

DESIGN AND EVALUATION OF QUICK SWITCHING SYSTEM WITH SINGLE SAMPLING PLAN AS REFERENCE PLAN UNDER THE CONDITIONS OF INTERVENED POISSON DISTRIBUTION

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ABSTRACT

Acceptance sampling plans are designed to state the sample sizes for inspection to determine the quality of any product. A combination of acceptance sampling plans with switching rules for changing from one plan to another are called acceptance sampling schemes. The basic and most widely used sampling scheme is quick switching system (QSS-1). The key objective of this article is to propose a methodology for selection of parameters of QSS-1 with single sampling plan (SSP) as reference plan when the number of defects follows Intervened Poisson distribution (IPD). Further, numerical illustrations are stated to describe the determination of QSS-1 with single sampling plan as reference plan under the conditions of Intervened Poisson distribution (IPD).

KEYWORDS: Acceptance Sampling Plan, Acceptance Sampling Schemes, Quick Switching System, Single Sampling Plan, Intervened Poisson Distribution

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INTRODUCTION

Acceptance sampling is a product control technique used to inspect the quality of incoming raw materials or semi-finished products or finished products against the specified quality standards. The acceptance sampling plan are primarily designed for incoming product inspection, but in recent years, it has become a vital practice with suppliers to improve their process performance. According to Schilling and Neubauer (2009), a good sampling plan should protect both the consumer against the dreadful conditions of accepting low quality level products as well as the producer in the sense that the products produced by the specified quality level will have a high chance of accepting the lot. A combination of acceptance sampling plans with switching rules for changing from one plan to another are called acceptance sampling schemes. The simplest sampling scheme is Quick Switching System (QSS) introduced by Dodge (1967) and further QSS was extensively studied by Romboski (1969). The basic QSS developed by Romboski (1969) is designated as QSS-1. In this system, when a rejection occurs in normal inspection, an immediate switching to tightened inspection is made. QSS-1 uses basic attribute sampling plans for lot inspections, called 'reference plan'.

Romboski (1969) has designed QSS-1 by taking single sampling plan as reference plan under Poisson model. Soundarajan and Aruminayagam (1988) constructed the tables for the selection of modified QSS-1. Further Aruminayagam (1991) formulated QSS-1 with Repetitive Group sampling plan as reference plan. Suresh (1993) proposed a procedure for selection of QSS-1 plans indexed through various quality limits. Further, Jayalakshmi

(2009) developed QSS-1 with different reference plans such as Special Type Double sampling plan, Repetitive Deferred sampling plan and Multiple Deferred sampling plan. Suresh and Kaviyarasu (2008) studied QSS-1 with Conditional Repetitive Group sampling plan as reference plans indexed through various quality limits. All these above mentioned reference plans are based on Poisson model. But, these models fail to investigate if there is any intervention happened during the production process. Shanmugam (1985) introduced Intervened Poisson distribution (IPD) for those instances when some intervention process alters the mean of the rare events.

IPD has a wide range of applicability in the areas such as agriculture, epidemiology, public health, process control, medicine, manufacturing, queueing studies etc. More real time illustrations can be found in Shanmugam (1985, 2001) and Huang and Fung (1989). The characteristics of the IPD are also presented by Shanmugam (1985, 1992), Scollnik produced the Bayesian analysis for IPD in 1995 and introduced intervened generalized Poisson distribution in 2006 which is a generalized version of the IPD. Dhanavathan (1998, 2000) introduced Compound Intervened Poisson distribution and studied its nature, characteristics and also estimated its parameters. Satheesh and Shibu (2011) introduced a modified version of IPD namely modified intervened Poisson distribution.

Designing of sampling plans under the conditions of IPD is very much important when there is a quality intervention taking place in the middle of the lot formation. The principle objective of this paper is to propose a design methodology for selection of parameters QSS-1 with single sampling plan (SSP) as reference plan when number of defects follows Intervened Poisson distribution (IPD).

Single Sampling Plan under the Conditions of Intervened Poisson Distribution

A single sampling plan (SSP) by attributes is specified by the parameters such as lot size (N), sample size (n) and acceptance number (c). The operating procedure for SSP is as follows;

- Take a random sample of n units from a lot of size N units.
- Determine $X = x$, the number of defective units from the sample.
- If $x \leq c$, accept the lot; otherwise reject the lot.

The OC function of SSP is defined as,

$$P_a(p) = P[x \leq c] \quad (1)$$

Where p symbolizes the lot proportion defectives. In practice, the values of $P_a(p)$ can be determined for different values of p using different probability models. As quoted by Schilling and Neubauer (2009), when n is large, p is small and $n/N \leq 0.10$ such that $np < 5$, the proportion defective p is distributed according to the Poisson distribution with mean np .

