International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)
ISSN (P): 2249-6890; ISSN (E): 2249-8001
Vol. 7, Issue 5, Oct 2017, 63-74

BUCKLING ANALYSIS OF LAMINATED COMPOSITE PLATES USING LAYERWISE HIGHER ORDER SHEAR DEFORMATION THEORY

S. LOKESH[1], GS VIVEK[2] & L. PRAVEEN[3]

[1,2]Department of Mechanical Engineering, Institute of Aeronautical Engineering, Dundigal, Hyderabad, India

[3] Department of Mechanical Engineering, MLR Institute of Technology, Dundigal, Hyderabad, India

ABSTRACT

This paper represents the investigation on the response of symmetric laminated composite plate. In this paper, an attempt is made to study the Buckling characteristics of a laminated composite plates, using layer wise HSDT. Buckling is one of the major modes of failure of laminated structures and it is necessary to predict the critical load of the structural component, for the easy replacement with good load bearing capacity. Studying buckling characteristics of laminated composite plates is essential, in order to have a deep insight of their behaviour for proper analysis and design. The derivation of equations of motion for layer wise higher order models is obtained, using the virtual work principle. Further, the analytical solutions are developed to find buckling characteristics of laminated composite plates. The solution predicts critical buckling load of laminated composite plate. The results obtained by using the present model are compared with the available literature.

KEYWORDS: Laminated Composite Plate, Layer Wise Higher Order Shear Deformation Theory, Buckling Analysis & Cross-Ply, Angle-Ply Laminates

Received: Jul 20, 2017; **Accepted**: Aug 10, 2017**; Published**: Aug 29, 2017; **Paper Id**.: IJMPERDOCT20177

Original Article

1. INTRODUCTION

Laminated composites with continuous fibres are widely being used in various engineering fields, such as aeronautical and aerospace industry, marine, aviation, civil, sport, as well as in other fields of modern technology and other applications. They are preferred due to their characteristics like, high stiffness to weight ratio, excellent fatigue strength, high energy absorption, self damping capacity, high strength to weight ratio, low weight, resistances to electrochemical corrosion and good resistance to corrosive agents, capable of being engineered according to requirements. It is a challenging task, to find the accurate prediction of the response characteristics of composite structures. Hence, it is necessary to analyze the buckling characteristics of laminated composite plates. In the present paper, the displacement model is developed using the Layer wise HSDT. The equations of motion are derived using the principle of virtual work. The solutions are obtained using Navier's method. Later buckling characteristics are determined using Layer wise HSDT. Finally, the accuracy of the proposed theory is verified based on the results available.

2. THEORETICAL FORMULATION AND DEVELOPMENT OF LAYERWISE HSDT

There has been a considerable progress in understanding behaviour of composite laminates. It is noticed that, anisotropic multilayered structures posses transverse discontinuous mechanical properties and higher transverse shear and transverse normal stress deformability. In order to model materials with such behaviour, two

different approaches have aroused, they are equivalent single-layer theories (ESL) and layerwise theories (LWT). In order to overcome the limitations of CLPT and FSDT, Higher-order Shear Deformation Theories (HSDT), which involve higher-order terms in Taylor's expansion of the displacements in the thickness coordinate were developed. These models can disregard shear correction factors and give more accurate and stable transverse shear stresses. It is noticed that, LWT models have some advantages over the conventional 3D models. First, they allow independent in-plane and through the thickness interpolation, the element stiffness matrix can be computed much faster. Second, even the volume of input data is reduced, LWT are capable of achieving the same level of solution accuracy, as a conventional 3D models.

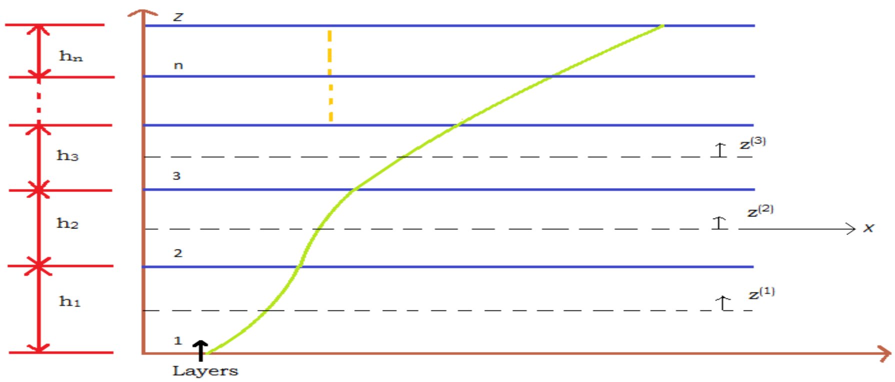

Figure 2.1: Layer Wise Kinematics of 'N' Layer Laminated Composite Plate

A rectangular plate having width a, length b and thickness h along x, y and z axes, respectively is considered. Figure 2.1, shows the one dimensional layer wise kinematics of three layered composite. The displacement components $u^{(2)}$, $v^{(2)}$ and $w^{(2)}$, along x, y and z directions, respectively for the middle layer are expanded using, Taylor's series in terms of thickness coordinate $z^{(2)}$.

