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UNSTEADY FREE CONVECTION FLOW OF NON- NEWTONIAN FLUID ALONG A CONTINUOUSLY MOVING STRETCHING SHEET WITH HEAT GENERATION AND RADIATION

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ABSTRACT

An unsteady MHD free convective flow of non-Newtonian fluids over a continuously stretching sheet has been analyzed to investigate the Radiation effects with Magnetic field and Heat generation. The governing non-linear partial differential equations of the problem have been transformed into a system of non-dimensional ordinary differential equations using dimensionless transformations. The resulting non- dimensional equations are then solved numerically, using explicit backward difference scheme. The results from numerical computations have been presented in the form of velocity and temperature profiles. The dimensionless velocity and temperature profiles are displayed graphically showing the effects for the different values of the involved parameters: Radiation parameter, Magnetic field parameter and Heat source parameter. The investigated results showed that the flow field is influenced by considering the parameters and the corresponding skin friction coefficient and nusselt number have been presented through graphs.

KEYWORDS: MHD, Stretching Sheet, Radiation and Heat Generation

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Original Article

INTRODUCTION

The effect of free convection and heat transfer on the flow of a non-Newtonian fluid along a stretching sheet has gained importance in many engineering areas and geophysics. The boundary layer concept of non-Newtonian fluid is too much important in case of various possible engineering applications such as wire drawing, metal and plastic extrusion, glass fiber production and paper production. The study of magneto hydrodynamic (MHD) flow plays an important role in agriculture, engineering and petroleum industries. In the above mentioned studies of MHD flow and heat transfer problems are restricted when technological processes take place at high temperature areas such as nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such high-tech areas. Many analytical and numerical studies have been analyzed to explain the various aspects of boundary layer flow.

The study of boundary layer flow over a stretching surface was first developed by Sakiadis [1]. He used similarity transformation to develop numerical solution. Erickson et al. [2] studied the velocity and temperature effects with heat and mass transfer in the boundary layer who extended the work of Sakiadis. Vajravelu and Hadyinicolaou [3] have studied the convective heat transfer in electrically conducting fluids near an isothermal stretching sheet and they studied the effect of internal heat generation or absorption. Cheng et al. [4] & A. Raptis [5] studied heat and mass transfer flow of free convection. Elableshy [6] studied heat transfer over a stretching sheet & Makinde [7] studied on mass transfer of free convection with radiation. Chaim [8] investigated the effect of heat transfer of electrically conducting fluids over a stretched surface in the presence of magnetic field. M.A. Samad et al. [9] have analyzed the effects of mass transfer on the free convection flow of non- Newtonian power-law fluids along a continuously moving stretching sheet. Recently, K.C. Saha [10] extended the work of M.A. Samad. In the present work, an attempt is taken to focus the effect of radiation, for more specific the problem has been extended for the case of unsteady non- Newtonian fluids to implement the effect of free convection with radiation and heat generation.

NOMENCLATURE

B Magnetic field strength C_f Skin friction coefficient C_p Specific heat at constant pressure F Magnetic Force g Acceleration due to gravity Gr Grashof number K Consistency coefficient k_1 Rosseland mean absorption coefficient L Characteristics length M Magnetic field parameter N Radiation parameter Nu Nusselt number n Power-law fluid index Pr Prandtl number Q_0 Heat generation constant Q Heat source parameter q_r Stream-Boltzman constant Re Reynolds number T Temperature T_w Temperature of the fluid near the boundary layer	T_∞ Temperature of the fluid away from the boundary layer u Velocity component along x direction U_0 Free stream velocity v_0 Velocity component along y direction x Coordinate along the plate y Coordinate normal the plate **Greek symbol** α Thermal diffusivity β Volumetric coefficient of thermal expansion μ Dynamic viscosity γ Kinematic viscosity ρ Density of the fluid σ Electric conductivity σ_1 Stefan-Boltzaman constant τ_w Local wall shear stress k Thermal conductivity

GOVERNING EQUATIONS

Let us consider an unsteady MHD free convective flow of a non- Newtonian fluid along a continuously moving stretching vertical sheet with heat generation and radiation. Here the system is taken to be Cartesian coordinate system with X-axis and Y-axis in the vertical and horizontal direction respectively. Two equal and opposite forces are introduced along the X-axis, so that the sheet is stretched keeping the origin fixed. The plate is maintained at a constant temperature T and the ambient temperature is T_∞. A strong magnetic field of strength B is imposed along the Y- axis and hence only the applied magnetic field B plays a role which gives rise to magnetic force in the X- direction.

