VIBRATION CONTROL OF A CANTILEVER BEAM OF VARYING ORIENTATION SUBJECT TO PARAMETRIC AND DIRECT EXCITATION

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ABSTRACT

The system of single degree of freedom of a cantilever beam of varying orientation subject to parametric and direct excitation is considered and studied. The vibration control of the equation of motion using absorber at the primary resonance \( \omega \equiv \Omega \). The method of multiple scale perturbation technique is applied to obtain the solution up to the second order approximations. The numerical solution of the system is obtained using Runge-Kutta fourth-order method. The stability and effects of different system parameters are studied numerically. Comparison of the approximate solution and numerical solution is obtained.

KEYWORDS: Vibration Control, Direct and Parametric Excitation, Phase Plane, Frequency Response Curves, Resonance Cases, Stability

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1. INTRODUCTION

A system of a cantilever beam subjected to vertical harmonic excitation at its base presented. External resonant excitations may be sources of undesirable flexural vibrations. Amer and Sayed [1] studied the response of one-degree of freedom, non-linear system under multi-parametric and external excitation forces simulating the vibration of the cantilever beam. Variation of some parameters leads to multi-valued amplitudes and hence to jump phenomena. Glabisz [2] studied the stability of one-degree of freedom system under velocity and acceleration dependent non-conservative forces. Eissa [3] investigated the non-linear mechanical oscillator subjected to parametric and external excitation forces.

Eissa and Amer [4] simulated the vibration control of a second order system simulating the first mode of a cantilever beam vibration subject to primary and sub-harmonic resonance. A control law based on cubic velocity feedback is proposed. Kamel and Amer [5] studied the behavior of one-degree of freedom system with different quadratic damping and cubic stiffness non-linearities simulating the axial vibration of a cantilever beam under multi-parametric excitation forces. Queini and Nayfeh [6] proposed a non-linear active control law to suppress the vibration of the first mode of a cantilever beam when subjected to a principal parametric excitation. The method of multiple scale perturbation technique is applied to solve the non-linear differential equations and obtain approximate solutions up to and including the second order approximations. Jain [7] shows the solutions of differential equations using Runge-Kutta fourth-order method. Nayfeh [8] studied the method of multiple scales is used to analyze the response of two-degree of freedom systems with quadratic non-linearities to a parametric harmonic excitation.
Sayam et al. [9] studied numerical simulations of the response of a uniform, cantilever beam subjected to a base excitation. Sayed and Hamed [10] studied the response of a two-degree-of-freedom system with quadratic coupling under parametric and harmonic excitations. The saturation phenomena is very useful in suppressing the undesired vibrations. Asfar et al. [11] studied the response of self-excited two-degree-of-freedom system to multi-frequency excitations. Orhan and Peter [12] studied the effect of excitation and damping parameters on the super harmonic and primary resonance responses of a slender cantilever beam undergoing flapping motion is investigated. This problem is cast into mathematical form using a nonlinear inextensible beam model which is subjected to time-dependent boundary conditions and linear or nonlinear damping forces. Eissa and Sayed [13] studied the effects of different active controllers on simple and spring pendulum at the primary resonance via negative velocity feedback or it's square or cubic. Sayed and Kamel [14] studied the vibration and dynamic chaos should be controlled in either structures or machines. An active vibration absorber for suppressing the vibration of the non-linear plant when subjected to external and parametric excitations.

Eissa and El-Ganaini [15, 16] studied the control of both vibration and dynamics chaos of mechanical system having quadratic and cubic non-linearities subjected to harmonic excitation using multi-absorber. Ashour and Nayfeh [17] also studied non-linear adaptive control of flexible structures using the saturation phenomenon. This phenomenon was utilized to suppress high-amplitude bending and torsional vibration modes of rectangular cantilever plates. Attilio [18] investigated the primary resonance of an externally excited van der pole oscillator under state feedback control with a time delay. Zhao and Xu. [19], applied the feedback control and saturation control to suppress the vibration of the primary system in a two-degree-of-freedom dynamical system with parametrically excited pendulum. Kamel et. al. [20-22] studied the vibration in ultrasonic machining that described by two different systems of non-linear differential equations. Mustafa and Sadri [23] studied the vibration control of a cantilever beam when subjected to direct and parametric excitation.