Displacement Field

$$u^{(k)}(x,y,z) = u_0(x,y) + z^{(k)}\theta_x(x,y) + \left(z^{(k)}\right)^2 u_0{}^*(x,y) + \left(z^{(k)}\right)^3 \theta_x{}^*(x,y) + \Phi_x$$

$$v^{(k)}(x,y,z) = v_0(x,y) + z^{(k)}\theta_y(x,y) + \left(z^{(k)}\right)^2 v_0{}^*(x,y) + \left(z^{(k)}\right)^3 \theta_y{}^*(x,y) + \Phi_y$$

$$w^{(k)}(x,y,z) = w_0(x,y)$$

$$\text{Where } \Phi_m = \begin{cases} \left(\sum_{i=\text{middle layer}}^{n} \frac{h_i}{2}\theta_{(m)}^{(i)}\right) \text{ for layers above middle layer} \\ -\left(\sum_{1}^{i=\text{middle layer}} \frac{h_i}{2}\theta_{(m)}^{(i)}\right) \text{ for layers below middle layer} \end{cases}$$

Where u_0, v_0 and w_0 are the displacements of the middle plane along x, y and z directions, respectively. The parameters θ_x and θ_y are the rotations of normal to middle plane of kth layer, about x and y axes, respectively (k=1, 2, 3...,) and $u_0{}^*$, $v_0{}^*$, $\theta_x{}^*$ and $\theta_y{}^*$ are the higher-order terms in the Taylor's series expansion and represent higher-order transverse cross sectional deformation modes, of the middle layer. Laminated plate is made of laminae, having a fibres oriented at an angle θ, measured from the material x to global x axis. The stress strain relations of the laminae are therefore, defined in material coordinate system as:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \\ \tau_{13} \\ \tau_{23} \end{Bmatrix}^k = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{21} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix}^k$$

In which

$\sigma = ((\sigma_1, \sigma_2, \tau_{12}, \tau_{13}, \tau_{23})^t$ are the stress components of k^{th} laminae in material coordinates

$\varepsilon = ((\varepsilon_1, \varepsilon_2, \gamma_{12}, \gamma_{13}, \gamma_{23})^t)$ are the strain vectors k^{th} laminae in material coordinates and

C_{ij}'s are matrix of material elastic coefficients for k^{th} laminae, given as,

$$C_{11} = \frac{E_1}{(1-\mu_{12}\mu_{21})}; \quad C_{12} = \frac{\mu_{21}E_1}{(1-\mu_{12}\mu_{21})};$$

$$C_{22} = \frac{E_2}{(1-\mu_{12}\mu_{21})};$$

$C_{33} = G_{12}$; $C_{44} = G_{23}$; $C_{55} = G_{13}$

Since, all quantities should be referred to a single coordinate system; it is needed to establish transformation relations among stresses and strains, in global system to the corresponding quantities in material (local) coordinate system. The constitutive matrix in global coordinate system will then be of the form:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^k = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{33} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix}^k$$

Where,

$Q_{11}=C_{11}*Cos[\theta]^4+2*C_{12}*Sin[\theta]^2*Cos[\theta]^2+4*C_{33}*Sin[\theta]^2*Cos[\theta]^2+C_{22}*Sin[\theta]^4;$

$Q_{12}= (C_{11}+C_{22} -4*C_{33})*Sin[\theta]^2*Cos[\theta] +C_{12} (Sin[\theta]^4+Cos[\theta]^4);$

$Q_{22}=C_{11}*Sin[\theta]^4+ (2*C_{12}+4*C_{33})*Sin[\theta]^2*Cos[\theta]^2+C_{22}*Cos[\theta]^4;$

$Q_{13}= (C_{11}-C_{12}-2*C_{66})*Sin[\theta]*Cos[\theta]^3+(C_{12}-C_{22}+2*C_{33})Sin[\theta]^3*Cos[\theta];$

$Q_{23}= (C_{11}-C_{12}-2*C_{33})*Sin[\theta]^3*Cos[\theta]+(C_{12}-C_{22}+2*C_{33})Sin[\theta]*Cos[\theta]^3;$

$Q_{33}= (C_{11}+C_{22}-2*C_{12}-2*C_{33})Sin[\theta]^2*Cos[\theta]^2+C_{33}(Sin[\theta]^4+Cos[\theta]^4);$

$Q_{44}=C_{44}*Cos[\theta]^2+C_{55}*Sin[\theta]^2$;

$Q_{45}=(C_{55}-C_{44})*Cos[\theta]*Sin[\theta]$;

$Q_{55}=C_{55}*Cos[\theta]^2+C_{44}*Sin[\theta]^2$;

3. BUCKLING ANALYSIS OF LAMINATED COMPOSITE PLATES BASED ON LAYERWISE HSDT

Buckling in structures is due to in-plane loads, represented by mechanical and thermal loads. Buckling is one of the major modes of failure of laminated structures and it is necessary to predict the critical load of the structural component, for the easy replacement with good load bearing capacity. The objective of buckling analysis, is to determine the critical buckling load. The buckled configuration exhibits the structure, when the in-plane load exceeds its critical buckling load and leads to instability. Hence, the effect of thickness ratio, aspect ratio and modulus ratio on the buckling of laminated composite plates are investigated

4. NAVIER SOLUTION USING HIGHER-ORDER DISPLACEMENT MODEL BASED ON LAYERWISE THEORY

In the Navier method, the displacements are expanded in a double Fourier series, in terms of unknown parameters. The choice of the trigonometric functions in the series is restricted to those which satisfy the boundary conditions of the problem. Substitution of the displacement expansions in the governing equations, results in an invertible set of algebraic equations, among the parameters of the displacement expansion.

The simply supported boundary conditions for the higher-order shear deformation theory are:

At edges x = 0 and x = a

$v_0 = 0,\ w_o = 0,\ \theta_y = 0,\ M_x = 0,\ v_0^* = 0,\ \theta_y^* = 0,\ M_x^* = 0,\ N_x = 0,\ N_x^* = 0,$

At edges y = 0 and y = b

$u_0 = 0,\ w_o = 0,\ \theta_x = 0,\ M_y = 0,\ u_0^* = 0,\ \theta_x^* = 0,\ M_y^* = 0,\ N_y = 0,\ N_y^* = 0,$

