MODELING THE NON-LINEAR ADJUSTMENT OF THE DINAR / EURO EXCHANGE RATE: AN APPLICATION OF THE STAR MODEL

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ABSTRACT

The aim of this study is to examine the dynamics of asymmetrically adjusting the Dinar / Euro exchange rate. After rejecting the hypothesis of a cointegrated linear relationship between exchange rate and its fundamentals, we concluded to a non-linear adjustment of the exchange rate to its equilibrium value. The study uses the Smooth Transition Autoregressive model (STAR) in which the transition from one regime to another is done smoothly. STAR models are more appropriate as they are used to describe asymmetric dynamics and define different regimes. Our results point to the existence of a nonlinear adjustment with an exchange rate mean reversion to its fundamental value.

KEYWORDS: Exchange Rate, Non-Linear Model, STAR Model

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1. INTRODUCTION

The recent literature on exchange rate dynamics abundantly pointed to the presence of nonlinear phenomena in exchange rates dynamics. In this regard, we can mention the lessons learned from behavioral finance theory and in particular investors’ heterogeneous expectations in currency markets [Frankel and Froot (1990), De Long, Shleifer, Summers and Waldmann (1990) De Grauwe and Grimaldi, (2006)]. We can also mention central banks’ intervention policies in the presence of target zones for exchange rates or the preservation measures against speculative attacks on currency (Flood and Marion, 1999). Indeed, these theoretical proposals are claimed to affect adjustment of exchange rate dynamics, making its reproduction difficult by linear modeling techniques and rejecting the assumptions of the efficient market hypothesis and the instantaneous, linear and continuous adjustment of exchange rates.

In this paper, we examine adjustment of exchange rate by focusing on the concepts of asymmetry and nonlinearity inherent to exchange rate dynamics. To this end, we will specify dynamics of exchange rates through the use of smooth transition autoregressive models (STAR). We believe that these models are appropriate to provide a productive modeling for the study of asymmetric fluctuations in exchange rates. They are, indeed, likely to reproduce regime behavior in the presence of heterogeneous transaction costs and the different expectations of economic operators.

This paper is structured as follows. The second section reviews the different studies on the nonlinear modeling of exchange rate dynamics. In the third section, we propose STAR to model the dinar / euro exchange rate. The fourth section details the empirical methodology, putting in focus the discontinuous and asymmetric character of exchange rate’s tendency towards its equilibrium value. The fifth section presents the main results and
2. THE NONLINEAR DYNAMICS OF EXCHANGE RATES: A LITERATURE REVIEW

In currency field, recent studies have largely explained the presence of non-linearity in exchange rates’ deviation from their equilibrium levels. Most of this studies found that non-linear models outperform the usual linear models [Suárez and López (2011), Kiliç R (2009)].

Indeed, Lardic et al (2003) believe that it is the heterogeneity of participants in the foreign exchange market, in particular investors who respond differently to "news", that generate a nonlinear dynamic for the exchange rate. Any deviation from its equilibrium value is supposed to be temporary since there are constraints on a return to long-term equilibrium. Similarly, it should be added that the exchange rate fluctuates within a band. The moment it reaches the extremes of the band, the Central Bank intervenes to keep it inside the band. Thus, we have two trends, a dynamic that represents the evolution of exchange rate at the band’s center and a dynamic that represents its evolution at the extremes. Authors like Flood and Taylor (1996) suggest that adjustment towards an equilibrium exchange rate may depend on the magnitude of the deviation from equilibrium. The adjustment process may be persistent. Thus, studies on Sub-Saharan countries have shown that for countries with overvalued exchange rates, their exchange rates’ return to equilibrium is difficult. However, return to equilibrium accelerates for countries with undervalued exchange rates. Dumas (1992) showed that adjustment towards equilibrium is not necessarily continuous. It may depend both on the magnitude of the deviations from equilibrium and on changes in economic fundamentals. The author showed that deviation of exchange rates from the PPP follows a non-linear process, while adjustment speed towards equilibrium depends on the magnitude of deviation.

