + \frac{d}{2} \cos \phi_i
\]

\[
y_{gi} = y_i + \frac{d}{2} \sin \phi_i
\]

Where, \( x_{ci} \) and \( y_{ci} \) represents the position of gravity centre of the \( i_{th} \) joint, with X and Y axis, respectively.

\[
x_{ci} = x_i - \frac{d}{2} \sin \phi_i \phi_i
\]

\[
y_{ci} = y_i + \frac{d}{2} \cos \phi_i \phi_i
\]

Where, \( x_{gi} \) and \( y_{gi} \) represents the position of gravity centre of the \( i_{th} \) joint, with X and Y axis, respectively.

\[
x_{gi} = x_i - \frac{d}{2} \cos \phi_i \phi_i^2 - \frac{d}{2} \sin \phi_i \phi_i
\]

\[
y_{gi} = y_i - \frac{d}{2} \sin \phi_i \phi_i^2 + \frac{d}{2} \cos \phi_i \phi_i
\]

Where \( x_{gi} \) and \( y_{gi} \) represents the position of gravity centre of the \( i_{th} \) joint, with X and Y axis, respectively.

Where \( i = 1, 2, 3, 4, 5, \ldots, n \)

The snake robot designed with \( n \) links, thus, the gravity centre of the entire snake robot can be derived as:

\[
M_t X_C = \sum_{i=1}^{n} m x_{gi} \quad M_t Z_C = \sum_{i=1}^{n} m z_{gi}
\]

Where, \( M_t \) represents the total mass of the snake robot. \( X_C \) and \( Y_C \) represents the position of the mass centre of the robot, with X and Y axis, respectively.

Using the equations (10) - (16), the position, velocity and acceleration of gravity centre of the entire snake robot body can be derived as follows:

\[
X_C = \frac{\sum_{i=1}^{n} m x_{gi}}{M_t} = \frac{1}{n} \sum_{i=1}^{n} x_i
\]

\[
y_{gi} = y_i + \frac{d}{2} \cos \phi_i \phi_i
\]
\[ Y_C = \sum_{i=1}^{n} \frac{m y_i}{M_i} = \frac{1}{n} \sum_{i=1}^{n} y_i \]  

(18)

Where, the \( X_C \) and \( Y_C \) represents the position of the gravity centre of the snake robot body, with X and Y axis, respectively.

\[ \dot{X}_C = \sum_{i=1}^{n} \frac{m x_i}{M_i} = \frac{1}{n} \sum_{i=1}^{n} \dot{x}_i \]  

(19)

\[ \dot{Y}_C = \sum_{i=1}^{n} \frac{m y_i}{M_i} = \frac{1}{n} \sum_{i=1}^{n} \dot{y}_i \]  

(20)

Where, the \( \dot{X}_C \) and \( \dot{Y}_C \) represents the velocity of the gravity centre of the snake robot body, with X and Y axis, respectively.

\[ \ddot{X}_C = \sum_{i=1}^{n} \frac{m \ddot{x}_i}{M_i} = \frac{1}{n} \sum_{i=1}^{n} \ddot{x}_i \]  

(21)

\[ \ddot{Y}_C = \sum_{i=1}^{n} \frac{m \ddot{y}_i}{M_i} = \frac{1}{n} \sum_{i=1}^{n} \ddot{y}_i \]  

(22)

Where, the \( \dddot{X}_C \) and \( \dddot{Y}_C \) represents the acceleration of the gravity centre of the snake robot body, with X and Y axis, respectively.

4. DYNAMICS OF THE SNAKE ROBOT

The dynamics of the snake robot is considered to be a combination of mechanism dynamics and environment constraints.

![Figure 3: Forces Acting on the Link of the Snake of Robot](image-url)
Consider the force diagram of the $i_{th}$ link of the snake robot, that is illustrated in Fig.3. Where, $N_{i,j+1}$ and $F_{i,j+1}$ are the supporting forces and friction forces, of the $i_{th}$ link and $(i+1)_{th}$ link, at the $(i+1)_{th}$ joint. The torques, internal forces, mass and moment of inertia of the $i_{th}$ link are, represented as $T_i, f_i, m_i, and I_i$, respectively. Finally, based on the newton’s second law, the motion for the $i_{th}$ link, with respect to the frame of reference on the $(i+1)_{th}$ link can be described as:

$$F_i + F_{i,j+1} + f_{ix} - f_{i+1x} = m_i \ddot{x}_i$$  \hspace{1cm} (23)  

$$N_{i,j} + N_{i,j+1} - mg + f_{iy} - f_{i+1y} = m_i \ddot{y}_i$$  \hspace{1cm} (24)  

$$\tau_i - \tau_{i+1} - (f_{iy} + f_{i+1y} + N_{i,j} - N_{i,j+1}) \cos \Theta_i/2 + (f_{ix} + f_{i+1x} + F_{i,j} - F_{i,j+1}) \sin \left( \Theta_i/2 \right) = I_i \ddot{\Theta}_i$$  \hspace{1cm} (25)  