When some intervention is applied to the production process with an intention to improve the quality, of the products during the observational period, then the mean of the rare event p may get altered. In such situation the appropriate probability distribution for number of defectives in the sample is modeled by IPD.

The OC function for SSP under the conditions of IPD (θ, ρ) can be defined as,

$$P_a(p) = \sum_{x=1}^c P(X = x | \theta, \rho) \quad (2)$$

$$P_a(p) = \sum_{x=1}^c \frac{[(1 + \rho)^x - \rho^x] \theta^x}{e^{\rho\theta} (e^\theta - 1)x!} \quad (3)$$

Where $\theta = np$ and ρ is the intervention parameter which is measured in percentage. Hence, by obtaining an estimated value for ρ , the probability of acceptance can be calculated using equation (3) for given values of n , c and p .

QSS-1 with SSP as Reference Plan under the Conditions of IPD

The QSS-1 proposed by Romboski (1969) is portrayed by five parameters namely sample size n , normal and tightened acceptance numbers say c_N and c_T and normal and tightened probability of acceptance P_N and P_T respectively. The parameters are determined in such a way that both producer as well as consumer is protected. The operating procedure for QSS-1 is as follows,

- Start and continue normal inspection until a lot is rejected.
- If a lot is rejected, switch to tightened inspection and continue tightened inspection until a lot is accepted.
- If a lot is accepted in step 2, switch to normal inspection and repeat the process as dictated.

It is clear that in QSS there must be flow of lots for which reference sampling plans are required to sentence about the individual lots. Thus, in this paper we consider single sampling plan under the conditions of Intervened Poisson distribution (IPD) as reference plan. Romboski (1969) has derived the probability of acceptance $P_a(p)$ for the scheme as,

$$P_a(p) = \frac{P_T}{1 - P_N + P_T} \quad (4)$$

Where p is proportion defectives, P_T is the probability of accepting the lots under tightened inspections and P_N is the probability of accepting the lots under normal inspections. Here, P_T and P_N can be determined by equation (3) for the corresponding acceptance numbers, c_T and c_N .

Determination of Plan Parameters

Determination of plan parameters is generally termed as ‘designing of sampling plans’. The prime objective of designing QSS-1 with SSP as reference plan under the conditions of IPD is to obtain an optimum combination of parameters (n , c_T , c_N) so that the scheme can protect both consumer as well as producer. This can be achieved by specifying (p_1 , α , p_2 , β) and applying the following conditions,

$$i) P_a(p_1) = 1 - \alpha \text{ and } ii) P_a(p_2) = \beta$$

Following unity value approach of Cameron (1952), the unity values for QSS-1 with single sampling plan as reference plan under the conditions of IPD are computed for various combinations of p_1 , α , p_2 and β through iterative procedure and tabulated for different sets of plan parameters viz., ρ , c_T and c_N in Table 1. The Table 1 also contains operating ratio $OR = np_2 / np_1$ which is a discriminating factor between the plans.

For the estimated values of ρ and for the known strength (p_1 , α , p_2 , β), the optimum plan parameters can be determined by employing the following strategy:

- Compute the operation ratio $OR = p_2 / p_1$.

- Select OR from the Table 1 in the column headed OR that is nearly equal to the computed ratio OR value.
- Determine the np_1 , np_2 , c_T and c_N values corresponding to the ratio OR value.
- The sample size n can be obtained by dividing np_1 by p_1 or np_2 by p_2 . The highest sample size is considered as optimum sample size.

Where np_1 is the acceptable quality limit and np_2 is limiting quality limit.

Numerical Illustration

Assume that the production process has an intervention in between the production. It is also assumed that the intervention parameter ρ is known and it is estimated as $\rho = 10\%$. Suppose that the required plan should have the strength $p_1 = 0.0025$, $\alpha = 0.05$, $p_2 = 0.062$ and $\beta = 0.10$ then to obtain an optimum plan, the operating ratio is computed as $OR = p_2/p_1 = 0.062/0.0025 = 24.8$. From Table 1 we select an $OR = 23.7$ which is near to the calculated OR . Corresponding to the OR the values c_N , c_T , np_1 and np_2 are chosen to be 3, 1, 0.0388 and 0.9196 respectively. Since, $np_1/p_1 = 15.52 \approx 16$ and $np_2/p_2 = 14.83 \approx 15$, the number of products to be inspected is 16 in each lot. Thus, the optimum plan for the given specifications is (16, 3, 1).