With the rejection of the linearity hypothesis of exchange rates, several empirical studies focused on examining the role of non-linear models in explaining deviation of exchange rates from their equilibrium. Sarantis (1999) used STAR models to explain the nonlinear behavior of the effective exchange rate of the eight most industrialized countries in the world. The author concluded to a parallelism between nonlinear dynamics and exchange rates’ large movements. This was the case of PPP adjustment to its equilibrium value. Such an adjustment is not continuous and depends on the magnitude of the deviation from equilibrium. Drawing on the work of Sarantis (1999), Dufrénot et al (2006) tried to examine whether, in the presence of transaction costs, exchange rate adjustment is non-linear? The examined sample was the four most developed countries; the United States, Japan, Germany and France. They specified a model explaining PPP’s deviation from equilibrium. Dufrénot et al (2004) used the BEER approach to build a model describing the evolution of equilibrium exchange rates based on fundamentals. These are the ratio between the prices of tradable and non-tradable goods, terms of trade, and interest rate differential. After estimating their model, applying ADF and PP tests on residuals indicated no cointegration between variables. Such a finding may be explained by several economic arguments like transaction costs, economic changes of variables that are incompatible with a linear adjustment and heterogeneity of participants. The STAR model, specifically the ESTAR model, has been used by these authors to address various issues related to adjustment speed, persistence degree of deviations from fundamentals. The results pointed to an adjustment mechanism that reveals a locally explosive dynamic and an overall mean reversion effect. The deviations are less persistent.

3. THE STAR MODEL FOR THE EURO / DINAR EQUILIBRIUM

3.1 The Determinants of Exchange Rates: A Model for the Fundamental Exchange Rate

Several authors showed a great interest to equilibrium real exchange rate and tried to identify the factors that determine exchange rate in the short, medium and long term. Traditionally, movements in inflation, interest rates, money...
supply and balance of payments explain dynamics of exchange rates. Thus, determining exchange rates by means of traditional theories (PPP, PTI, the monetary model, the portfolio model) come short in explaining and predicting changes in exchange rates. This traditional approach rests on simplifying assumptions, which is a major limitation against its use, both theoretically and empirically.

Reacting to empirical failures of traditional theories to determine the exchange rate, other recent alternative approaches seem to be more appropriate to economic reality. We can mention the FEER, BEER and the NATREX, which are macroeconomic in nature. Indeed, Edwards (1989) presented a general equilibrium approach in which exchange rate is determined by economic variables that determine the internal and external balance of the economy. Among these variables, Edwards (1989) retained terms of trade, technological progress, and flow of capital. Similarly, Williamson (1994) defines the fundamental real exchange rate (FEER) as an exchange rate which jointly ensures an internal balance given by a non-inflationary growth and an external balance given by a sustainable amount of the balance of payment. Stein (1991, 1995) speaks of the natural exchange rate (NATREX) which ensures equilibrium of the balance of payment. Stein (1991, 1995) retains capital productivity, capital intensity and external debt as the main determinants of NATREX.

These approaches are intended to identify a long-term reference value as a function of fundamental variables that should converge to the actual exchange rate. They highlight the role played by a number of fundamentals in determining the equilibrium exchange rate. These theoretical models allow for deducting a reduced equation of real exchange rates using a number of fundamentals.

Referring to the fundamental approach to equilibrium exchange rate applied to developing countries, we propose the following equation to explain the dynamics of the real exchange rate of the Dinar.

\[ \log(IRDTE) = a_0 + a_1 TET + a_2 GGDPT + a_3 M2GDPT + a_4 DIPPI + a_5 BOCGDPT + a_6 DINF + Z_t \]

- **IRDTE**: the index of the real exchange rate of the dinar against the euro;
- **TET**: Terms of trade in Tunisia;
- **GGDPT**: ratio of public spending to GDP in Tunisia;
- **M2GDPT**: ratio of money supply within the meaning of M2 to GDP in Tunisia.
- **DIPPI**: the difference in the price index of industrial production between Tunisia and the euro zone;
- **BOCGDPT**: ratio of current account to GDP in Tunisia,
- **DINF** is difference in inflation between Tunisia and the euro zone;
- **Z_t** is error term.