As, the head and the tail of the snake robot are free, there are two equations:

$$f_{ix} = f_{iy} = f_{n+1x} = f_{n+1y} = 0$$  \hspace{1cm} (26)  

$$\tau_1 = 0 = \tau_{n+1}$$  \hspace{1cm} (27)  

From the above equations (23), (24), (26). We obtain,

$$\sum_{i=1}^{n+1} F_i = \sum_{i=1}^{n} m_i \ddot{x}_i$$  \hspace{1cm} (28)  

$$\sum_{i=1}^{n} m_i \ddot{y}_i = 0$$  \hspace{1cm} (29)  

On substituting the equations (14),(15), into (28) and (29) we obtain:

$$n \ddot{x}_1 - \sum_{i=1}^{n} \left[ \sum_{k=1}^{i-1} (d \cos \Theta_k \Theta_k^2 + d \sin \Theta_k \Theta_k) + \frac{d}{2} \cos \Theta_k \Theta_k^2 + \frac{d}{2} \sin \Theta_k \Theta_k \right] = \sum_{i=1}^{n+1} F_i$$  \hspace{1cm} (30)  

$$n \ddot{x}_1 - \sum_{i=1}^{n} \left[ \sum_{k=1}^{i-1} (d \sin \Theta_k \Theta_k^2 - d \cos \Theta_k \Theta_k) + \frac{d}{2} \sin \Theta_k \Theta_k^2 - \frac{d}{2} \cos \Theta_k \Theta_k \right] = 0$$  \hspace{1cm} (31)  

Equations (30) and (31), represent the relation between the acceleration of the head link 1 and the angular acceleration.

The generalized formulae, in Equations (23) and (24), from the tail to the head, the relation between the internal force and external force, are obtained for each and every link.

$$f_{xi} = \sum_{j=1}^{n} m \ddot{x}_j - \sum_{j=i+1}^{n+1} F_j - F_{i,j}$$  \hspace{1cm} (32)  

$$f_{yi} = \sum_{j=1}^{n} m \ddot{y}_j - \sum_{j=i+1}^{n+1} N_j - N_{i,j} + (n + 1 - i) mg$$  \hspace{1cm} (33)  

Consider the torque equation (25) and substituting (32) and (33) into it, we have:

$$\tau_i - \tau_{i+1} + \left( 2 \sum_{j=i}^{n} m \ddot{x}_j - 2 \sum_{j=i+1}^{n} F_j - m \ddot{x}_i \right) \cos \Theta_i/2 - \left( 2 \sum_{j=i}^{n} m \ddot{y}_j - 2 \sum_{j=i+1}^{n} N_j - m \ddot{y}_i \right) + (2n + 1 - 2i) mg \right) d \Theta_i/2 = I_i \ddot{\Theta}_i$$  \hspace{1cm} (34)  

Where, $F_i$ and $N_j$ represent the friction force of the $j_{th}$ joint and the following conditions are satisfied.

$$F_j = F_{i,j} + F_{j-1,j}$$  \hspace{1cm} (35)  

$$N_j = N_{i,j} + N_{j+1,j}$$  \hspace{1cm} (36)  

And, $F_i$ and $N_i$ are environmental constraints that are analysed in the next section.
Finally, on simplifying the equation (34) we obtain.

\[ \sum_{i=1}^{n} \left( 2 \sum_{j=i+1}^{n} m \dddot{x}_j - 2 \sum_{j=i+1}^{n} F_j + m \dddot{x}_i \right) \sin \Theta_i/2 - \left[ 2 \sum_{j=i+1}^{n} m \dddot{y}_j - 2 \sum_{j=i+1}^{n} N_j + m \dddot{y}_i + (2n + 1 - 2i)mg \right] \cos \Theta_i/2 = \sum_{i=1}^{n} l_i \dddot{\Theta}_i \]  

(37)

The above equation (34) is a linear equation with one unknown variable \( \Theta_1 \). On solving this equation, the rotational acceleration of the first joint can be obtained. And substituting it into equations (30) and (31), the linear acceleration of the head link, i.e. link 1 can be correspondingly derived. The rotational velocity, angle and moving velocity of the first joint can be obtained, through integration. Finally, the joint torques can be obtained, by substituting the values in equation (34).

