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EPQ MODEL FOR AN ITEM UNDERGOES NON- INSTANTANEOUS DETERIORATION RECEIVES PRICE DISCOUNT PERMITS DELAY IN PAYMENTS

C. SUGAPRIYA

Department of Mathematics, Queen Mary's College, Chennai, Tamil Nadu, India

ABSTRACT

In this paper, an EPQ model for a single product subject to exponential deterioration under production-inventory policy using permissible delay in payments is discussed. It is assumed that a single machine produces a single product over an infinite planning horizon and the production cost is considered constant. The optimal production cycle time is derived under conditions for continuous review, deterministic demand and no shortage.

KEYWORDS: EPQ, Non-Instantaneous Deterioration, Price Discount, Production Rate, Demand Rate, & Permit Delay in Payments

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Original Article

1.0 INTRODUCTION

Kuo-Lung Hou [4] discussed the EPQ model with an underlying fact that setup cost and process quality are functions of capital expenditure. An efficient procedure is developed to find the optimal production run length, setup cost and process quality.

Yang P.C and Wee H. M [6] developed a single-vendor, multi-buyers, production–inventory policy for a deteriorating item with a constant production and demand rate.

Geetharamani G, et al [3] presented a cost optimization model for a multi item inventory system for deterministic demand with backlogging.

In recent research, the extensive use of trade credit as an alternative has been addressed by Goyal [2] who developed an EOQ model under the conditions of permissible delay in payments.

Misra [5] first studied the EPQ model for deteriorating items with the varying and constant rate of deterioration. Gary C. Lin, Dennis E. Kroll and C.J. Lin [1] obtained common production cycle time for an ELSP with deteriorating items.

K. Jeyaraman and C. Sugapriya [7] developed an EPQ for non instantaneous deteriorating items using price discount.

In this paper, an EPQ model for a single product subject to exponential deterioration under production-inventory policy using permissible delay in payments is discussed. The product is provided with price discount for its deterioration rate. In the next section, assumptions and notations for the development of the model are given. The cycle time of the model is derived in the Section 3. An illustrative numerical example and final concluding

remarks are given in the subsequent sections.

1.1 Model Development

It is a production model. The production starts at t = 0 and continues until t = T_1. It reaches the maximum $I_1(T_1)$ at T_1. During this time period [0, T_1], the inventory increases at the rate p-d and there is no deterioration. After the maximum inventory is reached, production is terminated and the deterioration starts. From this point, the on-hand inventory diminishes to the extend of the demand plus the loss due to the deterioration. Production is resumed when all the on hand inventories are depleted as time T . Then an identical production run begins. The production rate of each product is independent of the production lot size. Since an exponential deterioration process is assumed, the inventory level of the system for the product at time t over period [0 , T] can be represented by the differential equations:

Error! Bookmark not defined. for $0 \le t \le T_1$ (1.1)

Error! Bookmark not defined. for $0 \le t \le T_2$ (1.2)

The boundary conditions associated with these equations are: at $I_1(0)$ =0, $I_2(T_2)$ =0.

$$I_1(t) = (p-d)t \qquad \text{for } 0 \le t \le T_1 \tag{1.3}$$

$$I_2(t) = \frac{d}{\theta}\left[e^{\theta(T_2-t)} - 1\right] \text{ for } 0 \le t \le T_2 \tag{1.4}$$

Setup Cost: The setup cost per unit time is given by

$$\text{SC} = \frac{A}{T} \tag{1.5}$$

Holding Cost: The holding cost per unit time is given by

$$\text{HC} = \frac{h}{T}\left[\int_0^{T_1} I_1(t)dt + \int_0^{T_2} I_2(t)dt\right] \tag{1.6}$$

Deterioration Cost: The number of decayed units in a cycle is the difference between the maximum inventory and the number of units used to meet the demand. Hence, the deterioration cost per unit time is given as

$$\text{DC} = \frac{k}{T}\left[I_2(0) - \int_0^{T_2} d.dt\right] \tag{1.7}$$

Price Discount: Price discount per unit time is

$$\text{PD} = \frac{kr}{T}\int_0^{T_2} d.dt_2 \tag{1.8}$$

Assuming $t\theta$ <1, an approximate value is obtained by neglecting the terms of degree greater than or equal to 2 in $t\theta$ in the Tailors expansions of the exponential function

Therefore, the total cost per unit time is given by

TVC (T) = SC+HC+DC+PD

$$\text{TVC (T)} = \frac{A}{T}+\frac{h}{2}\left(\frac{pT_1^2}{T}+dT-2dT_1\right)+\frac{k\theta T_2^2 d}{2T}+\frac{krdT_2}{T} \tag{1.9}$$

Consider the equations (1C.1) and (1C.2) to express T_1 and T_2 in terms of T in the equation (1.8). The third and higher powers of θT terms are neglected for small values of θT. Then equation (1.9) becomes

$$TVC(T)=\frac{A}{T}+\frac{h}{2}\left(dT-\frac{d^2T}{p}\right)+\frac{kd\theta T}{2}\left(\frac{p-d}{p}\right)^2+krd\left(\frac{p-d}{p}\right) \tag{1.10}$$

