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INVESTIGATIONS OF THE INVARIANCE OF THE OUTPUT OF A COMPLEX ELECTRIC SYSTEM BASED ON THE TECHNOLOGY OF EMBEDDING SYSTEMS

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ABSTRACT

In this paper, consider the application of embedding technology for the study of the invariance of the output of complex controlled electrical systems under small perturbations, as stationary deterministic multidimensional dynamical systems. The regulator synthesis technique based on the modern matrix theory is presented. The results of the synthesis of the regulator of the model of a multi-machine electrical system are obtained, which allow analyzing the influence of the parameters of the electric system regime.

KEYWORDS: Electric System, Regulator Synthesis, Technology of Embedding Systems, Invariance, Canonizer & Zero Divisors

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Original Article

1. INTRODUCTION

One of the most important properties of a dynamical system is its invariance. As noted in [1], the problem of invariance is the problem of determining such structures and parameters of control systems in which the influence of arbitrarily changing external disturbances and the system's own parameters on the dynamic characteristics of control processes can be partially or completely compensated. The automatic control systems created in this way have very high levels of accuracy and quality and are also less susceptible to various kinds of noise or interference.

This problem was quite extensive, for the first time it was posed by a professor G.V. Shchipanov [2], and to this day a wide discussion of its application continues [3-6].

It should be noted that there are different types of invariant systems that differ in both the principles of construction and functionality.

This is due to the fact that input-output sets of complex electric power systems (EPS) are subject to ambiguous transformations due to the presence of zero divisors and non-commutativity of operators [7], i.e. algebraic singularities of the system under investigation, which are characteristic only of multidimensional systems.

The invariance problem is posed as follows. In the case of the representation of the system under study in the state space [8]:

$$\dot{x} = Ax + Bu + Sw,$$
$$u = -Kx, \tag{1}$$
$$y = Cx,$$

where *x, u, y, w* are the state, control, output and disturbance vectors of the system, respectively; *A, B, C, S* are matrices with constant numerical elements of appropriate dimensions; *K* is a matrix with constant numerical elements, called a regulator. For the invariance of the output of the controlled system under investigation, the transfer matrix from the perturbation *w(p)* to the output of the system *y(p)* with the model in the state space must be identically equal to zero:

$$F_y^w(p) = C(pI_n - A_y)^{-1} S = 0, \tag{2}$$

Where A_y=*A+BK* is the dynamics matrix of the system with a regulator. The main task is to find a regulator (synthesizing) that ensures the fulfillment of the condition (2). However, as noted in [9], certain difficulties arise in solving this problem, since in (2) there is an operation of inversion of the matrix, and, as a rule, polynomial.

2. MATHEMATICAL MODEL OF THE REGULATOR

Necessary and sufficient conditions under which identity (2) holds are satisfied if the requirements of the theorem [10] are satisfied that the system (1) for given matrices *A, B, C,* and *S* has invariance to perturbations in the sense of the identity, if and only if the following condition is satisfied:

$$\overline{\overline{C}^R \pi}^L S = 0, \tag{3}$$

whereπ is the matrix of maximum column rank corresponding to the condition:

$$\overline{\overline{C}^R \pi}^L A_y \overline{C}^R \pi = 0, \tag{4}$$

in which the identity is satisfied:

$$\overline{\overline{\overline{C}^R \pi}^L B}^L \, \overline{\overline{C}^R \pi}^L A \overline{C}^R \pi = 0, \tag{5}$$

and the system is closed by any controller from the set:

$$\{K\}_{\gamma,\chi} = -\left(\overline{\overline{C}^R \pi}^L B\right)^{\sim} \overline{\overline{C}^R \pi}^L A \overline{C}^R \pi \left(\overline{C}^R \pi\right)^{\sim} + \overline{\overline{\overline{C}^R \pi}^L B}^K \chi + \gamma \overline{\overline{C}^R \pi}^L, \tag{6}$$

whereχandγarematricesofgivensizeswitharbitrarynumericelements, $\overline{C}^R$ - rightdivisorofmatrixzero*C*, $\overline{\overline{C}^R \pi}^L$ - left divisor of matrix zero $\overline{C}^R \pi$, matrices with the upper notation (~) are summary canonizers of corresponding matrices, double and triple dashes over matrices denote the repeated determination of the corresponding zero-divisor of the maximum rank from the combination of matrices consisting of this bar.

An algorithm that makes it possible to form a matrix π of maximal rank satisfying condition (4) in a finite number of steps is as follows [11].

1. The Condition is Checked

$$\overline{CB}^L CA\overline{C}^R = 0. \quad (7)$$

If this condition is met, it is assumed$\pi = \pi_0 = I_{(n\text{-}rankC)}$.