Now considering the above assumptions the governing boundary layer equations becomes:

$$\frac{\partial v_0}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} = \frac{K}{\rho}\frac{\partial}{\partial y}\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y} + g\beta(T - T_\infty) - \frac{\sigma B^2 u}{\rho} \tag{2}$$

$$\frac{\partial T}{\partial t} + v_0 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}(T - T_\infty) - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

The appropriate initial and boundary conditions are:

$$u = 0, T = 0 \ at\ t = 0 \tag{4a}$$

$$\left.\begin{array}{l} u = U, T = T_w \text{ at } y = 0 \\ u = 0, T = T_\infty \text{ as } y \to \infty \end{array}\right\} \text{at } t > 0 \tag{4b}$$

The effect of the second term on the right side of the momentum equation is due to buoyancy force so here we consider the case $s = 1.0$ i.e the free convection flow (buoyancy effect is dominant). To describe the radiactive heat flux the Rosseland approximation is used which is negligible in the X-direction compared to the Y-direction in the energy equation. The radiative heat flux q_r is described by the Rosseland approximation such that

$$q_r = \frac{4\sigma_1}{3k_1}\frac{\partial T^4}{\partial y} \tag{5}$$

Ihe temperature difference are sufficiently small so T^4 can be expressed in a Taylor series and then neglecting higher-order terms we have the following approximations:

$$T^4 \approx 4T_\infty^4 - 3T_\infty^4 \tag{6}$$

Now the last term of equation (3) is obtained as folowing

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma_1 T_\infty^3}{3k_1}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

Introducing q_r in (3), we obtain the following governing boundary layer equations:

$$\frac{\partial T}{\partial t} + v_0\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}(T - T_\infty) + \frac{1}{\rho c_p}\frac{16\sigma_1 T_\infty^3}{3k_1}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

NON-DIMENSIONALIZATION

The solutions of the governing equations (2) and (8) under the boundary condition (4) will be based on a finite difference scheme it is required to make the said equations dimensionless. For this purpose it has been now introduced the following dimensionless variables;

$$Y = \frac{y}{L}, U' = \frac{u}{U_0}, V' = \frac{v_0}{U_0}, t' = \frac{tU_0}{L},\ T' = \frac{T - T_\infty}{T_w - T_\infty} \tag{9}$$

$$y = YL, u = U'U_0, v_0 = V'U_0, t = \frac{t'L}{U_0}, T - T_\infty = T'(T_w - T_\infty) \tag{10}$$

Now, we substitute the values of the above dimensionless variables into the equations (1), (2) and (8) and by simplifying we obtain the following nonlinear coupled partial differential equations with initial and boundary conditions are:

$$\frac{\partial V'}{\partial Y} = 0 \tag{11}$$

$$\frac{\partial U'}{\partial t'} + V'\frac{\partial U'}{\partial Y} = \frac{1}{Re}\frac{\partial}{\partial Y}\left|\frac{\partial U'}{\partial Y}\right|^{n-1}\frac{\partial U'}{\partial Y} + GrT' - MU' \tag{12}$$

$$\frac{\partial T'}{\partial t'} + V'\frac{\partial T'}{\partial Y} = \left(\frac{3N+4}{3N}\right)\frac{1}{RePr}\frac{\partial^2 T'}{\partial Y^2} + QT' \tag{13}$$

$$U' = 0, T' = 0 \textit{ at } t' = 0 \tag{14a}$$

$$\left.\begin{array}{l} U' = 1, T' = 1, Y = 0 \\ U' = 0,\ T' = 0, Y \to \infty \end{array}\right\} \text{at } t' > 0 \tag{14b}$$

Where

$Re = \frac{U_0^{2-n}L^n}{K/\rho}$ is the local Reynolds number.

$Gr = g\beta(T_w - T_\infty)\frac{L}{U_0^2}$ is the Grashof number.

$M = \frac{\sigma B^2 L}{\rho U_0}$ is the Magnetic field parameter.

$P_r = \frac{K}{\alpha\rho}(\frac{U_0}{L})^{n-1}$ is the Prandtl number.

$Q = \frac{Q_0 L}{\rho C_p}$ is the Heat source parameter.