Now suppose we use $n = 20$, $c = 1$ as the tightened plan and $n = 20$, $c = 2$ as the normal plan and both the plans have intervention of 5% then these plans have lot tolerance percent defective (LTPD) of 16.91% and 24.92% respectively, when using single sampling plan under the conditions of Intervened Poisson distribution. While, in QSS-1 format, the LTPD is 6.96% which is much lower than the LTPD obtained by individual tightened and normal plans. Also, the tightened and normal plans have an acceptable quality limit (AQL) of 0.46% and 2.76% respectively, while QSS-1 AQL is 0.23% which is again lesser than the AQLs of normal and tightened plans. Thus, the combined sampling scheme QSS-1 with single sampling plan as reference plan under the conditions of IPD is better than the individual sampling plan under normal and tightened sampling plans.

CONCLUSIONS

Quick Switching System is very much essential when there is a flow of lots that are expected to have a good quality level. During the production there is a possibility of intervention with the aim of improving the quality. In order to study such intervention in the lots, SSP under the conditions of IPD will be more useful. Thus, the proposed QSS-1 with SSP under the conditions of IPD as reference plan will be more effective to inspect the continuous stream of lots in which there includes an intervention during the production process.

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APPENDIX

Table 1: Unity values and Operating Ratios for Selection of QSS-1 with SSP as Reference Plan for given Producer’s Risk $\alpha = 0.05$ and Consumer’s risk $\beta = 0.10$ under the Conditions of Intervened Poisson Distribution

C_N	C_T	ρ	$np_{0.95}$	$np_{0.50}$	$np_{0.10}$	OR	ρ	$np_{0.95}$	$np_{0.50}$	$np_{0.10}$	OR		
2	1	0.001	0.0506	0.6157	1.5228	30.09	0.05	0.4883	1.7499	3.6084	7.39		
3	1		0.0503	0.5559	1.1648	23.16		0.0458	0.5095	1.0716	23.4		
3	2		0.4192	1.7529	3.3321	7.949		0.3909	1.6463	3.1406	8.034		
4	1		0.0503	0.5495	1.1068	22.00		0.0458	0.504	1.0206	22.28		
4	2		0.4083	1.5676	2.5715	6.298		0.3811	1.4732	2.4233	6.359		
4	3		0.9670	2.8190	4.8371	5.002		0.9115	2.6724	4.5957	5.042		
5	1		0.0502	0.5488	1.0963	21.84		0.0458	0.5035	1.0116	22.09		
5	2		0.4078	1.5259	2.3783	5.832		0.3804	1.4345	2.2427	5.896		
5	3		0.9370	2.5473	3.8429	4.101		0.8833	2.4137	3.6482	4.130		
5	4		1.5801	3.8345	6.2193	3.936		1.4975	3.6471	5.9229	3.955		
2	1		0.01	0.0497	0.6052	1.4972		30.12	0.1	0.0424	0.5172	1.2816	30.23
3	1			0.0494	0.5467	1.1466		23.21		0.042	0.4693	0.99	23.57
3	2	0.4131		1.7325	3.2956	7.978	0.3658	1.5477		2.9615	8.096		
4	1	0.0494		0.5406	1.0899	22.06	0.042	0.4646		0.9447	22.49		

4	2		0.4026	1.5496	2.5433	6.317		0.3567	1.3859	2.2851	6.406
4	3		0.9564	2.7912	4.7912	5.010		0.8599	2.5344	4.3695	5.081
5	1		0.0494	0.5399	1.0798	21.86		0.0419	0.4641	0.937	22.36
5	2		0.4025	1.5084	2.3529	5.846		0.3561	1.35	2.1165	5.944
5	3		0.9264	2.5219	3.8058	4.108		0.8336	2.2891	3.4659	4.158
5	4		1.5648	3.7991	6.1629	3.938		1.4202	3.4716	5.6462	3.976
2	1		0.0391	0.4785	1.1847	30.30		0.0363	0.445	1.1022	30.36
3	1		0.0388	0.4349	0.9196	23.70		0.0360	0.4051	0.8582	23.84
3	2		0.3432	1.4584	2.7979	8.152		0.3217	1.3796	2.6486	8.233
4	1		0.0388	0.4307	0.8789	22.65		0.0360	0.4013	0.8213	22.81
4	2		0.3346	1.3068	2.1592	6.453		0.3144	1.2351	2.0443	6.502
4	3	0.15	0.8135	2.4085	4.1614	5.115	0.20	0.7715	2.2929	3.9692	5.145
5	1		0.0388	0.4303	0.8722	22.48		0.0359	0.4009	0.8154	22.71
5	2		0.3343	1.2735	2.0015	5.987		0.3135	1.2040	1.8964	6.049
5	3		0.789	2.1749	3.2985	4.181		0.7481	2.0701	3.1442	4.203
5	4		1.3496	3.3112	5.3924	3.996		1.2850	3.1633	5.1585	4.014