The simply supported boundary conditions shown in equations are considered for solutions of laminated composite plates, using displacement model. The boundary conditions are satisfied by the expansions:

$$u_0(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}(t)\cos\alpha x \sin\beta y$$

$$v_0(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}(t)\sin\alpha x \cos\beta y$$

$$w_0(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}(t)\sin\alpha x \sin\beta y$$

$$\theta_x(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} X_{mn}(t)\cos\alpha x\,\sin\beta y$$

$$\theta_y(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Y_{mn}(t)\,\sin\alpha x\,\cos\beta y$$

$$u_0^*(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}^*(t)\,\cos\alpha x\,\sin\beta y$$

$$V_o^*(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}^*(t)\,\sin\alpha x\,\cos\beta y$$

$$\theta_x^*(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} X_{mn}^*(t)\;\cos\alpha x\,\sin\beta y$$

The mechanical loads are also expanded in double Fourier sine series as:

$$q\,(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{mn}(t)\,\sin\alpha x\,\sin\beta y$$

Where Q_{mn} (z, t) = $\frac{4}{ab}\int_0^a\int_0^b q(x,y,t)\;\sin\alpha x\,\sin\beta y\,dxdy$ Where $\alpha = \frac{m\pi}{a}$ and $\beta = \frac{n\pi}{b}$

The governing equations of motion are obtained as:

$$\begin{aligned}&\left(A_{11}\alpha^2+A_{33}\beta^2\right)U_{mn}+\left(A_{12}+A_{33}\right)V_{mn}\,\alpha\beta+\left(B_{11}\alpha^2+B_{33}\beta^2\right)X_{mn}+\left(B_{12}+B_{33}\right)Y_{mn}\,\alpha\beta+\\&\left(A_{14}\alpha^2+A_{36}\beta^2\right)U_{mn}^*+\left(A_{15}+A_{36}\right)V_{mn}^*\,\alpha\beta+\left(B_{14}\alpha^2+B_{36}\,\beta^2\right)X_{mn}^*+\left(B_{15}+B_{36}\right)Y_{mn}^*\,\alpha\beta=\\&\left(I_1\ddot{U}_{mn}+I_2\,\ddot{X}_{mn}+I_3\ddot{U}_{mn}^*+I_4\,\ddot{X}_{mn}^*\right)-(N^1)\end{aligned}$$

$$\begin{aligned}&\left(A_{21}+A_{33}\right)U_{mn}\alpha\beta+\left(A_{33}\,\alpha^2+A_{22}\beta^2\right)V_{mn}+\left(B_{21}+B_{33}\right)X_{mn}\,\alpha\beta+\left(B_{33}\,\alpha^2+B_{22}\,\beta^2\right)Y_{mn}+\\&\left(A_{24}+A_{36}\right)U_{mn}^*\alpha\beta+\left(A_{36}\alpha^2+A_{25}\beta^2\right)V_{mn}^*+\left(B_{24}+B_{36}\right)X_{mn}^*\,\alpha\beta+\left(B_{36}\,\alpha^2+B_{25}\beta^2\right)Y_{mn}^*=\\&\left(I_1\ddot{V}_{mn}+I_2\,\ddot{Y}_{mn}+I_3\,\ddot{V}_{mn}^*+I_4\,\ddot{Y}_{mn}^*\right)-(N^2)\qquad)\end{aligned}$$

$$\begin{aligned}&\left(L_{11}\;\alpha^2+L_{22}\;\beta^2\right)W_{mn}+L_{11}\;\alpha X_{mn}+L_{22}\;\beta Y_{mn}+2L_{13}\;\alpha U_{mn}^*+\\&2L_{24}\;\beta V_{mn}^*+3L_{15}\;\alpha X_{mn}^*+3L_{26}\;\beta Y_{mn}^*=\left(I_1\ddot{W}_{mn}+Q_{mn}\right)\end{aligned}$$

$$\begin{aligned}&\left(B_{11}\alpha^2+B_{33}\beta^2\right)U_{mn}+\left(B_{21}+B_{33}\right)V_{mn}\,\alpha\beta+L_{11}W_{mn}\alpha+\left(D_{11}\alpha^2+D_{33}\beta^2+L_{11}\right)X_{mn}+\\&\left(D_{12}+D_{33}\right)Y_{mn}\,\alpha\beta+\left(B_{41}\alpha^2+B_{63}\beta^2+2L_{13}\right)U_{mn}^*+\left(B_{51}+B_{63}\right)V_{mn}^*\,\alpha\beta+\\&\left(D_{14}\alpha^2+D_{36}\,\beta^2+3L_{15}\right)X_{mn}^*+\left(D_{15}+D_{36}\right)Y_{mn}^*\,\alpha\beta=\\&\left(I_2\ddot{U}_{mn}+I_3\,\ddot{X}_{mn}+I_4\,\ddot{U}_{mn}^*+I_5\,\ddot{X}_{mn}^*\right)-(M^1)\end{aligned}$$

$$\begin{aligned}&\left(B_{12}+B_{33}\right)U_{mn}\alpha\beta+\left(\mathrm{B}_{33}\,\alpha^2+B_{22}\beta^2\right)\mathrm{V}_{\mathrm{mn}}+\mathrm{L}_{22}W_{mn}\beta+\left(\mathrm{D}_{21}+D_{33}\right)X_{mn}\,\alpha\beta+\\&\left(\mathrm{D}_{33}\,\alpha^2+D_{22}\,\beta^2+L_{22}\right)Y_{mn}+\left(B_{42}+B_{63}\right)U^*_{mn}\alpha\beta+\\&\left(B_{63}\alpha^2+B_{52}\beta^2+2L_{24}\right)V^*_{mn}+\left(D_{36}+D_{24}\right)X^*_{mn}\,\alpha\beta+\\&\left(D_{36}\,\alpha^2+D_{25}\beta^2+3L_{26}\right)Y^*_{mn}=\left(I_2\ddot{V}_{mn}+I_3\,\ddot{Y}_{mn}+I_4\,\ddot{V}^*_{mn}+\mathrm{I}_5\,\ddot{\mathrm{Y}}^*_{\mathrm{mn}}\right)\text{- }(\mathrm{M}^2)\end{aligned}$$