$N = \frac{KK_1}{4\sigma_1 T_\infty^3}$ is the Radiation number.

FINITE DIFFERENCE TREATMENT

The set of non- linear partial differential equations (1) - (3) with initial and boundary conditions are difficult to solve them analytically since they are non- linear and coupled. For simplicity the explicit finite difference scheme has been used to solve momentum and energy equations subject to the conditions. The process is repeated in time and provided the time-step is sufficiently small. Figure 1 shows the velocity and temperature profiles for different step sizes. Here to satisfy the convergence criterion of 10^{-4} we have chosen step size $h = 0.10$ in all cases. In order to verify the effects of step size $h = 0.13, h = 0.10, h = 0.08$ the programming code is run our model with three different step sizes each case we find excellent agreement among them.

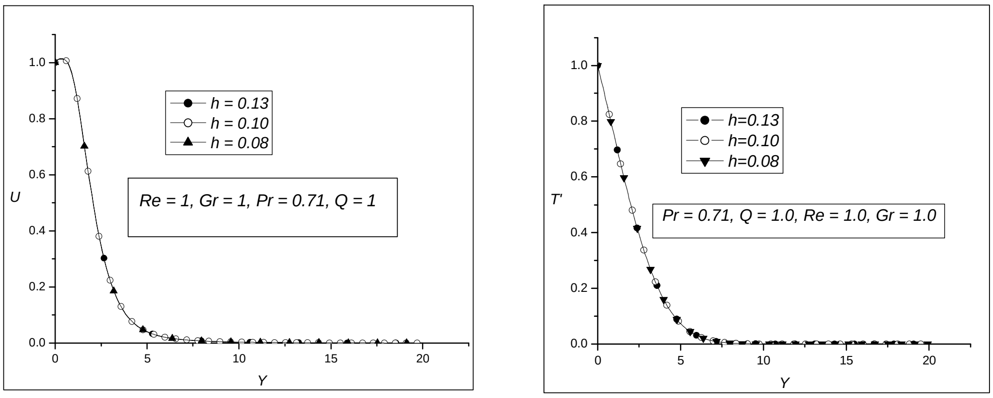
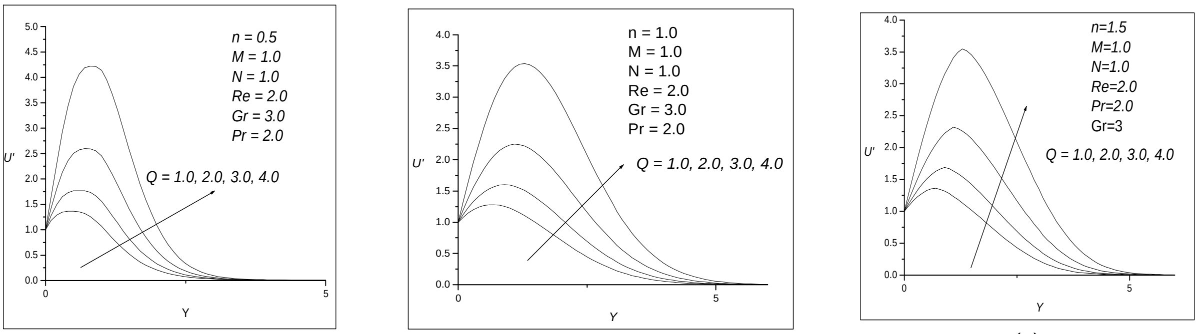