4.1. Dynamic Analysis with Environmental Constraints

In this subject of dynamics, deals with the calculation of interaction forces, between the environment and the snake body. To derive these forces, the supporting force and the friction force are needed, which are a function of the kind of locomotion of the snake body, the number of segments and the shape number of the body.

![Wave-Like Motion of the Snake Robot Body in One Segment](image)

**Figure 4: Wave-Like Motion of the Snake Robot Body in One Segment**

The above Figure 4, shows the travelling wave-like motion of the body, in one segment. In this case, it is assumed that, there are at least two supporting points with the ground. The joints contacting the ground are u and u’. The body is considered to be stable, if the centre of mass lies in between the two supporting joints.

Considering the FBD of the above body, the external force acting on the snake body are the gravity force \( G \), Normal or supporting force \( N \) and the friction force \( F \). The supporting forces balance out the gravity and the driving force of the snake is the friction force. Based on the moment and force balance principle, we obtain:

\[ N_u = \frac{1}{\lambda} \left[ (G + M_i \ddot{Y}_c)(x_c - x_u) - M_i \dddot{x}_c \right] \]  

(38)

\[ N_{u'} = G - N_u \]  

(39)

Here, \( \lambda \) is the distance between two supporting points. \( M_i \dddot{x}_c \) and \( M_i \dddot{y}_c \) are the inertial forces of the body, along the X-axis and Y-axis. In this analysis, viscous friction of the environment is neglected and only, the coulomb friction model is used, to represent the environment dynamics:

\[ F_l = N_c (\mu \cdot sgn(\dot{V})) \]
Where, $\mu$ is the friction coefficient between the contact point and the supporting plane. From the above equations, it can be observed that, the driving force which is the friction force is related with the position and the acceleration of the snake robot. Thus, the snake robot is controlled by its body shapes and joint torques, according to the kinematics and mechanism dynamics, jointly. In the further section, we analyse the obtained simulation results and discuss the relationship between the joint torque and the body shape, along with the environmental coefficient.

5. SIMULATION OF THE SNAKE ROBOT USING VREP SOFTWARE

The design and control methodology are very significant elements, in the robot simulators. In recent years, the robots are operating and employing with extremely complex control methods, when compared to the robots in the earlier generations. Virtual Robot Experimentation Platform (V-REP) is known as the Swiss army knife, among the simulators in robotics, as there isn’t any simulator with such number of functions, features or more elaborate API’s. V-REP is based on a distributed control architecture, where each object can be individually controlled, using an embedded script, a plug in, a Robotic Operating System (ROS) node, a remote API client, or a custom solution. Controllers can be written in various languages like C/C++, Python, Java, Lua, Matlab, Octave or Urbi. A child script is a simulation script, which represents a small code or program, written in Lua facilitating handling of a particular function, in a simulation. Child scripts are attributed to a certain object, in the scene and are recognized from their script icon, in the scene hierarchy.

In this present work, V-REP software is used for the simulation. The designed model of the snake robot imported from the solid works to the V-REP software shown in Fig.5. The snake robot consists of 10 links and 9 joints and each link is equipped with a servo motor, using the revolute joint shown in Fig.6. Every two links of robot forms one respond unit shown in Fig.7. A snapshot of the snake robot simulation is shown in Fig.8. Graphical representation (Fig.9) has implemented, to display the amount of torque acting on the each joint of the snake robot. The values of torque for joints are obtained from the V-REP simulation. As can be seen, maximum joint torques for joints 4, 2, 1, 9, 6, 8, 7 are in increasing order except joint, which displays lesser torque, due to the joint 5 is the nearest joint to the centre of gravity, the required maximum torques increases.

![Figure 5: Designed Model of the Snake Robot](image-url)
Figure 6: Link of the Snake Robot Equipped with Servo Motor

Figure 7: Respondable Unit of the Snake Robot

Figure 8: Simulation of the Snake Robot

Figure 9: Amount of the Torque Acting on the Each Joint of the Snake Robot
6. CONCLUSIONS

In this study, starting with the analysis of the wavelike locomotion, then moving on to the kinematics and dynamics involved, within the motion and finally the simulation. The results obtained from the theoretical models were then validated through simulation. The Joint torques obtained from the simulation study were analysed and then, concluded that, the different joints follow a periodic curve for the motion. The initial torque of the joint 7 is found to be high, among other joints and continues to reach higher torque, compared to other joints in the robot.

REFERENCES