3. MODELLING OF NONLINEARITY: STAR SPECIFICATION

STAR models as proposed by Luukonen, Saikkonen and Teräsvirta (1988), Teräsvirta and Anderson (1992) have been developed to respond to criticism about the abrupt transition between regimes in the TAR models. In practice, these models remain the most commonly used to describe the evolution of nonlinear economic and financial phenomena.

The STAR model of order p can be written as follows:
\[ Z_t = \left( \sum_{i=1}^{p} a_i^{(1)} Z_{t-i} \right) \left( 1 - F(s_t; \gamma, c) \right) + \left( \sum_{i=1}^{p} a_i^{(2)} Z_{t-i} \right) F(s_t; \gamma, c) + \varepsilon_t \]

\( F(s_t; \gamma, c) \) is a continuous function of the transition from one state to another such that \( 0 \leq F \leq 1 \) is transition speed; \( s_t \) is a stationary transition variable.

The STAR model may be designed as a model with two regimes each associated with the extreme values of the transition function \( F(s_t; \gamma, c) = 0 \) and \( F(s_t; \gamma, c) = 1 \) with a continuous transition of one of the regimes to the other. The transition function allows a series ‘dynamic to move gradually from one regime to another, allowing the model to be interpreted in two distinct ways. First, as a model implying the existence of two separate extreme regimes through which the system moves gradually, second, as a continuum of different regimes.

The parameter \( \gamma \) that determines the transition speed between the regime’s two extremes, is respectively represented by the coefficients \( (a_i^{(1)} and a_i^{(2)} \) \( i=1, 2, \ldots, p)\). The transition is abrupt, as the smoothing parameter is large. However, when \( \gamma \) is zero, the transition function becomes constant, and the STAR model is reduced to a linear AR model.

Teräsvirta and Anderson (1992) proposed two types of transition functions:

The logistics function to achieve the LSTAR model (Logistic STAR), written as:

\[ F(s_t; \gamma, c) = \left( 1 + \exp\left( -\gamma(s_t - c) \right) \right)^{-1} \]

The exponential function to form the model ESTAR (Exponential STAR):

\[ F(s_t; \gamma, c) = 1 - \exp\left( -\gamma(s_t - c)^2 \right) \]

The LSTAR model is superior in being used for detecting asymmetry when expansion or recession phases go through different dynamics. However, the ESTAR model can detect symmetric dynamics during expansion and recession phases and a different dynamic for intermediate phases, e.g. a mild recession phase that separates a strong expansion phase and an accelerated recession.

4. METHODOLOGY

4.1. Estimating the Linear Model and Cointegration Test

We use the fundamental equation of exchange rates to estimate the non-linear adjustment of the exchange rate to its equilibrium value. The aim is to model the exchange rate of the Tunisian Dinar / Euro to its fundamentals and try to explain the persistent deviations during the 1980-2008 period. Deviations sometimes result from policy or from an undervaluation of the exchange rate. Traditional techniques are sometimes limited or even insufficient to justify long periods of overvaluation or undervaluation of the national currency against other foreign currencies. Deviations from equilibrium thus remain persistent with a very slow convergence speed. We estimate the long-run relationship between variables by an ordinary least squares method. Our results are reported in the following table.
Table 1: Long-Term Coefficients and Unit Root Test on Residuals

<table>
<thead>
<tr>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
<th>ADF</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.054</td>
<td>-0.0067</td>
<td>0.011</td>
<td>0.028</td>
<td>-0.0056</td>
<td>0.0035</td>
<td>-0.008</td>
<td>-3.31</td>
<td>-3.45</td>
</tr>
<tr>
<td>(17)</td>
<td>(-4.73)</td>
<td>(1.18)</td>
<td>(9)</td>
<td>(-4.3)</td>
<td>(1.02)</td>
<td>(-2.3)</td>
<td>(-3.45)</td>
<td>(-3.45)</td>
</tr>
</tbody>
</table>

The figures in parentheses are t-Student.