$$\begin{aligned}&\left(A_{41}\alpha^2+A_{63}\beta^2\right)U_{mn}+\left(\mathrm{A}_{42}+A_{63}\right)\mathrm{V}_{\mathrm{mn}}\,\alpha\beta+2\mathrm{L}_{31}\,W_{mn}\alpha+\left(\mathrm{B}_{41}\alpha^2+B_{63}\beta^2+2L_{31}\right)X_{\mathrm{mn}}+\\&\left(\mathrm{B}_{42}+B_{63}\right)Y_{mn}\,\alpha\beta+\left(A_{44}\alpha^2+A_{66}\beta^2+4L_{33}\right)U^*_{mn}+\left(A_{45}+A_{66}\right)V^*_{mn}\,\alpha\beta+\\&\left(B_{44}\alpha^2+B_{66}\,\beta^2+6L_{35}\right)X^*_{mn}+\left(B_{45}+B_{66}\right)Y^*_{mn}\,\alpha\beta=\\&\left(I_3\ddot{U}_{mn}+I_4\,\ddot{X}_{mn}+I_5\,\ddot{U}^*_{mn}+\mathrm{I}_6\,\ddot{\mathrm{X}}^*_{\mathrm{mn}}\right)\text{- }(\mathrm{N}^{*1})\end{aligned}$$

$$\begin{aligned}&\left(A_{51}+A_{63}\right)U_{mn}\alpha\beta+\left(\mathrm{A}_{63}\,\alpha^2+A_{52}\beta^2\right)\mathrm{V}_{\mathrm{mn}}+2L_{42}\,\mathrm{W}_{\mathrm{mn}}\beta+\left(\mathrm{B}_{51}+B_{63}\right)X_{mn}\,\alpha\beta+\\&\left(\mathrm{B}_{63}\,\alpha^2+B_{52}\,\beta^2+2L_{42}\right)Y_{mn}+\left(A_{54}+A_{66}\right)U^*_{mn}\alpha\beta+\\&\left(A_{66}\alpha^2+A_{55}\beta^2+4L_{44}\right)V^*_{mn}+\left(B_{54}+B_{66}\right)X^*_{mn}\,\alpha\beta+\\&\left(B_{66}\,\alpha^2+B_{55}\beta^2+6L_{46}\right)Y^*_{mn}=\left(I_3\ddot{V}_{mn}+I_4\,\ddot{Y}_{mn}+I_5\,\ddot{V}^*_{mn}+\mathrm{I}_6\,\ddot{\mathrm{Y}}^*_{\mathrm{mn}}\right)\text{- }(\mathrm{N}^{*2})\end{aligned}$$

$$\begin{aligned}&\left(B_{14}\alpha^2+B_{36}\beta^2\right)U_{mn}+\left(\mathrm{B}_{24}+B_{36}\right)\mathrm{V}_{\mathrm{mn}}\,\alpha\beta+3L_{51}\,\mathrm{W}_{\mathrm{mn}}\alpha+\left(\mathrm{D}_{41}\alpha^2+D_{63}\beta^2+3L_{51}\right)X_{\mathrm{mn}}+\\&\left(\mathrm{D}_{42}+D_{63}\right)Y_{mn}\,\alpha\beta+\left(B_{44}\alpha^2+B_{66}\beta^2+6L_{53}\right)U^*_{mn}+\left(B_{54}+B_{66}\right)V^*_{mn}\,\alpha\beta+\\&\left(D_{44}\alpha^2+D_{66}\,\beta^2+9L_{55}\right)X^*_{mn}+\left(D_{45}+D_{66}\right)Y^*_{mn}\,\alpha\beta=\\&\left(I_4\ddot{U}_{mn}+I_5\,\ddot{X}_{mn}+I_6\,\ddot{U}^*_{mn}+\mathrm{I}_7\,\ddot{\mathrm{X}}^*_{\mathrm{mn}}\right)\text{- }(\mathrm{M}^{*1})\end{aligned}$$

$$\begin{aligned}&\left(B_{15}+B_{36}\right)U_{mn}\,\alpha\beta+\left(\mathrm{B}_{36}\,\alpha^2+B_{25}\beta^2\right)\mathrm{V}_{\mathrm{mn}}+3\mathrm{L}_{62}\,W_{mn}\beta+\left(\mathrm{D}_{51}+D_{63}\right)X_{mn}\,\alpha\beta+\\&\left(\mathrm{D}_{63}\,\alpha^2+D_{52}\,\beta^2+3L_{62}\right)Y_{mn}+\left(B_{45}+B_{66}\right)U^*_{mn}\alpha\beta+\\&\left(B_{66}\alpha^2+B_{55}\beta^2+6L_{64}\right)V^*_{mn}+\left(D_{54}+D_{66}\right)X^*_{mn}\,\alpha\beta+\\&\left(D_{66}\,\alpha^2+D_{55}\beta^2+9L_{66}\right)Y^*_{mn}=\left(I_4\ddot{V}_{mn}+I_5\,\ddot{Y}_{mn}+I_6\,\ddot{V}^*_{mn}+\mathrm{I}_7\,\ddot{\mathrm{Y}}^*_{\mathrm{mn}}\right)\text{- }(\mathrm{M}^{*2})\end{aligned}$$