In the present paper we investigate the problem of suppressing the vibration of a non-linear system of a cantilever beam of varying orientation subject to parametric and direct excitation. The multiple time scale perturbation technique has been applied to solve those differential equations up to the second approximation. All possible resonance cases are extracted and investigated at this approximation order, and the resulting different resonance cases are reported numerically. The stability of the steady state solution near the selected resonance case is studied applying both frequency response equations and phase-plane technique.

2. MATHEMATICAL MODELING

The equation of suppressing the vibration of a non-linear system of a cantilever beam of varying orientation subject to parametric and direct excitation is given by:

\[ \dot{U} + \alpha_1 U + 2\varepsilon \mu \dot{U} + \varepsilon \alpha_2 U^3 - \varepsilon U \dot{U}^2 - \varepsilon \alpha_2 U \dot{U}^2 = \varepsilon f_{0} \overline{\Omega}^2 \sin(\overline{\Omega t}) \cos(\alpha) + \varepsilon U f_{0} \overline{\Omega}^2 \sin(\overline{\Omega t}) \sin(\alpha) + T \]

where \( U \) is the generalized co-ordinate, dot the differentiation with respect to time, \( \mu, \alpha_2 \) are the viscous damping coefficients, \( \alpha_1 \) is the coefficient of the nonlinearity, \( \alpha \) is the orientation angle, \( f_{0}, \overline{\Omega} \) are the forcing amplitude and frequency, respectively, \( \varepsilon \) is a small perturbation parameter, \( T = -\varepsilon G U^3 \) is a control input, and \( G \) is a positive constant.

To obtain the solution of Eq. (1), the method of multiple time scale is used, and \( U \) is expanded as

\[ T(\varepsilon,t) = u_0(T_0,T_1) + \varepsilon u_1(T_0,T_1) + O(\varepsilon^2) \]

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where \( T_0 = t \) represents a fast time scale, \( T_1 = \varepsilon t \) represents a slow time scale and the time derivatives became

\[
\frac{d}{dt} = D_0 + \varepsilon D_1 + \ldots, \quad \frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \ldots.
\]

(3)

Substituting from equations (2), (3) into equation (1) and equating the coefficients of the same power of \( \varepsilon \) in both sides we get:

\[
O(\varepsilon^0): (D_0^2 + \omega^2)u_0 = 0
\]

(4)

\[
O(\varepsilon)(D_0^2 + \omega^2)u_1 = -2D_0 D_\mu u_0 - 2\mu(D_\mu u_0) - \alpha\mu u_0^3 + \alpha_2\mu_0(D_\mu u_0)^2 + \alpha_2\mu_0^2 D_0^2 u_0
\]

\[
- G(D_\mu u_0)^3 + u_0 f_1 \ddot{\Omega}^2 \sin(\alpha) \sin(\ddot{\Omega}T_0) + f_2 \ddot{\Omega}^2 \cos(\alpha) \sin(\ddot{\Omega}T_0)
\]

(5)

The general solution of equation (4) can be written in the form:

\[
u_0(T_0, T_1) = A(T_1)e^{i\alpha T_0} + A^*(T_1)e^{-i\alpha T_0}
\]

(6)

where \( A(T_1) \) is a complex-valued quantity that will be determined by imposing the solvability condition at the next level of approximation.