We applied the two ADF (Augmented Dickey Fuller) and PP (Philipppe Perron) tests on the error series $z_t$. The results show the presence of a unit root. Indeed, the critical values of both ADF and PP tests are respectively equal to -3.31 and -2.64. Both values are higher than the critical values of ADF and PP with a 5% risk thresholds. This means that the hypothesis of adjustment towards a long-term equilibrium, often seen as continuous with a constant rate, is no longer valid. Deviations from fundamentals vary, which may be inconsistent with a linear adjustment of exchange rates. This result argues in favor of a non-linear modeling of exchange rate dynamics. Theoretically, this may be justified by transaction costs or the heterogeneity of participants in the foreign exchange market, by degree of risk aversion and forecasting problems.

4.2. Nonlinearity Tests

Generally, there may be two categories of linearity tests: Linearity tests of a very well specified alternative and linearity tests of other unspecified alternatives. Tsay (1986) presented a test to check the null hypothesis of linearity against an alternative hypothesis of nonlinearity. However, the test received criticism, since all nonlinear models may be converted to linear models by introducing stress. Testing linearity is therefore about testing this set of constraints. Failure to reject $H_0$ does not necessarily imply that the process is linear, but it may be poorly specified. However, there are general linearity tests. Two types of tests will be retained here; the CUSUM test and the BDS test. These tests allow for testing the presence of any non-linearity. They are known by their robustness and their efficiency.

a. The CUSUM Test

The recursive residuals test, good for to checking the presence or absence of a structural change in a regime, allows to locate a break date, unlike other tests like the show test. These tests then allow for accepting or rejecting a structural change hypothesis, but they do not indicate a break date. The CUSUM test recursively calculates regression and residuals coefficients and uses the logic of CUSUM-type tests adapted to threshold models. Generally, detecting a threshold goes through analyzing the graph representing the different recursive statistics considered a function of an orderly transition variable. This method is commonly used to identify thresholds.

![Graph 1: The CUSUM Test](image-url)
The graph represents residuals in full continuous lines, while the dotted lines represent residuals’ confidence level sat a 95% level. Interpretation of the test is simple: when the full line curve crosses one of the two dotted lines, we can say that the parameters are unstable at a 5% level. In the graph, we retain the years 1985, 1992, 1994 and the years after 2002 as a structural change period. These years reflect the exchange rate policy conducted in Tunisia as a function of the macroeconomic fundamentals that changed following the reforms decided in Tunisia in 1986. The periods that range between (1982, 1986) and (1989, 2000) indicate a tendency of overvaluation of the Tunisian Dinar. However, periods that range between (1986, 1988), in particular the years after putting into effect the Euro, show a movement towards undervaluation. This might have led to large deviations and the return to equilibrium is slow.

Indeed, during 85-86, the country has signed macroeconomic restructuring agreements and implemented a structural adjustment program (SAP). Aware of the need for a constant real effective exchange rate, the Tunisian government has decided to devaluate the national currency to improve competitiveness of its domestic products. According to a study carried out by the IEQ (2003), devaluation significantly reduced the overvalued dinar and generated a movement towards an overvaluation estimated at 4.8% in 1987 and 1.2% in 1988. In 1992, the authorities decided to convert the dinar and launched the interbank foreign exchange market in 1994. In this market, banks can exchange currency or make investments on behalf of their customers. Convertibility remains partial for some sectors of the economy. It relies, therefore, on the availability of foreign exchange reserves to support the exchange rate when in crisis. The 1996-2000 period represents Tunisia’s performance. Because of a strong growth, low interest rates and a controlled inflation, the country looked like a favorable context. In this period, there was an appreciation of the Dollar against the Euro, the currency of Tunisia’s primary economic partner. The price of raw materials, rare in the country, increased, leading to an improvement in the purchasing power of the other Arab countries, considered the second market for Tunisian exports after Europe [JP Domecq (2002)]. In 2001, with new international economic conditions, there was a decline in economic growth in Europe and worldwide, a drought that lasted for long periods during the past decade, reducing GDP growth from 6.1% by the end of 1999 to 4.7% in 2001 (GDP, excluding agriculture, continued its growth), the sharp increase in oil prices resulted in instability of exchange rates in the world including Tunisia. However, what made the situation worse in 2008 is the international financial crisis, which affected most international exchange markets and consequently the Dinar through the Euro and the Dollar, the two currencies to which the National currency is attached.