The analytical solutions for buckling problems of laminated composite plates, can be obtained by setting the distributed load over the surface of the plate and time derivatives to zero:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} & S_{17} & S_{18} & S_{19} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} & S_{27} & S_{28} & S_{29} \\ S_{31} & S_{32} & S_{33}+\mathrm{K} & S_{34} & S_{35} & S_{36} & S_{37} & S_{38} & S_{39} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} & S_{47} & S_{48} & S_{49} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} & S_{57} & S_{58} & S_{59} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} & S_{67} & S_{68} & S_{69} \\ S_{71} & S_{72} & S_{73} & S_{74} & S_{75} & S_{76} & S_{77} & S_{78} & S_{79} \\ S_{81} & S_{82} & S_{83} & S_{84} & S_{85} & S_{86} & S_{87} & S_{88} & S_{99} \\ S_{91} & S_{92} & S_{93} & S_{94} & S_{95} & S_{96} & S_{97} & S_{98} & S_{99} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ Y_{mn} \\ U_{mn}^{*} \\ V_{mn}^{*} \\ X_{mn}^{*} \\ Y_{mn}^{*} \end{Bmatrix} +$$

$$\begin{bmatrix} m_{11} & 0 & 0 & m_{14} & 0 & m_{16} & 0 & m_{18} & 0 \\ 0 & m_{22} & 0 & 0 & m_{25} & 0 & m_{27} & 0 & m_{29} \\ 0 & 0 & m_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ m_{41} & 0 & 0 & m_{44} & 0 & m_{46} & 0 & m_{48} & 0 \\ 0 & m_{52} & 0 & 0 & m_{55} & 0 & m_{57} & 0 & m_{59} \\ m_{61} & 0 & 0 & m_{64} & 0 & m_{66} & 0 & m_{68} & 0 \\ 0 & m_{72} & 0 & 0 & m_{75} & 0 & m_{77} & 0 & m_{79} \\ m_{81} & 0 & 0 & m_{84} & 0 & m_{86} & 0 & m_{88} & 0 \\ 0 & m_{92} & 0 & 0 & m_{95} & 0 & m_{97} & 0 & m_{99} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ Y_{mn} \\ U_{mn}^{*} \\ V_{mn}^{*} \\ X_{mn}^{*} \\ Y_{mn}^{*} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Where, $K = N_{cr}(\lambda_1 \alpha^2 + \lambda_2 \beta^2)$

The in-plane loading conditions for laminated composite plates are given as:

Uni-axial compression (UAC): $\lambda_1 = -1, \lambda_2 = 0$

Biaxial compression (BAC): $\lambda_1 = -1, \lambda_2 = -1$

The elements S_{ij} and (i=1,2….9 and j=1,2…9) are given and solutions for the equations for each m,n gives U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}, $U_{mn}^{*}, V_{mn}^{*}, X_{mn}^{*}, Y_{mn}^{*}$, which are used to compute $u_o, v_o, w_o, \theta_x, \theta_y, u_o^{*}, v_o^{*}, \theta_x^{*}, \theta_y^{*}$

$S_{11} = A_{11}*\alpha^2 + A_{66}*\beta^2$; $S_{12} = (A_{12}+A_{66})*\alpha*\beta$;

$S_{13} = 0$; $S_{14} = B_{11}*\alpha^2 + B_{66}*\beta^2$; $S_{15} = (B_{12}+B_{66})*\alpha*\beta$;

$S_{16} = A_{14}*\alpha^2 + A_{36}*\beta^2$; $S_{17} = (A_{13}+A_{36})*\alpha*\beta$;

$S_{18} = B_{14}*\alpha^2 + B_{36}*\beta^2$; $S_{19} = (B_{15}+B_{36})*\alpha*\beta$;

$S_{21} = (A_{21}+A_{66})*\alpha*\beta$; $S_{22} = A_{66}*\alpha^2 + A_{22}*\beta^2$;

$S_{23} = 0$; $S_{24} = (B_{12}+B_{66})*\alpha*\beta$;

$S_{25} = B_{66}*\alpha^2 + B_{22}*\beta^2$; $S_{26} = (A_{24}+A_{36})*\alpha*\beta$;

$S_{27} = A_{36}*\alpha^2 + A_{25}*\beta^2$; $S_{28} = (B_{24}+B_{36})*\alpha*\beta$;

$S_{29} = B_{36}\ \alpha^2 + B_{25}*\beta^2$; $S_{31} = 0$; $S_{32} = 0$;

$S_{33} = L_{11}*\alpha^2 + L_{22}*\beta^2$; $S_{34} = L_{11}*\alpha$;

$S_{35} = L_{22}*\beta$; $S_{36} = 2*L_{13}*\alpha$;

$S_{37} = 2*L_{24}*\ \beta$;

$S_{38} = 3*L_{15}*\alpha$; $S_{39} = 3*L_{26}*\ \beta$;

$S_{41} = B_{11}*\alpha^2 + B_{66}*\beta^2$; $S_{42} = (B_{21}+B_{66})*\alpha\ *\beta$;

$S_{43} = L_{11}*\alpha$; $S_{44} = D_{11}*\alpha^2 + D_{33}*\beta^2 + L_{11}$;

$S_{45} = (D_{12}+D_{33})*\alpha\ *\beta$; $S_{46} = B_{41}*\alpha^2 + B_{63}*\beta^2 + 2L_{13}$;

$S_{47} = (B_{51}+B_{63})*\alpha\ *\beta$; $S_{48} = D_{14}*\alpha^2 + D_{36}*\beta^2 + 3L_{15}$;

$S_{49} = (D_{15}+D_{36})*\alpha\ *\beta$;

$S_{51} = (B_{12}+B_{66})*\alpha\ *\beta$; $S_{52} = B_{66}*\alpha^2 + B_{22}*\beta^2$;

$S_{53} = L_{22}\ *\beta$; $S_{54} = (D_{21}+B_{33})*\alpha\ *\beta$;

$S_{55} = D_{66}*\alpha^2 + D_{22}*\beta^2 + L_{22}$; $S_{56} = (B_{42}+B_{63})*\alpha\ *\beta$;

$S_{57} = B_{63}*\alpha^2 + B_{52}*\beta^2 + 2L_{24}$; $S_{58} = (D_{24}+B_{36})*\alpha\ *\beta$;

$S_{59} = D_{36}*\alpha^2 + D_{25}*\beta^2 + 3L_{26}$;

$S_{61} = A_{41}*\alpha^2 + A_{63}*\beta^2$; $S_{62} = (A_{42}+A_{63})*\alpha\ *\beta$;

$S_{63} = 2*L_{31}*\alpha$; $S_{64} = B_{41}*\alpha^2 + B_{63}*\beta^2 + 2*L_{31}$;