Substituting equation (6) into equation (5) we get:

\[
(D_0^2 + \omega^2)u_1 = [-2i \alpha(D_1A) - 2i \omega \mu A - 3 \alpha \omega^3 A^3 - 2 \alpha_2 \omega^3 A^2 \dddot{A} - 3iG \omega^3 A^2 \dddot{A}]e^{i\alpha T_0}
\]

\[
+ [-\alpha \omega^3 A^3 - 2 \alpha_2 \omega^3 A^3 + iG \omega^3 A^3]e^{3i\alpha T_0} - [if_2 \ddot{\Omega}^2 \cos(\alpha)]e^{i\dddot{\Omega}T_0}
\]

\[- \frac{if_1 \ddot{\Omega}^2 \sin(\alpha)}{2}e^{i\dddot{\Omega}T_0} - \frac{if_1 \ddot{\Omega}^2 \sin(\alpha)}{2}e^{-i\dddot{\Omega}T_0}
\]

\[
+ [2i \alpha(D_1A) + 2i \omega \mu A - 3 \alpha \omega^3 A^3 - 2 \alpha_2 \omega^3 A^2 \dddot{A} + 3iG \omega^3 A^2 \dddot{A}]e^{-i\alpha T_0}
\]

\[
+ [-\alpha \omega^3 A^3 - 2 \alpha_2 \omega^3 A^3 + iG \omega^3 A^3]e^{-3i\alpha T_0} + \frac{if_2 \ddot{\Omega}^2 \cos(\alpha)}{2}e^{-i\dddot{\Omega}T_0}
\]

\[
+ \frac{if_1 \ddot{\Omega}^2 \sin(\alpha)}{2}e^{-i\dddot{\Omega}T_0} + \frac{if_1 \ddot{\Omega}^2 \sin(\alpha)}{2}e^{i\dddot{\Omega}T_0}
\]

(7)

Eliminating the secular terms the general solution of equation (7) is

\[
u_1(T_0, T_1) = E_1 e^{i\alpha T_0} + E_2 e^{3i\alpha T_0} + E_3 e^{i\dddot{\Omega}T_0} + E_4 e^{i(\alpha + \dddot{\Omega})T_0} + E_5 e^{i(\alpha - \dddot{\Omega})T_0} + cc
\]

(8)

where \( E_i \) (\( i = 1, \ldots, 5 \)) are complex functions in \( T_1 \) and \( cc \) is complex conjugate of the preceding terms. From above
– proposed solution, the reported resonance cases are: primary resonance \( \tilde{\Omega} \equiv \omega \) and sub-harmonic resonance \( \tilde{\Omega} \equiv 2\omega \).

3. STABILITY ANALYSIS

The system is solved numerically using Runge-Kutta fourth-order method, hence the different resonances cases are deduced to obtain the worst cases one of the worst cases has been chosen to study the system stability. The selected resonance case is the primary case \( \tilde{\Omega} \equiv \omega \). In this case we introduce the detuning parameter \( \sigma \) according to:

\[
\tilde{\Omega} = \omega + \varepsilon \sigma
\]  

(9)

Substituting Eq. (9) into Eq. (7) and eliminating the secular terms leads to:

\[
2i \omega (D_A + \mu A) + (3\alpha_1 + 2\alpha_\omega \omega^2 + 3iG \omega^3)A^2 + \frac{i\omega^2 \cos(\alpha)}{2} e^{i\sigma T} = 0
\]  

(10)

Substituting the polar form:

\[
A = \frac{1}{2} a(T)e^{i(\phi(T))}
\]  

(11)

where \( a \), \( \beta \) are unknown real-valued function. Inserting equation (11) into equation (10) and separating the real and imaginary parts we have the following:

\[
a' = -\mu a - \frac{3}{8} G \omega^2 a^3 - \frac{1}{2} f_{\omega z} \omega \cos(\alpha) \cos(\phi)
\]  

(12)

\[
a\phi' = a\sigma - \frac{1}{4} \alpha_\omega \omega^3 - \frac{3}{8\omega} \alpha_\omega a^3 + \frac{1}{2} f_{\omega z} \omega \cos(\alpha) \sin(\phi)
\]  