b. The BDS Test

This test was introduced by Grassgerber and Procaccia (1983) and then developed by Brock, Dechert and Scheinkman (1987) to test the null hypothesis \( H_0 \) that the observations are independently and identically distributed against an unspecified alternative hypothesis. The idea is to form an m-history of a series \((X_t)\) with n components:

\[
X_t^m = (X_{t+1}, X_{t+2}, \ldots, X_{t+m-1})
\]

Where \( m \) is the space dimension in which the pair of points that fall within an \( r \) radius distance, defined in advance, is located.

The full correlation of \( C_{m,N}(r) \), then measures the number of pairs \((X_t, X_r)\), which are located within a distance of less than \( r \).

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Where $\varnothing(k)$ is the Heaviside function, which takes 1 if $k \geq 0$ and 0 otherwise. $d(i, j)$ measures the Euclidean distance between points $i$ and $j$. For $N$ large ($N \to +\infty$), then this proportion is the probability that two vectors misalign with less than $r$. If the variables are independent and identically distributed (iid), each of the individual probabilities is reduced to $C_i(r)$, then we deduce $C_{m,N}(r) = C_i(r)^m$.

Bearing in mind that the random variables are iid, Brock, Dechert, and Sheinkman showed that under the null hypothesis of independence of the examined series, the $W$-statistic, called BDS statistic, and defined by:

$$W_{m,N}(r) = \sqrt{N} \frac{C_{m,N}(r) - (C_i(r))^m}{\sigma_m}$$

Asymptotically moves towards the standard normal distribution. This statistic is often used to test the null hypothesis that the series is independently and identically distributed, against an alternative that may result from a stochastic non-stationarity or a non-linear dependence. Therefore, and in order to detect non-linearity, it was seen necessary to start with the series stationarity before applying the BDS test.

The BDS test uses an asymptotic distribution of the variables and is particularly effective for large sample sizes. However, Brock, Hsieh and LeBaron (1988), examining samples of three different sizes (100, 500, 1000), totaling 5000 experiments with $m$ values ranging from 1 to 10 and a radius taken between 0.25 and two times the standard deviation, reached the following conclusions:

As sample size is large, the values of radius $r$, for which BDS statistics are calculated, are approached by statistical distributions.

The higher the $m$ value, the less efficient is BDS, especially for small statistical samples.

In this study, we tried to apply the robust BDS test instead of other alternatives (ARCH, EGARCH ...). We retained an $m$ value equal to 6 for the test to be more effective. As for the stationarity problem, since $z_t$ is non-stationary, we opted for the first difference of $z_t$ because it is stationary. The results are reported in the table below:

<table>
<thead>
<tr>
<th></th>
<th>BDS Statistic</th>
<th>Std. Error</th>
<th>Z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.088755</td>
<td>0.010003</td>
<td>8.873022</td>
<td>0.0000</td>
</tr>
<tr>
<td>3</td>
<td>0.152575</td>
<td>0.016058</td>
<td>9.501706</td>
<td>0.0000</td>
</tr>
<tr>
<td>4</td>
<td>0.202282</td>
<td>0.019322</td>
<td>10.46899</td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>0.232480</td>
<td>0.020354</td>
<td>11.42171</td>
<td>0.0000</td>
</tr>
<tr>
<td>6</td>
<td>0.253905</td>
<td>0.019842</td>
<td>12.79651</td>
<td>0.0000</td>
</tr>
<tr>
<td>Raw epsilon</td>
<td>0.0248382</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pairs within epsilon</td>
<td>9031.0000</td>
<td>V-statistic</td>
<td>0.7072598</td>
<td></td>
</tr>
<tr>
<td>Triples within epsilon</td>
<td>798133.00</td>
<td>V-statistic</td>
<td>0.5531462</td>
<td></td>
</tr>
<tr>
<td>Dimension</td>
<td>$C(m,n)$</td>
<td>$c(m,n)$</td>
<td>$C(1,n-(m-1))$</td>
<td>$c(1,n-(m-1))$</td>
</tr>
<tr>
<td>2</td>
<td>3736.0000</td>
<td>0.601030</td>
<td>4449.0000</td>
<td>0.715734</td>
</tr>
</tbody>
</table>
Table 2: Contd.,