$S_{65} = (B_{42}+B_{63})*\alpha\ *\beta$; $S_{66} = A_{44}\ \alpha^2 + A_{66}*\beta^2 + 4*L_{33}$;

$S_{67} = (A_{45}+A_{66})*\alpha\ *\beta$; $S_{68} = B_{44}\ \alpha^2 + B_{66}*\beta^2 + 6*L_{35}$;

$S_{69} = (B_{45}+B_{66})*\alpha\ *\beta$;

$S_{71} = (A_{51}+A_{63})*\alpha\ *\beta$; $S_{72} = A_{63}*\alpha^2 + A_{52}*\beta^2$;

$S_{73} = 2*L_{42}*\ \beta$; $S_{74} = (B_{51}+B_{63})*\alpha\ *\beta$;

$S_{75} = B_{63}*\alpha^2 + B_{52}*\beta^2 + 2L_{42}$; $S_{76} = (A_{54}+A_{66})*\ \alpha\ *\beta$;

$S_{77} = A_{66}*\alpha^2 + A_{55}*\beta^2 + 4*L_{44}$; $S_{78} = (B_{54}+B_{66})*\alpha\ *\beta$;

$S_{79} = B_{66}*\alpha^2 + B_{55}*\beta^2 + 6*L_{46}$;

$S_{81} = B_{14}*\alpha^2 + B_{36}*\beta^2$; $S_{82} = (B_{24}+B_{36})*\alpha\ *\beta$;

$S_{83} = 3*L_{15}*\alpha$; $S_{84} = D_{41}*\alpha^2 + D_{63}*\beta^2 + 3L_{51}$;

$S_{85} = (D_{42}\ +D_{63})*\alpha*\beta$; $S_{86} = B_{44}\ \alpha^2 + B_{66}*\beta^2 + 6*L_{53}$;

$S_{87} = (B_{54}+B_{66})*\alpha\ *\beta$; $S_{88} = D_{44}\ \alpha^2 + D_{66}*\beta^2 + 9*L_{55}$;

$S_{89} = (D_{45}+D_{66})*\alpha *\beta;$

$S_{91}= (B_{15}+B_{36})*\alpha *\beta;\ S_{92} = B_{36}\ \alpha^2 + B_{25}*\beta^2;$

$S_{93} = 3*L_{62}*\beta;\ S_{94} = (D_{51}+D_{63})*\alpha *\beta;$

$S_{95} = D_{63}*\alpha^2 + D_{52}*\beta^2 + 3L_{62};\ S_{96} = (B_{45}+B_{66})*\alpha *\beta;$

$S_{97} = B_{66}*\alpha^2 + B_{55}*\beta^2+ 6*L_{46};\ S_{98} = (D_{45}+D_{66})*\alpha *\beta;$

$S_{99} = D_{66}*\alpha^2 + D_{55}*\beta^2+ 9*L_{66};$

5. RESULTS

The following material properties are considered for laminated composite plates to perform the analysis.

Material: Graphite Epoxy

Young's Modulus: E_1 =25 Gpa, E_2 =1Gpa

Shear Modulus: $G_{12} = G_{23} = G_{13}$ =0.5Gpa,

Poisson's Ratio: $\mu_{12} = \mu_{23} = \mu_{13} = 0.25$

The numerical results obtained from the buckling analysis are shown in graphs

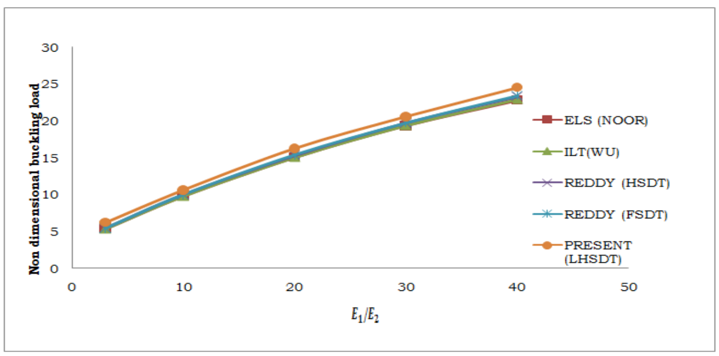

Figure.5.1: Comparison of Non- Dimensional Uni-Axial Buckling Load of a Simply Supported Cross-Ply Plate $(0^0/90^0/0^0)$ with Different Modular Ratios

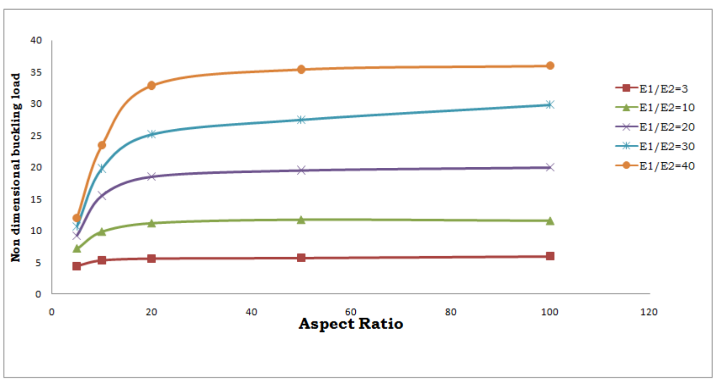

Figure.5.2: Effect of Aspect Ratio and Modular Ratio on Uni-Axial Buckling Load of a Simply Supported Cross Ply $(0^0/90^0/90^0/0^0)$ Laminated Composite Plate

