ABSTRACT

Due to the increased penetration of distributed generators into the distribution system as well as due to various power electronic devices in the power network, there exists strong disturbances in electrical waveforms i.e., in amplitude, phase and frequency. These fluctuations need to be supervised and monitored for efficient energy management, safety and also protection purposes. Nowadays, this task is performed by Phasor Measurement Units (PMUs), which measure the phasor of voltage and current waveforms on a common timescale synchronized to the Coordinated Universal Time (UTC). Phasor Measurement Units (PMUs) are also expected to quickly measure fundamental frequencies and rate of change of such frequencies (ROCOF) by accurate parameter estimation algorithms. Although Discrete Fourier Transform (DFT) is the most commonly used phasor estimation algorithm, it fails when the signal frequency is varied from its nominal frequency. In practice the frequency of the power system cannot be maintained strictly at 50 Hz. It varies when power imbalance exists in the system. These variations have a significant effect on the phasor estimation. In this paper recursive DFT algorithm is modified to give a better performance during off-nominal frequency variations. Then the performance of the proposed algorithm is compared with Least Square Error (LSQ), RDFT algorithms, which are investigated in MATLAB environment.

KEYWORDS: Electrical Waveforms, Phasor Measurement Units, Phasor Estimation, Recursive DFT Algorithm

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I. INTRODUCTION

Synchronized Phasor Measurement Units (PMUs), since their introduction in 1980s gave impetus to large-scale implementation of wide-area measurement systems (WAMS) using PMUs and phasor data concentrators (PDCs) in a hierarchical structure. Initially PMU technology applications are mostly concerned with the validation of system models, accurate postmortem analysis. With widespread application, they are able to perform linear state estimation and track dynamic phenomena in real time. This lead to the PMUs utility to improve protection and control functions [12].

In Power Systems, high voltage transmission and distribution lines are important links from the generating units to the end users. Protection of these lines using relays plays a vital role from the view point of security, economics and quality of power supplied. Hence correct action of relays is important in power systems. Due to DC offset present in fault current waveforms the line relays tends to overreach. So, these DC offset components has to be removed from these waveforms [13].
As per [2] DFT based phasor estimation is the most commonly used technique in currently available PMUs. Two classifications of power system transients are electromagnetic transients and electromechanical transients. Electromechanical transients are characterized by magnitude and phase angle modulation of power system voltages and current with low frequency signals corresponding to the movement of rotors of large electric machines around the synchronous speed. Electromechanical transients are characterized by step changes in the magnitude and phase angles of the waveform. They contain sustained harmonics and non-harmonic content. Disturbances in phasor estimations due to harmonics can be eliminated by DFT type phasor estimators.

With the advent of advanced microcontrollers and digital signal processors (DSPs) in relay implementation, DFT filter has gained importance for measuring fundamental and harmonic content of a waveform. DFT filter can be implemented in recursive and non-recursive forms. Recursive DFT is commonly used algorithm. Errors are introduced during phase and magnitude estimation using DFT filter due to sampled signal dynamics and frequency deviations [5].

Although DFT algorithm is the most widely used one, it suffers from certain drawbacks. Off-nominal frequency variations of the sampled signal would result in inaccurate phasor estimate. It is because the signal samples taken by PMU are not in phase with the nominal signal. This is one of the main drawbacks to be taken into account.

In this paper, performance evaluation of recursive Discrete Fourier Transform for phasor estimation under various power disturbance waveforms is analyzed. This is carried to validate the performance of recursive DFT as a phasor estimator in a PMU. Basic concept of phasor is covered in section II. Recursive DFT mathematics is reviewed in section III. In Section IV one of the basic methods to measure frequency is revised. Section V describes the motivation for proposal of phasor estimation algorithm during off-nominal frequency variations. Simulation results comparing the proposed algorithm with LSQ [9] and RDFT [6] algorithms are carried out in Section V. Finally Section VI concludes up with the key points analyzed in this paper.

II. CONCEPT OF PHASOR

A sinusoidal signal can be formulated as

\[ v(t) = V_m \cos(2\pi f t + \phi) \]  

(1)

Where \( V_m \) is the amplitude of the signal, \( f \) is the frequency and \( \phi \) is the angular position with respect to an arbitrary time reference. A phasor is the representation of a sinusoidal signal by its magnitude and phase angle at a given frequency. The phasor representation \( V \) of the sinusoid of (1) is given by

\[ V = \left( \frac{V_m}{\sqrt{2}} \right) e^{j\phi} \]  

(2)