<table>
<thead>
<tr>
<th></th>
<th>3203.0000</th>
<th>0.524652</th>
<th>4391.0000</th>
<th>0.719247</th>
<th>0.372077</th>
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</thead>
<tbody>
<tr>
<td>4</td>
<td>2795.0000</td>
<td>0.466222</td>
<td>4297.0000</td>
<td>0.716764</td>
<td>0.263940</td>
</tr>
<tr>
<td>5</td>
<td>2498.0000</td>
<td>0.424397</td>
<td>4231.0000</td>
<td>0.718824</td>
<td>0.191917</td>
</tr>
<tr>
<td>6</td>
<td>2241.0000</td>
<td>0.387850</td>
<td>4133.0000</td>
<td>0.715299</td>
<td>0.133945</td>
</tr>
</tbody>
</table>

The above results point to a dependence between change series. Indeed, all the correlation coefficients are statistically significant at a 95% confidence level with a lag order of 2 to 6. This dependence between series does not support inefficiency of markets, and it tells us nothing because of forecasting errors that remain unpredictable. However, it may point to a possible nonlinearity of the $z_t$ process.

4.3. The Choice between the LSTAR and ESTAR Models: Teräsvirta Test

Granger and Teräsvirta’s framework (1993) is a perfect setting to detect periods of under or over valuation of exchange rates against their equilibrium values. These authors proposed to replace the indicator function of an abrupt transition in a process by a more smooth transition between regimes.

Teräsvirta (1994) estimated and distinguished between types of models in steps. First, the author specified the model’s lag order, then estimated the transition variable and finally the transition function. The choice of the transition variable is made possible by applying an LM-type test, assuming under the null hypothesis $H_0$ that the model is linear against an alternative hypothesis $H_1$ that the model is a STAR-type. To choose between LSTAR and ESTAR, Teräsvirta proposed to test a set of three nested null hypotheses, which will eventually retain one of these models. The non-linear specification chosen by this author is:

$$Z_t = a_t^{(0)} + \sum_{i=1}^{p} a_t^{(1)}Z_{t-i} + \sum_{i=1}^{p} a_t^{(2)}Z_{t-i}Z_{t-d}^2 + \sum_{i=1}^{p} a_t^{(3)}Z_{t-i}Z_{t-d}^3 + \sum_{i=1}^{p} a_t^{(4)}Z_{t-i}Z_{t-d}^4 + \epsilon_t$$

Test $F_1$:

$$H_{01}: a_t^{(4)} = 0 \quad \text{for } i = 1, 2, \ldots, p$$
$$H_{01}: a_t^{(4)} \neq 0$$

Test $F_2$:

$$H_{02}: a_t^{(3)} = 0 / a_t^{(2)} = 0 \quad \text{for } i = 1, 2, \ldots, p$$
$$H_{02}: a_t^{(3)} \neq 0 / a_t^{(2)} = 0$$

Test $F_3$:

$$H_{03}: a_t^{(2)} = 0 / a_t^{(3)} = a_t^{(4)} = 0 \quad \text{for } i = 1, 2, \ldots, p$$
$$H_{03}: a_t^{(2)} \neq 0 / a_t^{(3)} = a_t^{(4)} = 0$$

