AN ANN BASED APPROACH TO ESTIMATE LONGITUDINAL DISPERSION COEFFICIENT USING DIMENSIONALLY CONSISTENT INPUT PARAMETERS

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ABSTRACT

Longitudinal dispersion coefficient is a fundamental parameter in hydraulic modeling of river pollution. A number of hydrodynamic parameters influence the longitudinal dispersion behavior in river. Many theoretical and empirical formulations have been proposed to determine the longitudinal dispersion coefficient. Attempts have also been made to estimate the longitudinal dispersion coefficient using artificial neural networks. In this paper a modified approach is presented for estimation of longitudinal dispersion coefficient in natural streams using artificial neural network. Dimensionally consistent parameter derived from the physiologic, hydraulic and hydrological parameters have been used as input to artificial neural networks. These estimated values of dispersion coefficients have been compared with the measured values as reported in literature and are found to be in a better fitment with the observed values.

KEYWORDS: Water Pollution, Longitudinal Dispersion Coefficient, Water Quality Modeling, Aquatic Ecosystem

INTRODUCTION

A Stream of pollutants, when discharged into a river, is subjected to different stages of mixing, as the current of flowing water transports these downstream. Advection plays an important role in the transportation of pollutants in the early stages of the transport process after the pollutant has been discharged into the river. Later, when the cross-sectional mixing is complete, the process of longitudinal dispersion becomes important. A number of hydrodynamic parameters influence the longitudinal dispersion behavior in river. A large number of researchers have contributed to the understanding of the mechanisms of longitudinal dispersion in rivers, beginning with the simplest dispersion of dissolved contaminants in pipe flow. Later, the concept of dispersion was extended to the mixing in open channels and further to natural streams. Many theoretical and empirical formulations have been proposed to determine the longitudinal dispersion coefficient.

The basic governing equation, which is based on the principle of conservation of mass and Fick's law is known as the 'Advection-Dispersion Equation' (ADE). Fick's law states that the flux of solute mass, that is, the mass of a solute crossing a unit area per unit time in a given direction, is proportional to the gradient of the solute concentration in that direction (Fischer et al, 1979).

\[
\frac{\partial C}{\partial t} = D_i \frac{\partial^2 C}{\partial x_i^2} - u_i \frac{\partial C}{\partial x_i} \pm kC
\]  

where, \(C\) = concentration \([ML^{-1}]\); \(t\) = time \([T]\); \(D_i\) = Dispersion coefficient in \(i^{th}\) direction (longitudinal, transverse and vertical direction) \([L^2T^{-1}]\); \(x_i\) = distance in the \(i^{th}\) direction \([L]\); \(u_i\) = velocity in the \(i^{th}\) direction \([LT^{-1}]\); and \(k\) = reaction/transformation rate \([T^{-1}]\).

To solve the A-D equation the values of dispersion coefficient in the longitudinal, transverse and the vertical
directions \((D_x, D_y, \text{ and } D_z)\) and the initial conditions and boundary conditions are required. Of these, initial and boundary conditions are obtainable through physical measurement. However, dispersion coefficients are not directly measurable and require to be estimated on the basis of some hypothesis involving measurable characteristics.

In this paper an approach is presented for estimation of longitudinal dispersion coefficient in natural streams using artificial neural network. Dimensionally consistent parameter derived from the physiologic, hydraulic and hydrological parameters have been used as input to artificial neural networks. These estimated values of dispersion coefficients have been compared with the measured values as reported in literature.

**MODELS FOR LONGITUDINAL DISPERSION COEFFICIENT**

Many theoretical and empirical formulations have been proposed to determine the longitudinal dispersion coefficient. Taylor (1954) introduced the concept of dispersion coefficient for longitudinal mixing in turbulent flow in a straight circular pipe. He derived the following equation:

\[
D = 10.1 \, a u_s
\]  

(2)

where, \(a\) is the radius of pipe, \(u_s\) is the shear velocity. The extension of Taylor's concept of longitudinal dispersion coefficient to open channels has been presented by many researchers (Elder, 1959; McQuivey and Keefer, 1974; Fischer, 1975; Liu, 1977). Fischer (1966, 1968) derived the following integral relation for dispersion coefficient in natural streams, with large width to depth ratio, using transverse velocity profile.

\[
D = -\frac{1}{A} \int_0^W \int_0^y \int_0^{y_t} u' h' dy' dy' dy
\]  