The magnitude of the phasor is the rms (root mean square) value of the sinusoid. As in steady state the frequency of phasor does not change, it is easier to represent signal and carry out computations in phasor form rather than in instantaneous form. But, during a disturbance in power system, frequency is not constant but time varying. Hence phasor concept cannot be applied under transient conditions. A recently published IEEE standard C37.118.1 [17] provides necessary guidelines on steady state and dynamic characteristics of phasor measurement.
III. RECURSIVE DFT ALGORITHM AND EFFECT OF OFF-NOMINAL FREQUENCY VARIATIONS

In this paper, recursive DFT algorithm is modified for phasor estimation in phasor estimator block in PMU architecture. But before that recursive DFT mathematics is revisited. In order to extract the fundamental component of input signal samples ($V_j$), a full-cycle recursive DFT filter can be applied according to the following forms [3], [6], and [15]:

$$C_j = C_{j-1} + \left(\frac{2}{N}\right) \cos \left(\frac{2\pi j}{N}\right)(V_j - V_{j-N})$$  \hspace{1cm} (3)

$$S_j = S_{j-1} + \left(\frac{2}{N}\right) \sin \left(\frac{2\pi j}{N}\right)(V_j - V_{j-N})$$  \hspace{1cm} (4)

Where $N$ is number of samples per cycle and $C_j$ and $S_j$ are the sine and cosine terms of the DFT expression. The root-mean-square (rms) value of the fundamental component of the input signal at sample $j$ ($P_j$) and its phase angle $\Phi_j$ are given by

$$P_j = \frac{1}{N} \sqrt{S_j^2 + C_j^2} \text{ and } \Phi_j = \tan^{-1} \frac{C_j}{S_j}$$  \hspace{1cm} (5)

The computed phasor $P_j$ is stationary in the complex plane. This simplifies the correlation of power system phasor with the estimated one by the filter. Taking the DFT filter sine coefficients as the reference signal, the terms $C_j$ and $S_j$ will be imaginary and real parts of the phasor, respectively. Alternatively, $P_j$ can be computed by

$$P_j = P_{j-1} + \frac{1}{N} \left(\frac{2}{N}\right)(V_j - V_{j-N})e^{-i\frac{2\pi j}{N}}$$  \hspace{1cm} (6)

Where $i$ is the imaginary number unit. Note that $P_j$ is equal to $P_{j-1}$ as long as the sample $V_j$ is equal to $V_{j-N}$.

Now if the input signal frequency deviated from 50 Hz by a small amount $\Delta f$ due to power imbalance in the system. The estimated phasor $P_j$ is given in equation (7)

$$P_{j(50+\Delta f)} = k_1 P e^{-i\frac{2\pi j}{50}} + k_2 P^* e^{-i\frac{2\pi j}{50}} e^{i\frac{2\pi j}{N}}$$ \hspace{1cm} (7)

Where: $P = P_j^{(50)}e^{i\frac{(1+N)\pi f}{50}}$ and $P^* = \text{Conjugate}(P)$

$$k_1 = \frac{\sin \left(\frac{\pi N f}{50}\right)}{N \sin \left(\frac{\pi f}{50}\right)} \text{ and } k_2 = \frac{\sin \left(\frac{\pi N f}{50}\right)}{N \sin \left(\frac{\pi f}{50}\right) - \frac{2\pi j}{N}}$$

It is to be reminded that the sampling clock frequency is dependent on nominal system frequency. The sampling frequency is kept constant during the frequency variations. When $\Delta f$ is very small, $k_1$ approaches 1.0 and $k_2$ approaches zero. This type of error is usually named off nominal. It increases with $\Delta f$ increase and vanishes when $\Delta f$ is zero. It is to be noticed that if the sampling frequency is changed in accordance with the system frequency then the error can be reduced. This requires frequency to be estimated and consequently changing the system nominal frequency. Following section describes commonly used frequency tracking technique.

IV. FREQUENCY TRACKING TECHNIQUE

One of the well known techniques for measuring frequency digitally is described [12]. This method starts with
counting the number of reference impulses between consecutive zero crossings of the signal. Now this number is related to a half of period of the signal and is to be multiplied by sampling period. Then on calculating reciprocal of the value results is frequency measured:

\[ f_{est} = \frac{1}{T_m} = \frac{1}{2M_{1/2}} = \frac{f_s}{2M_{1/2}} \]  

(8)

Where \( M_{1/2} \) is a number of reference samples during half period of the signal, \( f_{est} \) is a measured frequency and \( f_s \) is a sampling frequency. Since actual zero crossing may appear at different moment than at sampling instant, it results in measurement error depending on sampling frequency. Then the estimated accuracy can be given by:

\[ \delta_{1/2} = \frac{1}{M_{1/2}} = \frac{2f_{est}}{f_s} \]  

(9)

From the above equation (9), it is evident that error in measured frequency can be reduced by increasing the sampling frequency. The other method is increasing the number of half periods of counting pulses. Now in the proposed method frequency is tracked by the technique described above. Estimation of phasor by proposed algorithm is given in next section.