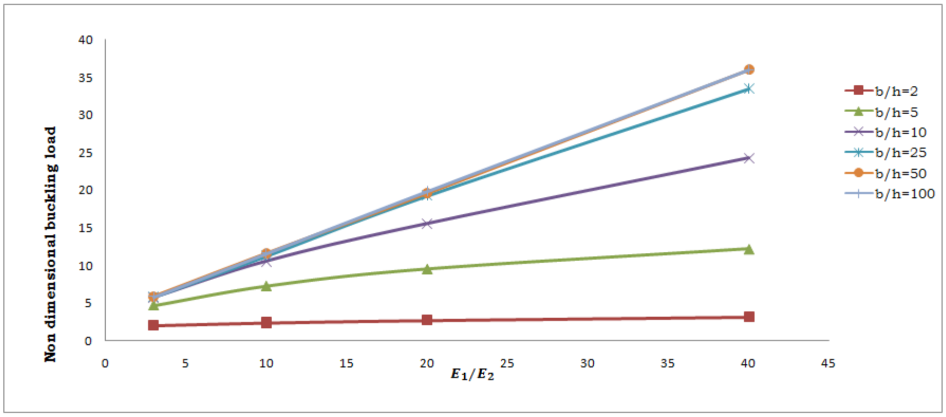

Figure.5.3: Effect of B/H Ratio and Modular Ratio on Non- Dimensional Uni-Axial Buckling Load of a Simply Supported Cross Ply $(0^0/90^0/90^0/0^0)$ Laminated Composite Plate

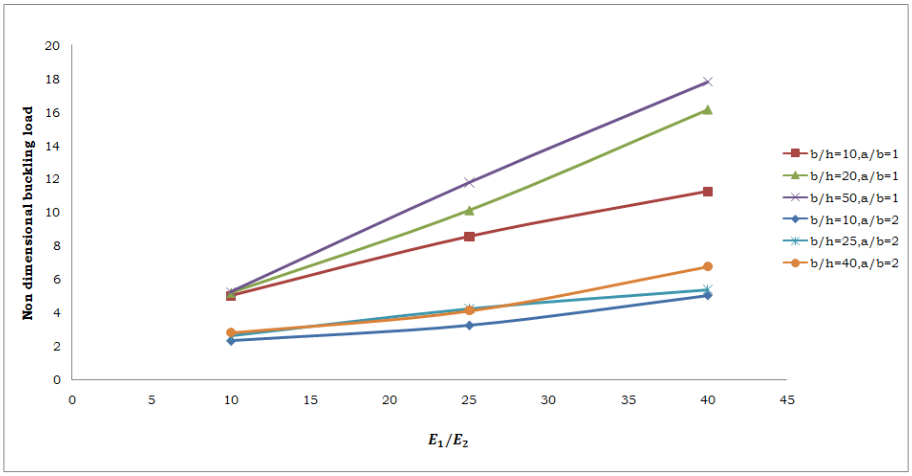

Figure.5.4: Effect of Aspect Ratio, Modular Ratio and A/B Ratio on Non- Dimensional Bi-Axial Buckling Load of a Cross-Ply Laminated Composite Plate

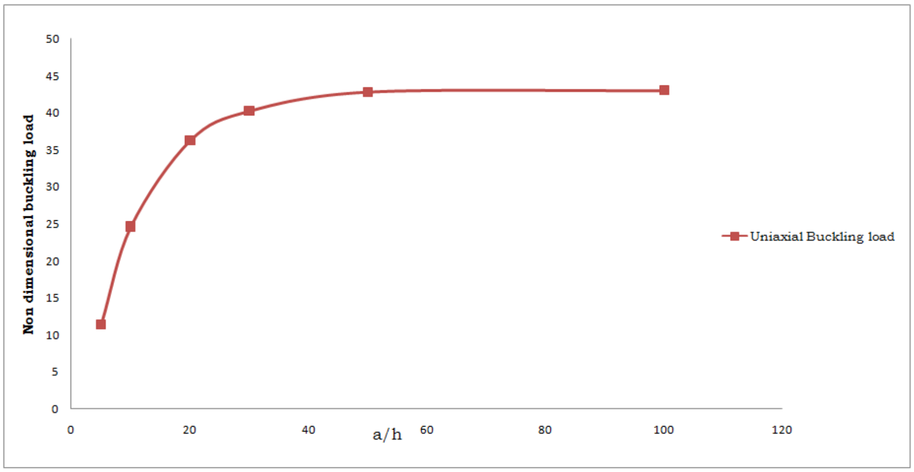

Figure.5.5: Effect of Aspect Ratio, on Non- Dimensional Uni-Axial Buckling Load of a Angle-Ply Laminated Composite Plate

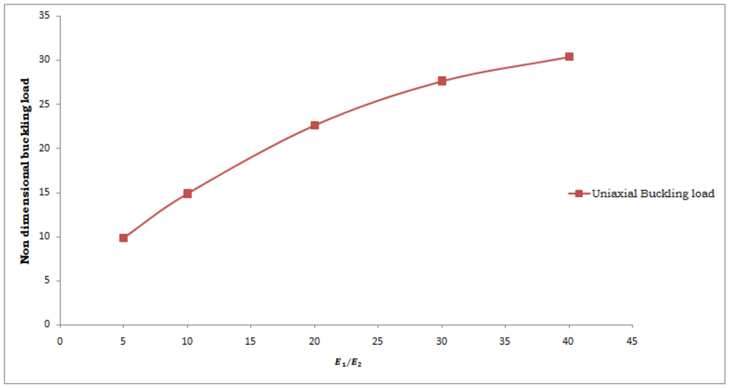

Figure 5.6: Effect of Modular Ratio, On Non- Dimensional Uni-Axial Buckling Load of a Angle-Ply Laminated Composite Plate