Figure 1: Velocity Profile and Temperature Profile for Different Step Sizes

RESULTS AND DISCUSSIONS

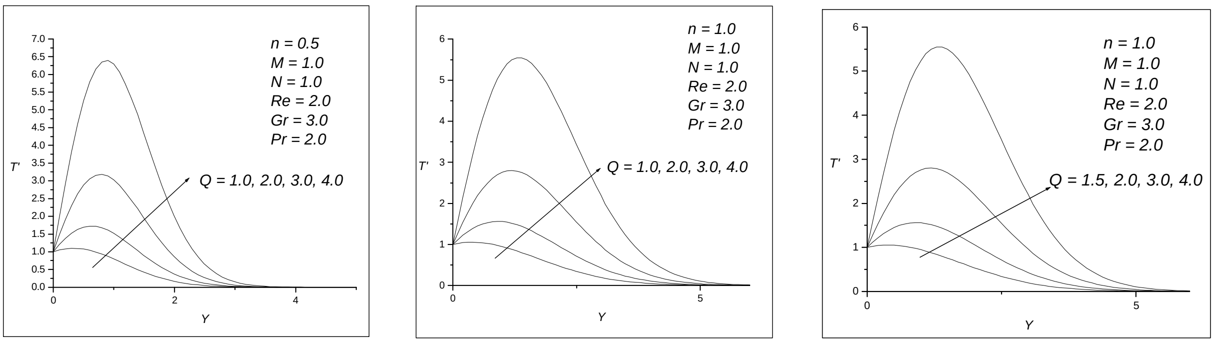
For the purpose of discussing the results found from the present investigation, the numerical calculations are presented in the form of non- dimensionsal velocity and temperature profiles. Numerical computations have been found for different values of the dimensionless parameters such as Magnetic field parameter (M), Heat source parameter (Q), Grashof number (Gr), Prandtl number (Pr) and Radiation parameter (N).



(a) (b) (c)

Figure 2: Velocity Profiles for Different Values of Heat Source Parameter (Q) of

(a) Pseudo- Plastic (b) Newtonian and (c) Dilatant's at Time t =1

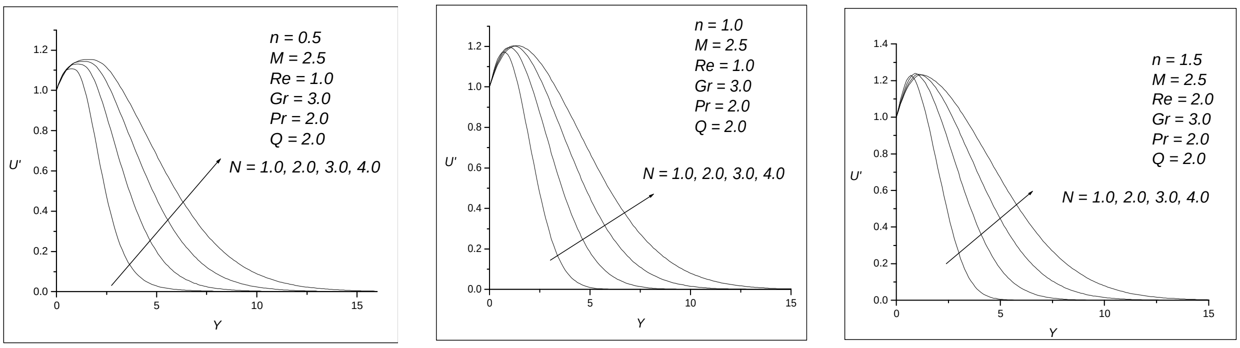


(a) (b) (c)

Figure 3: Temperature Profile for Different Values of Heat Source Parameter (Q) of

(a) Pseudo-Plastic (b) Newtonian and (c) Dilatant's at Time t =1

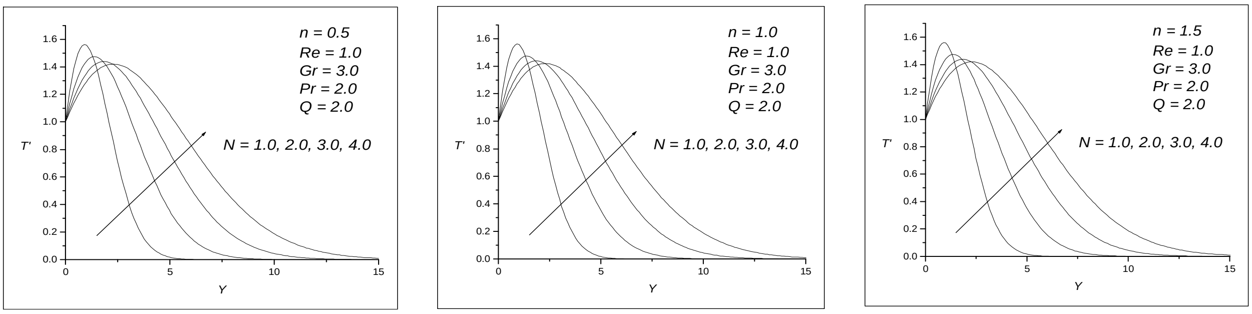
The effects of heat source parameter (Q) on the dimensionless velocity for Pseudo- plastics, Newtonian fluids and Dilatant's are discussed in Figure 2a, Figure 2b and Figure 2c respectively. It is observed that the velocity profiles increase very rapidly and overshoot with the increase of Heat source parameter(Q). For $Q = 4.0$, we get the highest peak near the wall due to the effect of Q on the stretching sheet and there is a sharp rise in velocity near the sheet. The average increase of velocity between $Q = 3.0$ to $Q = 4.0$ is 12.26% in case of Pseudo-plastics where it is 16.37% for Newtonian fluids and 15.72% for Dilatant's. From the above discussion we observe that Heat parameter (Q) is too much effective on the velocity of Newtonian fluids. Figure 3a, Figure 3b and Figure 3c shows the effects of Heat source parameter on the dimensionless temperature where figure 3a, figure 3b and figure 3c represents the temperature profiles for pseudo-plastics, Newtonian fluids and Dilatants.