1.1.1 Case1: M<T

The optimal production cycle time is greater than the credit period, the interest charged by the supplier

$$I_{E1} = \frac{kI_e}{T}\left[\int_0^M d.dt\right] \tag{1.11}$$

The interest charged by the supplier from the customer per unit time

$$I_P=\frac{kI_p}{T}\int_M^T I_2(t)dt \tag{1.12}$$

$$\text{TVC (T)} = \frac{A}{T}+\frac{h}{2}\left(dT-\frac{d^2T}{p}\right)+\frac{kd\theta T}{2}\left(\frac{p-d}{p}\right)^2+krd\left(\frac{p-d}{p}\right)+\frac{kI_e dM}{T}$$

$$-kI_p d\left[\left(\frac{p-d}{p}\right)T+\frac{M^2}{2T}-\frac{T}{2}-\frac{(p-d)}{p}M\right] \tag{1.13}$$

To minimize the total cost per unit time TVC (T), differentiate TVC (T) with respect to T and set the result equal to zero. The resulting equations are represented as

$$\sqrt{\frac{2A+2kI_e dM-kI_p dM^2}{hd\left(1-\frac{d}{p}\right)+\frac{k\theta d(p-d)^2}{p^2}-2kI_p d\left(1-\frac{d}{p}\right)}}=T \tag{1.14}$$

$$\frac{d^2TVC}{dT^2} = \frac{2A}{T^3} + \frac{2kdMI_e}{T^3} - \frac{kI_p M^2 d}{T^3} > 0.$$

The second derivative is positive. T is given by the equation (1.14) is the maximum

1.1.2 Case2: M = T

In this study, the customer need not pay interest. The loss of revenue by the supplier

$$I_{E2} = \frac{kI_e}{T}\int_0^T d.dt \tag{1.15}$$

$$TVC(T) = \frac{A}{T} + \frac{h}{2}\left(dT - \frac{d^2T}{p}\right) + \frac{kd\theta T}{2}\left(\frac{p-d}{p}\right)^2 + krd\left(\frac{p-d}{p}\right) + kd\,I_e \tag{1.16}$$

To minimize the total cost per unit time TVC_2 (T), differentiate TVC_2 (T) with respect to T and set the result equal to zero. The resulting equation is represented as

$$T = \sqrt{\frac{2A}{hd\left(1 - \frac{d}{p}\right) + kd\theta\left(\frac{p-d}{p}\right)^2}} > 0 \tag{1.17}$$

And the second derivative is found to be positive. Hence, T given by the equation (1.17) is the minimum.

1.1.3 Numerical Example for Case 1

Consider the following data for the given EPQ model. The data is A = $1000/set up, P =100 units/unit time, d =30 units/unit time, k = $300/ unit time, M =1 unit time, I_p= 0.14/ unit time, I_e= 0.12/ unit time, h = 0.05 units/unit time, θ = 0.2 and r = 0.1/ unit. The obtained results are: i) production cycle time T = 2.766 unit time, ii) total cost TVC (T) = $1045. 6, iii) production run time T_1= 0.8298 unit time.

1.1.4 Numerical Example for Case 2

Consider the following data for the given EPQ model. The data are A = $1000/set up, P =100 units/unit time, d =30 units/unit time, k = $300/ unit time, I_p= 0.14/ unit time, I_e= 0.12/ unit time, h = 0.05 units/unit time, θ = 0.2 and r = 0.1/ unit. The obtained results are: i) production cycle time T = 1.505 unit time, ii) total cost TVC (T) = $3038. 9, iii) production run time T_1= 0.4515 unit time.

1.2 CONCLUSIONS

The Economic Production Quantity (EPQ) model for non-instantaneous deterioration was developed. The price discount with constant production and demand rate, extending the facility of permissible delay in payments is considered. It is assumed that a single machine produces a single product over an infinite planning horizon. The production cost is considered as constant and the expenditure for the producer per unit time is minimized.

1.3 REFERENCES

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APPENDICES 2C

To minimize the total cost per unit time TVC, the equation (1.3) and (1.4) have to be expressed in terms of T So that there is only one variable T in the equation. At the moment when production run is terminated during a cycle. Therefore, $I_1(T_1) = I_2(0)$.

$$(p-d)T_1 = \frac{d}{\theta}\left(e^{\theta T_2} - 1\right)$$

Applying Taylor's expansion and approximation

$$(p-d)T_1 = d\left(T_2 + \frac{\theta T_2^{\,2}}{2}\right)$$

$$T_2 = \frac{p-d}{d}\,\mathrm{T} \qquad (1C.1)$$

$$T = T_1 + T_2 = \frac{p}{d}T_1 \qquad (1C.2)$$

and $e^{\theta T_2} = 1 + \theta T_2 + \dfrac{(\theta T_2)^2}{2}$