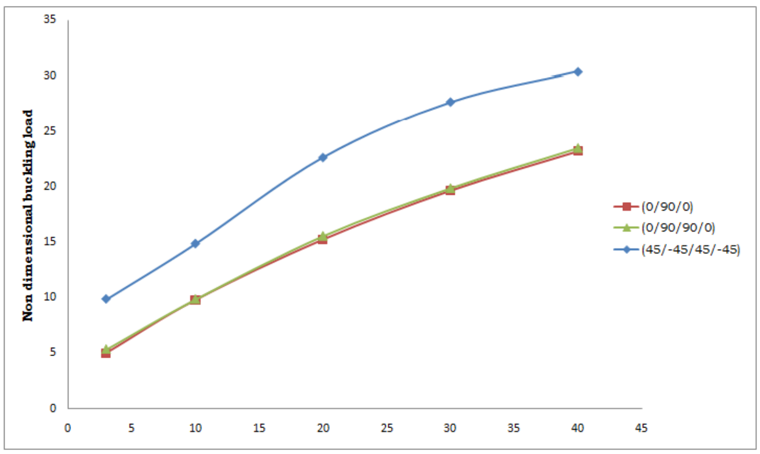

Figure5.7: Variation of the Critical Non- Dimensional Uni-Axial Buckling Load with The in-Plane Ply Modulus Ratio E_1/E_2 in the Range[1,40],A/H=10 With Different Ply Stacking Sequences

6. CONCLUSIONS

A Layer wise higher order shear deformation theory, is successfully developed for the Buckling analysis of laminated composite plates. Many problems are solved covering different features of laminated composites, such as ply orientations aspect ratio, thickness ratio and loading etc. The analysis has verified that, the present LHSDT is capable to predict the buckling response of laminated composite plates, as that of the other theories. It has been shown that, the effects of side-to-thickness ratio, aspect ratio, modular ratio play an important on non-dimensional buckling loads of laminated composite plates. The information derived from the study can help designers in choosing a particular side-to-thickness ratio, aspect ratio, modulus ratio in the material for designing a laminated composite plate.

REFERENCES

1. *Noor AK. Stability of multi-layered composite plates. Fibre Sci Technol1975;8:81–9.*
2. *Reddy JN, Phan ND. Stability and vibration of isotropic, orthotropic and laminated plates according to a higher-order shear deformation theory. J. Sound Vibr 1985;98(2):157–70.*
3. *Lee J, Gurdal Z, Griffin OH. Layer-wise approach for the bifurcation problem in laminated composites with delaminations. AIAA Journal 1993;31(2):3318.*

4. *Wu CP, Chen WY. Vibration and stability of laminated plates based on a local high order plate theory. J Sound Vib 1994; 177(4):503–20.*

5. *Andrzej T. Baranski, Sherrill B. Biggers,(2000) Postbuckling analysis of laminated composite plates using a Higher Order Zigzag Theory. Mechanics of Composite Materials and Structures, 7:285–314, 2000*

6. *A.R. Setoodeh, G. Karami. Static, free vibration and buckling analysis of anisotropic thick laminated composite plates on distributed and point elastic supports using a 3-D layer-wise FEM Engineering Structures 26 (2003) 211–220*

7. *Dorđe Vuksanović, Marina Ćetković Analytical Solution For Multilayer Plates Using General Layerwise Plate Theory Architecture and Civil Engineering Vol. 3, No 2, 2005, pp. 121 – 136*

8. *M. Cetkovic´, Dj. Vuksanovic Bending, free vibrations and buckling of laminated composite and sandwich plates using a layerwise displacement model Composite Structures 88 (2009) 219–227*

9. *M. Kharazi and H. R. Ovesy, Postbuckling behavior of composite plates with through-the-width delaminations, Thin-Walled Structures, 46 (7-9): (2008) 939-946*

10. *Na WJ, Reddy JN. Delamination in cross ply laminated beams using the layer wise theory. Asian Journal of Civil Engineering (Building and Housing)2009;10(4):451–80.*

11. *Zhang Y, Wang S. Buckling, post-buckling and delamination propagation in debonded composite laminates Part 1: Theoretical development. Composite Structures 2009;88 (1):121–30.*

12. *Ovesy HR, Kharazi M, Taghizadeh M. Semi-analytical buckling analysis of clamped composite plates containing embedded rectangular and circular delaminations. Mechanics of Advanced Materials and Structures 2010;17(5):343–52*

13. *Lars Fiedler,Walter Lacarbonara, Fabrizio Vestroni ; A generalized higher-order theory for buckling of thick multi-layered composite plates with normal and transverse shear strains Composite Structures 92 (2010) 3011–3019*

14. *Hosseini-Toudeshky H, Hosseini S, Mohammadi B. Buckling and delamination growth analysis of composite laminates containing embedded delaminations. Applied Composite Materials 2010;17(2):95–109*

15. *V. Rastegar and M. A. Ahmadfard, A two-dimensional layerwise-finite element thermal buckling analysis of laminated composite beams, Proc. of the ASME 2011 International Mechanical Engineering Congress and Exposition, Denver, USA (2011).*

16. *Topal, Application of layerwise optimization method to maximum buckling load design of laminated thin plates, Proc. of the 6th International advanced technologies symposium (IATS'11), Elazıg, Turkey (2011) 44-47*

17. *Neeraj Grover, D.K. Maiti, A new inverse hyperbolic shear deformation theory for static and buckling analysis of laminated composite and sandwich plates. Composite Structures 95 (2012)*

18. *Singh and Chakrabarti Buckling analysis of laminated composite plates using an efficient C0 FE model 1(2012) 1 – 13*

19. *Hamid Reza Ovesy, Sayyed Amir Mahdi Ghannadpour, Ehsan Zia-Dehkordi Buckling analysis of moderately thick composite plates and plate structures using an exact finite strip Composite Structures 95 (2013) 697–704*

20. *Arash Ranjbaran, Mahsa Kharazi, Buckling analysis of sandwich plate using layerwise theory Journal of Mechanical Science and Technology (2014)*

21. *S. M. R. Khalilia, M. M. Kheirikhahb (2014), Buckling analysis of the sandwich plates using improved higher order theory*

22. *Riccardo Vescovini, Lorenzo Dozioa (2015), Exact refined buckling solutions for laminated plates under uniaxial and biaxial loads Composite Structures 127 (2015) 356–368*