Rejecting $H_{01}$ can be interpreted as a rejection of the ESTAR model and retention of the LSTAR model since the cubical terms $Z_{t-d}Z_{t-d}^3$ are present only in the regression of this type of model. Failure to reject $H_{02}$ implies that the LSTAR model is retained. Rejecting $H_{02}$Following the acceptance of $H_{02}$, leads to retaining the LSTAR model. Failure to reject $H_{03}$ under certain conditions may lead to retaining the ESTAR model. Applying Teräsvirta’s linearity test on the residual $Z_t$, retains the ESTAR model instead of the linear model with the lowest p-value (0.039).
Simulating the performance of this procedure, Teräsvirta (1994) found that if the LSTAR model is retained, then the results are good. They are worse with the ESTAR model and the observations are distributed asymmetrically around the threshold. Accordingly, it may be said that the LSTAR and ESTAR models are almost identical and that they should be estimated simultaneously.

In this study, we have attempted to estimate the two models combined with the aforementioned models, namely the TSTAR, GBELL and GAUSS models. Indeed, we believe, there is no one test that can better estimate the superiority of one model over another, given the presence of nuisance parameters that are not easily identifiable. According to Ben Salem and Perrodin (2001/2), the literature on the study of non-linearity is expanding, unlike research on the specification of these models which is still in its infancy.

5. ESTIMATING NONLINEAR MODELS

In this section, we will present a non-linear specification to account for the dynamic evolution of exchange rates. The results will be interpreted in view of ultimately deriving the model that best fits the studied context.

5.1. Models and Constraints

To specify the non-linear adjustment of exchange rates to fundamentals, to detect patterns during downward and upward phases, to account for transaction costs and large breaks in adjustment or in the evolution of variables, the five aforementioned specifications will be retained.

The ESTAR model:

\[ Z_t = b_0 + b_1 Z_{t-1} + (b_0 + b_1 Z_{t-1})(1 - \exp(-\gamma(s_t - c)^2)) + \varepsilon_t \]

The LSTAR model:

\[ Z_t = b_0 + b_1 Z_{t-1} + (b_0' + b_1' Z_{t-1})(1 + \exp(-\gamma(s_t - c)^2))^{-1} + \varepsilon_t \]

We applied these different specifications processed by MATLAB software, which remains one of the most effective tools for numerical calculations and graphic visualization. The \( \gamma \) coefficient that represents adjustment speed between the two regimes is always positive. When \( \gamma \) moves away from zero, the model is no longer autoregressive. According to several studies like that of Eleftherios G (2010), a \( \gamma \) value leading to an optimum should range between \([1, 6]\]. To this end, we varied these parameters in a loop with a step of 0.5 to finally choose the \( \gamma \) value that corresponds to the most significant result.

The parameter \( c \) is the transition threshold. It represents the system’s change. The assigned threshold allows for a first economic interpretation of exchange rate regimes that define the dynamics of the process. We should then look for an effective method to detect the threshold \( c \). Since for a zero value for this parameter, we define two growth schemes, one is positive and one is negative, and referring to Eleftherios G, (2010), we set \( c \) between \([-0.3, 0.3]\) with a step of 0.01. As for the parameter \( b \) of the TSTAR model, it is defined positive with an interval of \([0.5, 2]\), with a step of 0.1.

\[ \text{This test was conducted using the RATS software, through null hypotheses tests, it allows for choosing the appropriate model to the } t \text{ series.} \]

\[ \text{This retained non-linear specification of the STAR model is introduced and developed for the first time by Chan and Tong (1987).} \]
The results of estimating the five specifications listed above, shown in the table below, took into account that these models should meet the following conditions for convergence reasons:

- The threshold \( c \), to be a realistic value, it should range between the lower and upper value of the \( Z_t \) series. This hypothesis is valid with the interval chosen to \( c \) being \([-0.3, 0.3]\).
- The coefficient of \( b_1' \) and the sum \( b_1 + b_4' \) should be negativeso that the non-linear process is validated. The \( \gamma \) coefficient should be positive.

This proposed method differs significantly from that presented by Teräsvirta (1994) which uses a sequential estimation procedure. According to this author, it consists in successively determine lag order, the transition variable and the transition function before proceeding to estimating the parameters. The choice of the transition variable is done by applying for each candidate variable an LM-type test, which will test the null hypothesis that the process is linear against an alternative hypothesis of a nonlinear STAR model.