(13)

where \( \phi = \sigma T \) \( - \beta \). For steady solutions \( a' = \phi' = 0 \) and equations (12) and (13) yields to:

\[
\mu a + \frac{3}{8} G \omega^2 a^3 = -\frac{1}{2} f_{\omega z} \omega \cos(\alpha) \cos(\phi)
\]  

(14)

\[
a\sigma - \frac{1}{4} \alpha_\omega \omega^3 - \frac{3}{8\omega} \alpha_\omega a^3 = -\frac{1}{2} f_{\omega z} \omega \cos(\alpha) \sin(\phi)
\]  

(15)

Squaring equations (14) and (15) and adding the results we get the corresponding frequency response equation (FRE) is:

\[
(\mu a + \frac{3}{8} G \omega^2 a^3)^2 + (a\sigma - \frac{1}{4} \alpha_\omega \omega^3 - \frac{3}{8\omega} \alpha_\omega a^3)^2 - \frac{f_{\omega z}^2 \omega^2 \cos^2(\alpha)}{4} = 0
\]  

(16)

3.1) Linear Solution

Let us consider \( A \) in the Cartesian form:
\[ A = \frac{1}{2} (p - iq)e^{i\sigma_t} \]  

(17)

where \( p \) and \( q \) are real values.

Substituting equation (17) into equation (10) and separating the real and imaginary parts we get:

\[ q' = \sigma p - \mu q \]

(18)

\[ p' = -\mu p - \sigma q - \frac{\int \omega \cos(\alpha)}{2} \]

(19)

The stability of the linear solution is obtained from the zero characteristic equation

\[ \begin{vmatrix} -\mu - \lambda & -\sigma \\ \sigma & -\mu - \lambda \end{vmatrix} = 0 \]

(20)

hence \( \lambda_{1,2} = -\mu \pm i\sigma \)

The linear solution is stable in this case if and only if \( \mu > 0 \), and otherwise it is unstable.

3.2) Non-Linear Solution

To determine the stability of the fixed points, one lets

\[ \alpha = a_0 + a_1, \quad \phi = \phi_0 + \phi_1 \]

(21)

where \( a_0, \phi_0 \) are solutions of equations (12) and (13) and \( a_1, \phi_1 \) are perturbations which are assumed to be small comparing to \( a_0, \phi_0 \).

Substituting equation (21) into equation (12) and (13) we obtain:

\[ a_1' = (-\mu - \frac{9}{8} \omega^2 \alpha_0^2) a_1 + \left(\frac{\int \omega \cos(\alpha) \sin(\phi_0)}{2}\right) \phi_1 \]

(22)

\[ \phi_1' = (\frac{\sigma}{a_0} - \frac{3}{4} \alpha \omega \omega_0 - \frac{9}{8a_0} \alpha a_0) a_1 + \left(\frac{\int \omega \cos(\alpha) \cos(\phi_0)}{2a_0}\right) \phi_1 \]

(23)

The stability of a given fixed points to a disturbance proportional to \( \exp(\lambda t) \) is determined by the roots of:

\[ \begin{vmatrix} -\mu - \lambda & -\frac{\int \omega \cos(\alpha) \sin(\phi_0)}{2} \\ \frac{\sigma}{a_0} - \frac{3}{4} \alpha \omega \omega_0 - \frac{9}{8a_0} \alpha a_0 & \frac{\int \omega \cos(\alpha) \cos(\phi_0)}{2a_0} \end{vmatrix} = 0 \]

(24)

Consequently, a non-trivial solution is stable if and only if the real parts of both eigenvalues of the coefficient matrix (24) are less than zero.
4. RESULTS AND DISCUSSIONS

The differential equation of the equation of motion is solved numerically using Runge-Kutta fourth-order method at non-resonance case.

From this figures the amplitudes of the equation of motion at angle orientation ($\alpha = 0$, $\alpha = \pi/6$) are about 0.3 and 0.2 respectively.