(a) (b) (c)

Figure 4: Velocity Profiles for Different Values of Radiation Parameter (N) of

(a) Pseudo- Plastic (b) Newtonian and (c) Dilatant's at Time t =1



(a) (b) (c)

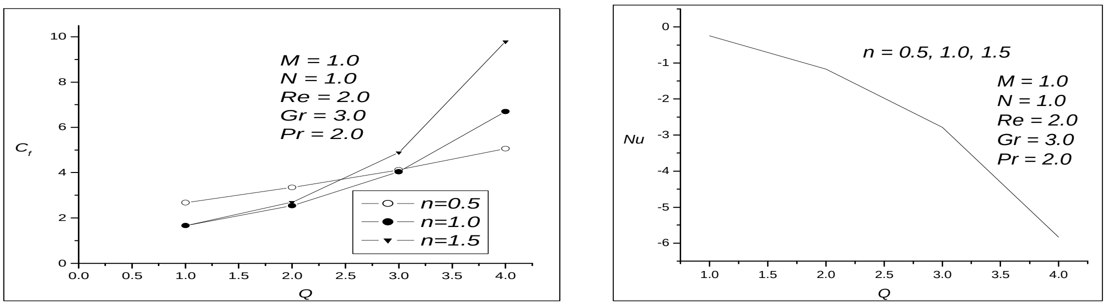
Figure 5: Temperature Profiles for Different Values of Radiation Parameter (N) of

(a) Pseudo-Plastic (b) Newtonian and (c) Dilatant's at Time t =1

We observed that the temperature profiles increase rapidly near the sheet as the Heat source parameter (Q) increase. In all cases of Newtonian and non- Newtonian fluids the average increasing rate of temperature for $Q = 1.0$ to 2.0 is 17.92%, for $Q = 2.0$ to 3.0 is 31.78% and at last the increasing rate is 60.56%. Hence it is clear from the figure that for $Q > 2.0$ there exists overshooting in the temperature profiles because the stretching sheet gains temperature from environment.