(3)

where, \(A = \text{area of cross-section}; u' = \text{deviation of the velocity from the cross-sectional mean velocity}; W = \text{width of channel}; y = \text{Cartesian coordinate in transverse flow direction}; \text{and } \theta = \text{transverse turbulent diffusion coefficient. To estimate the dispersion using Eq. 3, coefficient transverse profiles of velocity as well as cross-sectional geometry are required. That is, dispersion coefficient, as given by Fischer is data intensive. Fischer (1975) suggested a simplified non-integral form of Eq. (3), to be:

\[
D = 0.011 \frac{u^2 W^1}{h u_s}
\]  

(4)

Several more studies have been carried out to empirically or experimentally determine the longitudinal dispersion as a function of hydraulic and geometric parameters. The results obtained from these studies fall in the range of observed values in the specific cases studied. However, in general, when applied to many of the observed cases in natural river flows, the deviation of computed values from the observed values often varies manifold. Attempts are continuing to improve the prediction of longitudinal dispersion as close to the observed data as possible (e.g. Sooky, 1969; Bansal, 1971; Chatwin and Sullivan, 1982; Magazine et al, 1988; Iwasa and Aya, 1991; Jobson, 1997; Sukhodolov, 1997, Swamee et al, 2000). Attempts are also made to take into account, the sinuosity of streams and also the effect of dead zones, in the computation of longitudinal dispersion coefficient. Seo and Cheong (1998) also developed a model to determine the value of \(D'\) using hydraulic data that can be more easily obtained from natural rivers.

\[
\frac{D}{h u_s} = 5.918 \left[ \frac{W}{h} \right]^{0.620} \left[ \frac{u}{u_s} \right]^{1.428}
\]  

(5)
Deng et al (2001) suggested an expression for the transverse mixing coefficient and used it to derive an expression for longitudinal dispersion coefficient through direct integration of Fischer's triple integral. They derived the following expression

\[
D = \frac{0.15}{8\varepsilon_{0}} \left( \frac{W}{h} \right)^{5/3} \left( \frac{u}{u_*} \right)^2 h u_*
\]  

(6)

Eq. (6) has a distinguishing feature that it involves the effect of transverse mixing and therefore it clarifies its dispersion mechanism. A comprehensive review of many such models has been presented by Ahsan (2004, 2007, and 2008).

**ANN BASED ESTIMATION OF LONGITUDINAL DISPERSION COEFFICIENT**

Tayfur and Singh (2005) constructed a three-layer feed forward artificial neural network model which had four neurons in the input layer, six neurons in the hidden layer, and one neuron in the output layer. The model was first trained and then verified. Training and testing of the model was accomplished by employing 71 sets of the of measured data from 29 rivers in the United States. The data sets were obtained from Deng et al. (2001). A large set of data has already been used by Tayfur and Singh (2005) in their study. The same data has been used for the present study also. The data sets as given in Deng et al. (2001) have been used. The measured data from 29 rivers in the United States is shown in Table 1.

**Development of Input Parameters**

In the present study some secondary parameters has been developed using the primary parameters from the procured data set, having the same dimension as of dispersion coefficient. Eight such parameters have been developed and used for preparing the ann model. The developed secondary parameters are as given in Table 1.

**Table 1. Derived Input Parameters**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Secondary Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B U</td>
</tr>
<tr>
<td>2</td>
<td>H \times U</td>
</tr>
<tr>
<td>3</td>
<td>B \times u*</td>
</tr>
<tr>
<td>4</td>
<td>H \times u*</td>
</tr>
<tr>
<td>5</td>
<td>B^2 \times U /H</td>
</tr>
<tr>
<td>6</td>
<td>H^2 \times U /B</td>
</tr>
<tr>
<td>7</td>
<td>(H^2 \times U^2) / (u^* \times B)</td>
</tr>
<tr>
<td>8</td>
<td>(B^2 \times U^2) / (u^* \times H)</td>
</tr>
</tbody>
</table>

**Data used for Training and Testing**

The primary parameters in the procured data have a large variation in the values of all parameters. The variation in the values is as shown in Table 2.

**Table 2. Summary of Training Data**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Parameter</th>
<th>Training Data</th>
<th>Testing Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>1</td>
<td>B (m)</td>
<td>12.8</td>
<td>253.6</td>
</tr>
<tr>
<td>2</td>
<td>H (m)</td>
<td>0.22</td>
<td>3.96</td>
</tr>
<tr>
<td>3</td>
<td>U (m/s)</td>
<td>0.13</td>
<td>1.74</td>
</tr>
<tr>
<td>4</td>
<td>u^* (m/s)</td>
<td>0.024</td>
<td>0.53</td>
</tr>
</tbody>
</table>
ANN Modeling

A three layered feed forward network with back propagation training algorithm has been adopted. Back propagation is the most commonly used supervised training algorithm in multilayered feed forward networks. In back propagation algorithm, information is processed in the forward direction from the input layer to the hidden layer and then to the output layer. The objective of a back propagation algorithm is, by minimizing a predetermined error, to determine the optimal weights which would generate an output vector as close as possible to the target vector. Network architecture includes: Input layer – four neurons; Hidden layer – six neurons; and Output layer – one neuron. Results obtained for various options used in ANN modeling are as given below.