IV. FREQUENCY TRACKED PHASOR ESTIMATION ALGORITHM

It has been observed from the equation (7) that errors are introduced in magnitude and phase estimation, when the system frequency deviates from the nominal frequency. These errors may be mainly due to the spectral leakage caused by severe distortions at the window edges. That is, if we adjust the window length to the time period of power system nominal frequency (20 ms), then there is exactly one cycle of signal inside the window. But if the system frequency deviates from the nominal frequency then the considered window does not contain one cycle of signal and is also clipped at the edges. This introduces additional frequency components in the spectrum of the windowed signal. This introduced frequency components are the spectral leakages.

If the window length is made exactly equal to the time period of the signal then there is no spectral leakage. In practice it is not possible to match the window length closely with the fundamental time period of the signal. If it is possible to match the window length closely to the fundamental time period of the signal, then some degree of spectral leakage can be reduced and there will be better phasor estimate.

![Figure 1: Proposed Algorithm for PMU](image-url)
In this proposed algorithm, initially frequency will be estimated by the technique described in the section III. This frequency will be used to set a new data window and sampling frequency according to the tracked system frequency. Then the phasor is estimated by Recursive Discrete Fourier Algorithm considering estimated frequency as the nominal system frequency. Figure 1 shows the flow of proposed algorithm for PMU.

In the following section simulation studies of the proposed algorithm when the system frequency is reduced by \pm 1\ Hz is carried out. Here we will not include harmonic distortions and decaying DC offsets in the simulated signal.

V. SIMULATION RESULTS

In order to test the performance of proposed algorithm for phasor estimation, the technique is programmed using a digital computer in MATLAB programming environment.

An input signal having a frequency of 50\pm 1\ Hz have been given, and the simulation results are compared with the conventional algorithms. The sampling rate is 16000 Hz (320 samples per cycle in a 50 Hz system). The fundamental frequency of the system is set to 50Hz.

We consider Total Vector Error (TVE) as the performance criteria for the given algorithm. It is an index which shows us the vector error between the theoretically computed phasor and the estimated one at a particular instant of time. It is defined as the square root of the squared sum of error between the real and imaginary parts of the estimated phasor to the actual phasor, which is given by the following equation:

\[
TVE = \sqrt{\frac{(X_r(n) - X_r)^2 + (X_i(n) - X_i)^2}{X_r^2 + X_i^2}}
\]  

(10)

Here \(X_r(n)\) is the real part of the estimated phasor, \(X_r\) is the real part of the actual phasor, \(X_i(n)\) is the imaginary part of the estimated phasor, \(X_i\) is the imaginary part of the actual phasor at a particular instant \(n\).

The test signal used for testing the algorithm is a fundamental voltage signal whose frequency is different from the power system nominal frequency. This is to study off-nominal frequency variations:

\[
x_1(t) = 440\sqrt{2} \sin \left(2\pi f_1 t + \frac{\pi}{6} \right)
\]  

(11)

Where \(f_1\) is 49, 50 Hz

The simulation results of estimated phasor magnitude, angle and TVE for proposed algorithm when the frequency is 49 Hz are compared with Least Square Error (LSQ) and Recursive DFT Algorithms in Figures 2, 3, 4.

![Figure 2: Plot of Estimated Phasor Magnitude by LSQ, RDFT, Proposed Algorithm when Frequency of Signal is 49 Hz](Image)
The simulation results of estimated phasor magnitude, angle and TVE for proposed algorithm, when the signal frequency is 51 Hz are compared with Least Square Error (LSQ) and Recursive DFT Algorithms in Figures 5, 6, 7.
In this paper frequency tracked phasor estimation algorithm was proposed and is subjected to test signal with different frequencies. Table 1 showing the comparative analysis between conventional phasor estimation algorithm.

<table>
<thead>
<tr>
<th>Phasor Estimator</th>
<th>Max.TVE(%) (49 Hz)</th>
<th>Max.TVE(%) (51 Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSQ</td>
<td>32.25%</td>
<td>35.13%</td>
</tr>
<tr>
<td>RDFT</td>
<td>199.2%</td>
<td>202.5%</td>
</tr>
<tr>
<td>Proposed Algorithm</td>
<td>16.69%</td>
<td>16.67%</td>
</tr>
</tbody>
</table>

From the simulation investigations, it is found that severe drifts in phasor magnitude and angle are reduced and Max TVE (%) is 16.69% for test signal with 49 Hz frequency and 16.67 % for test signal with 51 Hz frequency. But it is not reduced to less than 1% as per IEEE standards. This small error in phasor estimation by proposed algorithm is obtained due to the accumulation of error during the estimation of phasor with initial samples when frequency is tracked. This algorithm need to be further modified to meet the standards.

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