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MAXIMAL POLYNOMIAL MODULUS LIMITS AND ZERO-FREE AREAS

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ABSTRACT

This work explores the space of all possible modulus sets for a given polynomial p, denoted by the notation M(p). In this study, we attempt a formulation of p that preserves certain features of M. (p). Simultaneously with the publication of the cubic polynomials p that by Jassim and London, Tyler discovered a quantic polynomial p. Our results were far more effective than any previous ones of their kinds. With careful planning, we crafted polynomials p and p such that their modulus M has both singleton components at a1, a2,.., and a and a discontinuity (p). Considering that there can be no more than a finite number of discontinuities in we conclude that the results are accurate. Let p(z) be a polynomial in n dimensions, with some zeroes at the point z0 C where |z0| = 1, and the remaining zeroes on or outside the perimeter of a given disc. For the location z=0 C, the symbol C is used. Here, we'll take a quick look at these polynomials and calculate their bounds.

KEYWORDS: Polynomials, Maximum Modulus, Zeros & Prescribed Disk

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Original Article

INTRODUCTION

The maximal modulus is symbolized by M(r, f), where f is a complete function.

= maximum | Zz |=r | f (Zz) |,

The maximum modulus set's closure may be tested with little effort. The scenario when f is a polynomial is of particular importance to us in this study. We focus on two "special" characteristics of the maximal modulus set.

The first involves what we call discontinuities. The simplest and least interesting of all the possible outcomes is that if f is a monomial, then M(x) = C. The union of all analytic except at their endpoints countable closed maximum curves is known as M(x). The union of all countable closed maximum curves that are analytic everywhere except at their endpoints is known as M(x).

In this case, we'll use Definition Accept that f is a complete function and that r is positive. Keep in mind that the same modulus might have more than one discontinuity in a maximum modulus set. Blumenthal [1] was the first to investigate these gaps; for more, see [2]. An unlimited number of discontinuities, making him the first person to provide such a function with discontinuities Blumenthal conjectured that there exists a cubic polynomial with this characteristic, but he did not provide any instances of polynomials with this feature.

In [5], such a polynomial is shown. Surprisingly, this appears to be the only discontinuity-free case of its kind in the published literature. Our first finding is a major generalisation of a result from [5] and provides supporting evidence for the central finding in [6].

1.2's Theorem Imagine there starting with a1 and ending with a. If this is the case, then M(p) has modulus a1, a2,.., a discontinuities if and only if p is a polynomials of degree 2n + 1.

Some of the analytic curves in the collection of curves with the largest modulus may be degenerate, or unique. Only in the case of a transcendental whole function f and a Our results demonstrate that this polynomial instance may be greatly fortified.

We now have Theorem 1.3 Let's pretend that there's a finite series of positive real numbers starting with a1 and ending with a. Then, for each of the points a1, a2, and an, M(p) consists of a singleton component. Remarks First, our findings are straightforward and basic, in contrast to those of [6], where the construction needed intricate and nuanced approximations.

Theorems 1.2 and 1.3 should not be interpreted as implying that the sets of maximal moduli do not include any extra discontinuities or singleton components; Figure. 1 shows that this is possible.

It's only logical to wonder whether similar outcomes are also attainable with lower-degree polynomials. The methods presented here do not seem adequate for accomplishing this goal. Finally, we prove the robustness of these constructs by demonstrating that the maximum modulus set of a polynomial has no more than a limited number of discontinuities.