<table>
<thead>
<tr>
<th>Option No.</th>
<th>Activation Function</th>
<th>Learning Rate</th>
<th>Epochs</th>
<th>Training RMSE</th>
<th>R²</th>
<th>Testing RMSE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>pl-pl-pl</td>
<td>0.04</td>
<td>1000</td>
<td>57.047</td>
<td>0.849</td>
<td>160.701</td>
<td>0.447</td>
</tr>
<tr>
<td>2</td>
<td>lg-lg-lg</td>
<td>0.04</td>
<td>5000</td>
<td>355.622</td>
<td>0.698</td>
<td>353.373</td>
<td>0.407</td>
</tr>
<tr>
<td>3</td>
<td>lg-lg-lg</td>
<td>0.04</td>
<td>5000</td>
<td>355.362</td>
<td>0.699</td>
<td>353.142</td>
<td>0.399</td>
</tr>
<tr>
<td>4</td>
<td>lg-lg-lg</td>
<td>0.04</td>
<td>1000</td>
<td>60.075</td>
<td>0.832</td>
<td>179.844</td>
<td>0.371</td>
</tr>
<tr>
<td>5</td>
<td>lg-lg-lg</td>
<td>0.04</td>
<td>5000</td>
<td>31.723</td>
<td>0.953</td>
<td>193.646</td>
<td>0.294</td>
</tr>
<tr>
<td>6</td>
<td>lg-lg-lg</td>
<td>0.04</td>
<td>1000</td>
<td>31.723</td>
<td>0.957</td>
<td>209.903</td>
<td>0.21</td>
</tr>
<tr>
<td>7</td>
<td>lg-lg-lg</td>
<td>0.04</td>
<td>5000</td>
<td>30.593</td>
<td>0.956</td>
<td>193.039</td>
<td>0.279</td>
</tr>
<tr>
<td>8</td>
<td>lg-lg-lg</td>
<td>0.04</td>
<td>1000</td>
<td>30.593</td>
<td>0.956</td>
<td>193.039</td>
<td>0.279</td>
</tr>
<tr>
<td>9</td>
<td>lg-lg-lg</td>
<td>0.04</td>
<td>5000</td>
<td>30.593</td>
<td>0.956</td>
<td>193.039</td>
<td>0.279</td>
</tr>
</tbody>
</table>

Various ANN models have been prepared and tested on the complete data set for training and testing as shown above. The better results were found for S.No. 5 Option 1 where RMSE value of 33.73 and R² value of 0.947 for training dataset has been achieved and RMSE value of 177.29 and R² value of 0.33 for testing dataset has been achieved. A comparison of results obtained from various models including the model suggested in the present study is given in Table 4. Therefore, a Three Layer neural network architecture with 4-6-1 neurons in corresponding layers using a pl-pl-pl activation function.
function may reliably be used to estimate the longitudinal dispersion coefficient.

### Table 4: Comparison of Results of Various Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Training Data</th>
<th>Testing Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE</td>
<td>R²</td>
</tr>
<tr>
<td>Deng et al</td>
<td>43.672</td>
<td>0.922</td>
</tr>
<tr>
<td>Fischer</td>
<td>788.58</td>
<td>0.266</td>
</tr>
<tr>
<td>Seo &amp; Cheong</td>
<td>74.962</td>
<td>0.899</td>
</tr>
<tr>
<td>Singh et al</td>
<td>Not reported</td>
<td>0.9</td>
</tr>
<tr>
<td>Present Study</td>
<td>33.738</td>
<td>0.947</td>
</tr>
</tbody>
</table>

### CONCLUSIONS

The objective of the present study was to develop an ANN based model to predict the longitudinal dispersion coefficient or natural streams. The data consisting of various parameters for 71 rivers (as mentioned in Deng et al, 2001) was collected. Some new parameters having the same dimensions as that of dispersion coefficient, using the primary parameters from the procured dataset were developed. Out of 71 set of data 49 were used for training the network and 22 were used for testing the network. Various models were prepared and tested on data set. ANN models developed for various options were compared with each other as well as with those developed by other researchers. ANN model developed in the present study exhibits significantly less RMSE and a better R² value as compared other models. Therefore, it is suggested that a three layer neural network architecture with 4-6-1 neurons in corresponding layers using a pl-pl-pl activation function may reliably be used to estimate the longitudinal dispersion coefficient.

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### REFERENCES