![Figure 1: The Basic Case of the Equation of Motion without Control at Angle Orientation ($\alpha = 0$)](image1)

![Figure 2: The Basic Case of the Equation of Motion without Control at Angle Orientation ($\alpha = \pi/6$)](image2)

4.1) Effect of Parameters

The effect of different parameters are studied numerically for the angle orientation $\alpha = 0$ in Figure 3 and for the angle orientation $\alpha = \pi/6$ as shown in Figure 4, the steady state amplitude is monotonic decreasing function of the viscous damping coefficients $\mu$, $\alpha_2$, and the nonlinear parameter $\alpha_1$ as shown in Figures. 3, 4 (a, b, c) respectively. But the steady state amplitude is monotonic increasing function of the excitation force $f_2$ as shown in Figure 3d, 4d.

![Figure 3](image3)

![Figure 4](image4)
4.2) Resonance Cases

The resonance cases of the equation of motion which is studied numerically obtain the worstcase. Hence the primary resonance \( \tilde{\Omega} \equiv \Omega \) is the worst resonance case. Figure 5, show the steady state amplitude of the equation of motion are increasing to about 2, 2.2 (6.6, 11 times) of the basic case at angle orientation \( \alpha = 0 \) and \( \alpha = \pi/6 \) respectively, which means the system needs to reduce the amplitude of vibration or controlled.
4.3) Effect of the Control

Figure 6, illustrate the vibration of the system control, which shows that the steady state amplitude of the equation of motion is decreased to about 0.14 and 0.35 at angle orientation $\alpha = 0$ and $\alpha = \pi/6$, and it is monotonic decreasing function of the positive constant $G$ which means the control is effective.
4.4) Frequency Response Curves

The frequency response is given by equation (11) is nonlinear algebraic equation of $a$ against the detuning parameter $\sigma$ which is solved by using numerical using Newton Raphson method. Figure 8, also the effect of different parameters are studied in Figures 8, 9. From Figures. 8, 9 (a, b) we can see that the amplitude of the system is monotonic decreasing functions of the damping coefficient $\mu$ and the positive gain $G$. Figures. 8c, 9c show that the amplitude is monotonic increasing of the excitation force $f_2$. But the amplitude is monotonic decreasing functions of the natural frequency $\omega$ and of the angle orientation $\alpha$ as shown in Figures. 8, 9 (d, e). Finally we obtain a comparison between the approximate solution and numerical solution which are a good agreement as shown in Figure 10.

Figure 8: Response Curves of the Equation of Motion at Angle Orientation $\alpha = 0$
Figure 9: Response Curves of the Equation of Motion at Angle Orientation $\alpha = \pi/6$

Figure 10: Comparison Between the Approximate Solution (-----) and Numerical Solution (_____ )
5. CONCLUSIONS

A non-linear controller method is used to suppress the vibration and equations of motion are given by the following system of differential equations. The method of multiple scale perturbation technique is used to derive two first-order differential equations of the amplitude and phase of the response. The stability and effects of different parameters are studied numerically. From the study the main results can be concluded as following:

- The steady state amplitude of system is monotonic decreasing functions of the viscous damping coefficients $\mu$, $\alpha_2$ and the nonlinear parameter $\alpha_1$.
- The steady state amplitude of the system is monotonic increasing function of the excitation force $f_1$.
- The worst resonance case is primary resonance $\Omega \equiv \omega$ which the steady state amplitude is increased to about 2 and 2.2 (6.6, 11 times) of the basic case at angle orientation $\alpha = 0$ and $\alpha = \pi/6$ respectively.
- The effectiveness of the controller $E_a$ are about 2.14 and 0.57 at angle orientation $\alpha = 0$ and $\alpha = \pi/6$ respectively.
- The approximate solution is a good agreement with the numerical solution as shown in Figure 1.

REFERENCES