5.2. The Results and Their Interpretation

The table below reports the sum of squares of residuals (SSR). The best model is the one that has the lowest SSR.

<table>
<thead>
<tr>
<th>Table 4: The Results</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td>Linear</td>
</tr>
<tr>
<td>( b_0 )</td>
</tr>
<tr>
<td>(1.28)</td>
</tr>
<tr>
<td>( b_1 )</td>
</tr>
<tr>
<td>(14.67)**</td>
</tr>
<tr>
<td>Nonlinear</td>
</tr>
<tr>
<td>( b_0' )</td>
</tr>
<tr>
<td>(-1.75)**</td>
</tr>
<tr>
<td>( b_1' )</td>
</tr>
<tr>
<td>(-1.75)**</td>
</tr>
<tr>
<td>Adjust Coef</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Threshold</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Test ARCH</td>
</tr>
<tr>
<td></td>
</tr>
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<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>Test Jung</td>
</tr>
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</tbody>
</table>

Figures in brackets are t-Student. A single asterisk indicates that the coefficient is significant at the 10\% threshold, two asterisks indicate that the coefficient is significant at the 5\% threshold. No asterisk indicates no significance.

For the estimation, we conducted the two tests of ARCH and Ljung-Box. The Ljung-Boxis designed to detect the presence autocorrelation between residuals, especially with a small sample size. The results indicate that all the models have almost the same Q Stat value of 59. This latter is much greater than the Chi-square critical value of 5.99. This implies rejecting the null hypothesis. Therefore, errors are autocorrelated. The ARCH test is introduced by Engel (1982) to test a homoscedasticity hypothesis against an alternative heteroscedasticity hypothesis. It follows from these test results that the values calculated for the five models are over 12, which also exceed the Chi-square critical value of 5.99, concluding to the rejection of a homoscedasticity hypothesis and supporting the heteroscedasticity hypothesis.

The t-Student test, indicates that all the estimated coefficients, except for constant of the ESTAR model and \( b_0 \) of the LSTAR model, are significant at the 5\% and 10\% thresholds. What is also remarkable in these regressions is that all the
coefficients of non-linearity of the models are significantly different from zero. This points to the importance of nonlinearities in determining the long-term exchange rate.

Given the conditions originally imposed on the validity of nonlinear models, it is clear that the sum of the coefficients $b_1 + b_2$ is negative for the first three models LSTAR and ESTAR. Deviations from equilibrium seem to be highly persistent.

The threshold value for the optimal solution is 0.02 for the two specifications. We found that significance of coefficients improved, moving from a negative $c$ value to a positive value. However, the coefficient reflecting transition speed is positive and varies between 5 for the LSTAR model and 1 for the ESTAR model.

6. CONCLUSIONS

In this study, we tried to explain the dynamics of the non-linear adjustment of exchange rate to its equilibrium using STAR models. These types of models are commonly used to account for the non-linear adjustment problem. They are good in taking into account asymmetry and regime breaks. The STAR model can detect nonlinearity and generate a smooth transition from one regime to another. The LSTAR model can detect phases of expansion or recession of a series with different dynamics. The ESTAR model also allows for detecting intermediate phases between a strong expansion phase and an accelerated recession. These two models were used to estimate the $Z_t$ residual. This residual is assumed to be a stochastic process resulting from the difference between the real exchange rate and its fundamentals.

By testing the non-linearity of this process through some statistical tests, we found nonlinear heteroscedastic dependence. The results point to the effectiveness of the LSTAR and ESTAR models in detecting non-linearity, as seen in the coefficients obtained on the constant and the $Z_{t-1}$ variable. Moreover, the linearity test of Teräsvirta was able to retain the ESTAR model over the other models.

REFERENCES


22. Hansen, B.E. (1996), «Inference when a Nuisance Parameter is not identified under the Null Hypothesis», Econometrica, 64, pp. 413-430.