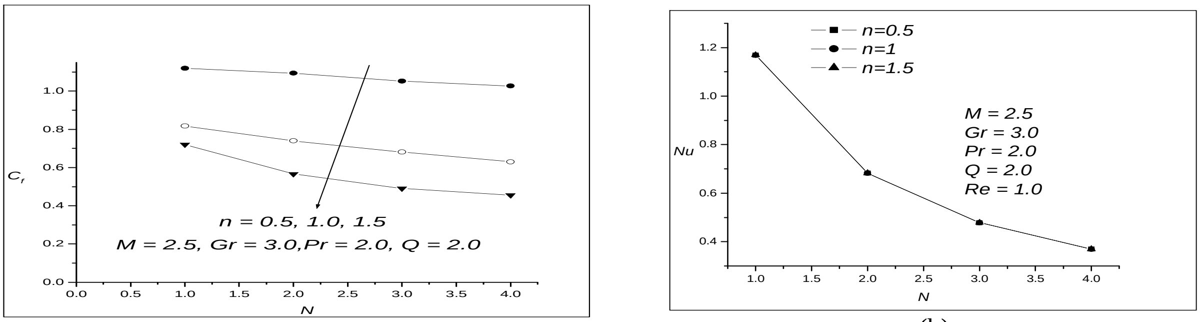
Figure 4a, Figure 4b and Figure 4c represents the velocity profiles of Pseudo-plastics, Newtonian fluids and Dilatant's where in these figures we discussed the effects of Radiation parameter (N) on the dimensionless velocity. It is clearly seen from the above three figures that the increasing Radiation parameter (N) lead to a increase in the velocity which confirmed with the fact that the velocity profiles rise near the stretching sheet for all values of N. We found that in each case of Pseudo-plastics, Newtonian fluids and Dilatant's velocity increases as Radiation parameter (N) increases. Physically, we know that higher radiation occurs when temperature is higher and ultimately velocity rises. In case of Pseudo-plastics the increasing rate of velocity between $N = 1.0$ to $N = 2.0$ is 6.83% and between $N = 3.0$ to $N = 4.0$ is 6.72%. In case of Newtonian the increasing rate of velocity between $N = 1.0$ to $N = 2.0$ is 7.00% and between $N = 3.0$ to $N = 4.0$ is 6.78%. In case of Dilatant's the increasing rate of velocity between $N = 1.0$ to $N = 2.0$ is 7.00% and between $N = 3.0$ to $N = 4.0$ is 6.83%. From the above discussion we observe that Radiation parameter (N) is too much effective on the velocity of Dilatant's. The effects of Radiation parameter (N) on the dimensionless temperature for Pseudo- plastics, Newtonian fluids and Dilatant's are discussed in Figure 5a, Figure 5b and Figure 5c respectively. It is observed from the above three figures that the increasing Radiation parameter (N) also lead to an increase in the temperature profiles and temperature profile first increase rapidly then start to decrease so we find a cross flow here. The average increasing rate of temperature between $N = 1.0$ to $N = 2.0$ is 16.79%, between $N = 2.0$ to $N = 3.0$ is 8.88% and between $N = 3.0$ to $N = 4.0$ is 5.35% and this increasing rate is same in all cases of Newtonian and non- Newtonian fluids and increasing rate of temperature is higher because higher radiation occurs at higher temperature.

We have observed from Figure 6a and Figure 6b which display the effects of Heat source parameter(Q) for selected values of power-law index (n) on the skin friction coefficient (C_f), Nusselt number (Nu). It is clearly seen from figures that due to the increase of heat source parameter(Q) the skin friction coefficient increases and Nusselt number (Nu) decreases when power-law index (n) is fixed but with the increase of power-law index (n) Nusselt number (Nu) is fixed. Figure 7a and Figure 7b depict the effects of Radiation parameter (N) on the skin friction coefficient (C_f), Nusselt number (Nu) at the selected values of n and keeping all other parameter fixed**.** We observe that the skin friction coefficient (C_f) decreases with the increase of Radiation parameter (N) and with the increase of power-law index(n).



(a) (b)

Figure 6: (a) Heat Source Parameter (Q) Effect on (a) Skin Friction Coefficient C_f and Nusselt Number (Nu) for Selected Values of n



(a) (b)

Figure 7: (a) Radiation Parameter (N) Effect on (a) Skin Friction Coefficient C_f and Nusselt Number (Nu) for Selected Values of n

It is very interesting that skin friction coefficient (C_f) is very high in case of Pseudo-plastic comparing to the other fluids and On the other hand the Nusselt number (Nu) decreases with the increase of Radiation parameter (N). In every case of Pseudo plastics, Dilatants and Newtonian fluids Nusselt number first increases with the increase of radiation parameter (N) then slowly decreases to zero which is an excellent agreement with velocity and temperature profiles.

CONCLUSIONS

By summarizing the present study we can make the following conclusions:

- There exists a significance effect of Heat source parameter on velocity and temperature profiles. Velocity and temperature of the flow field increase as the Heat source parameter increases. A strong Heat source parameter is applied to increase the wall temperature of Pseudo-plastic fluids.
- Velocity profile and temperature profile of the flow field increases with the increase of Radiation number. So, thermal radiation parameter can be used effectively to control the entire flow field. Radiation parameter can be applied to increase the velocity of the Dilatant's fluids. Hence the heat transfer rate of Dilatant fluids is greater than Pseudo-plastic.
- Using Heat source parameter we can control skin friction coefficient and Nusselt number. Skin friction coefficient increases and Nusselt number decreases with the increase of Heat source parameter.
- There exists a significant influence of Radiation parameter on skin friction coefficient and Nusselt number. With the increase of Radiation parameter skin friction coefficient and Nusselt number is also decreases.

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