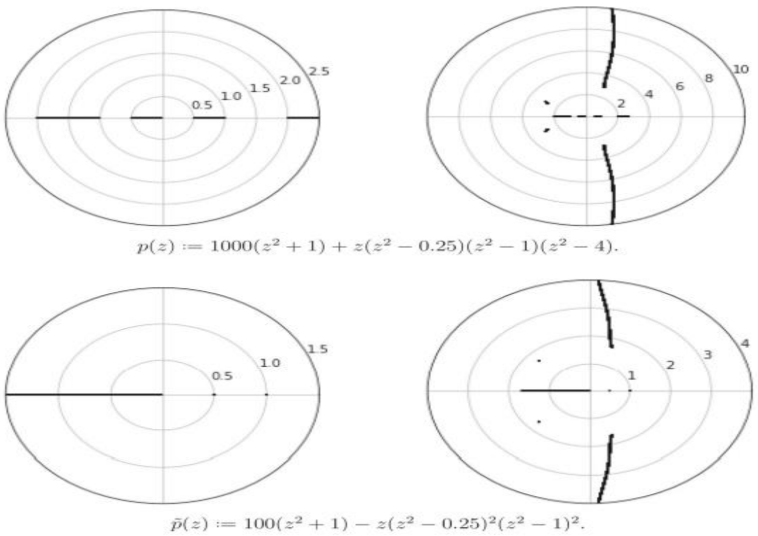

Figure 1

Figure 1 Images created by a computer showing the moduli space moduli 0.5 and 1 as per Theorem 1.3. When you zoom out, you'll see that these groups of maximum moduli seem to have even more gaps (seen on the right)

P is arbitrary, restricting the proof to the correct half-plane has no effect on the generality of the conclusion. Thus, we must show that if an is sufficiently big,

M(p), Re z 0,

and

R |z| R Im z = 0;

That is, z is a maximally improbable number.

Pick the range 004. We first demonstrate that, for sufficiently big values of a > 0,

This proves our first thesis. In this paper, we prove that the point(s). The next step is to prove that these points lie on the actual line by raising an as needed. Eq. (2.4) and the shape of imply that,for0, as a,

=(2 2) |p(rei)| 2 8a2r 2 cos 20 + O(a).

We may conclude that the point above is a local maximum by raising it one more time, if required. Applying our initial hypothesis, the conclusion is obvious.

Keep in mind that as we go through each of the ak, p(r) flips its sign, and a new r-value is obtained (as r grows from zero). Let p be the polynomial in which a > 0 exists (2.2). The algorithm is then deduced from the preceding remark and Lemma 2.3.

Our approach is conceptually similar to Green's algorithm. To do this, we will divide [0, 2] k into several smaller cubes and then assess p in each of the centres. Then, we use the newly discovered bound to determine whether each of the smaller cubes should be retained or discarded, and we continue to subdivide until an adequate level of precision is achieved. Let's assume that the largest is p.

In other words,

let H = [d, d] k

such that

|hj | h for j = 1... k.

Suppose the cube

C = [t H, t + H]

Has a centre at t. For the case when t0 C, the greatest modulus of p occurs at e it0, we have

Now, we know that the greatest modulus cannot happen in a particular cube if for any centre t, |p(e it)| p cos dh 2. We also compute upper limits on the modulus of p, which we find to be p kpk p sec dh 2. This rejection criterion may be used to develop a method for finding the largest possible modulus of a multivariate polynomial p on the polydisk shown here. Assume a degree d homogeneous multivariate polynomial over k variables, and aim for a precision greater than 0. The highest modulus of p on the polydisk, or a close approximation thereto, is the output.

Solving for the largest possible modulus on the real sphere If you're wondering whether or not the same approach we used to find the largest modulus of a polynomial p on the polydisk can be used to the k-dimensional real sphere, the answer is yes.

It can be shown that k-dimensional actual range is defined by

Sr = xR k, where

X kh=2 |zj | 2 = 2.

It is important to emphasise that we are not presuming that the sphere itself serves as the locus of the highest modulus of a genuine ball. For a real sphere, we provide an expression for its largest modulus.

After that, we'll establish a mapping,

M: t [2, 2],

k−2 × [−π, π]

Where

w1 = sin(t1),

wj = sin(tj), and

wk = z S k (t1,..., tk1)

For j = 2,

j Y1 i=1 cos(ti)... k

wk = k

Y1 i=1 cos(ti)

This may be demonstrated through a simple but laborious induction. By using this map, a new trigonometric polynomial is defined that may be used to prove Steckin's Lemma.

A real trigonometric polynomial of degree d is defined as if and only if p is analytic and p(z0) = p(M(t0)). Now, using the same formulas as in Section 4, we derive the

The same subdivision procedure as in Section 5 may be used after we know what d is to discover and approximating the maximal modulus of p on the real sphere. To demonstrate what degree q attains, some evidence is required.

CONCLUSIONS

Polynomials with some zeros at a point and the remainder on or outside a defined disc have not received a lot of attention. The purpose of this study is to compare the class of polynomials [M(p, R)]s to the class of polynomials ||p||s for all R 1 and s N.

REFERENCES

1. *Blumenthal, O.: Sur le mode de croissance des functionsentières. Bull. Soc. Math. Fr. 35, 213–232 (1907)*
2. *Butzer, P., Volkmann, L.: Otto Blumenthal (1876–1944) in retrospect. J. Approx. Theory 138(1), 1–36 (2006)*
3. *Hardy, G.H.: The maximum modulus of an integral function. Q. J. Math. 41, 1–9 (1909)*
4. *Hayman, W.K.: A characterization of the maximum modulus of functions regular at the origin. J. Anal. Math. 1, 135–154 (1951)*
5. *Jassim, S.A., London, R.R.: On the maximum modulus paths of a certain cubic. Q. J. Math. Oxf. Ser. (2) 37(146), 189–191 (1986)*
6. *Pardo-Simón, L., Sixsmith, D.J.: Variations on a theme of Hardy concerning the maximum modulus. Bull. Lond.Math. Soc. 52, 1134–1147 (2020)*

7. *Sixsmith, D.J.: Maximally and non-maximally fast escaping points of transcendental entire functions. Math. Proc. Camb. Philos. Soc. 158(2), 365–383 (2015)*
8. *Tyler, T.F.: Maximum curves and isolated points of entire functions. Proc. Am. Math. Soc. 128(9), 2561–2568 (2000)*
9. *Valiron, G.: (1949) Lectures on the general theory of integral functions. Chelsea*
10. *J.J. Green, Calculating the Maximum Modulus of a Polynomial Using Steckin's Lemma, SIAM J. Numer.Anal. Volume 36, Issue 4, pp. 1022-1029 (1999)*
11. *Krantz, S. G., The Maximum Modulus Principle and Boundary Maximum Modulus Theorem,5.4.1 and 5.4.2 in Handbook of Complex Variables. Boston, MA: Birkhuser, pp.*
12. *R. E. Edwards, Fourier Series 1, Graduate Texts in Math. 2nd ed., 64, Springer-Verlag, New York, 1979*
13. *N. C. Ankeny and T. J. Rivlin, On a theorem of S. Bernstein, Pacific J. Math., 5 (1955), 849–852.*
14. *A. Aziz, Growth of polynomials whose zeros are within or outside a circle, Bull. Aust. Math. Soc., 35 (1987), 247–256.*
15. *A. Aziz and Q. M. Dawood, Inequalities for a polynomial and its derivative, J. Approx. Theory, 54 (1988), 306–313.*
16. *N. K. Govil and R. N. Mohapatra, Markov and Bernstein Type inequalities for Polynomials, J. Inequal. Appl., 3 (1999), 349–387.*
17. *N. K. Govil and E. R. Nwaeze, Bernstein type inequalities concerning growth of polynomials, in Mathematical Analysis, Approximation Theory and their Applications, Springer International Publishing, (2016), 293–316.*
18. *G. V. Milovanovic, D. S. Mitrinovi ´ c and Th. M. Rassias (Eds), Topics in Polynomials: Ex-´ tremal Problems, Inequalities, Zeros, World Scientific, Singapore, (1994).*
19. *K. M. Nakprasit and J. Somsuwan, An upper bound of a derivative for some class of polynomials, J. Math. Inequal., 11(1) (2017), 143–150.*
20. *E. R. Nwaeze, A Note on a result due to Ankeny and Rivlin, Appl. Math. E-Notes, 16 (2016), 170–175.*
21. *Q. I. Rahman and G. Schmeisser (Eds), Analytic Theory of Polynomials, Oxford University Press, New York, (2002).*
22. *W. M. Shah, Extremal Properties and Bounds for the Zeros of Polynomials, Ph. D. Thesis, University of Kashmir, (1998).*