2. If Condition (7) is Not Satisfied, then the Matrix π_1 is Determined by the Formula

$$\overline{\overline{CB}^L CA\overline{C}^R}^R = 0. \quad (8)$$

If $\pi_1 = 0$, then the system does not have invariance, the algorithm stops. Otherwise, condition (4) is checked for $\pi = \pi_1$.

3. The matrix π_i for i> 1 is Determined by the Formula

$$\pi_i = \overline{\overline{\overline{\overline{C}^R \pi_{i-1}}^L B}^L \overline{\overline{C}^R \pi_{i-1}}^L A\overline{C}^R}^R \quad (9)$$

and the condition (5) is satisfied.

4. The algorithm stops at the k-th step when the condition (7) is first satisfied. The matrix π of maximal rank has the value π_k.

Consequently, the verification of the invariance of the output system (1) is reduced to the reduced iterative process of determining $\pi$and the calculation of the transfer matrix $F_y^w(p)$ is not required.

3. EXAMPLE

Apply the above method of determining the invariance of a dynamical system by output using the example of a model of an adjustable electrical system (Figure 1), the parameters of which are given in [3], without taking into account the damper coefficients of the generators. When solving the invariance problem, the matrix canonization method is used.

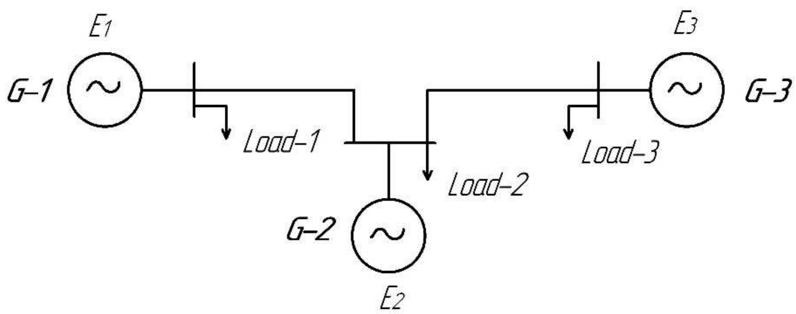

Figure 1: Diagram of a Three-Generator Electrical System

The matrices of the intrinsic dynamics of the model studied by EPS have the form [3]:

$$A=\begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\-57.2&27.53&33.42&0&0&0\\39.33&-92.08&50.53&0&0&0\\23.61&50.53&-95.15&0&0&0\end{bmatrix}, \quad (10)$$

$$C=[\Delta\delta_1\ 0\ 0\ 0\ 0\ 0]=[1\ 0\ 0\ 0\ 0\ 0],\ \text{S}=\begin{bmatrix}0&0&0\\1&0&0\\1&0&1\\0&0&0\\0&1&0\\0&0&1\end{bmatrix}, \quad (11)$$

$$B=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\-2.0837&0&0\\0&-2.0678&0\\0&0&0.97\end{bmatrix}. \quad (12)$$

The calculation shows that with the adopted parameters of the regime (the base version) the system is not stable, as can be seen from the spectrum of the matrix (10) of the intrinsic dynamics of the investigated EPS: 0 ± 11.9757i, 0 ± 9.7538i, 0 ± 2.4239i.

To check the required conditions for the invariance of the output of the EPS and in the final result of determining the parameters of the controller (6), we find successively the corresponding matrices.

Condition (7) requires the definition of the right divisor of matrix *C* and the left divisor of the matrix *CB*, which we obtain as a result of the canonization of these matrices:

$$\overline{C}^R=\begin{bmatrix}0&0&0&0&0\\1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix},\ CB=[0\ 0\ 0],\ \overline{CB}^L=\overline{[0\ 0\ 0]}^L=1,$$

$$CA\overline{C}^R=[0\ 0\ 1\ 0\ 0],\ \overline{CB}^L CA\overline{C}^R=[0\ 0\ 1\ 0\ 0]\neq 0.$$

Condition (7) does not hold, therefore, by formula (8) we define the matrix π_1:

$$\pi_i = \overline{\overline{C B}^L C A \overline{C}^R}^R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Check the fulfillment condition (5) for $\pi = \pi_1$:

$$\overline{C}^R \pi_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ \overline{C}^R \pi_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\overline{\overline{C}^R \pi_1}^L B = \begin{bmatrix} 0 & 0 & 0 \\ 2.0837 & 0 & 0 \end{bmatrix}, \ \overline{\overline{\overline{C}^R \pi_1}^L B}^L = [1 \ \ 0],$$

$$\overline{\overline{C}^R \pi_1}^L A \overline{C}^R \pi_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 33.42 & 27.53 & 0 & 0 \end{bmatrix}, \ \overline{\overline{\overline{C}^R \pi_1}^L B}^L \overline{\overline{C}^R \pi_1}^L A \overline{C}^R \pi_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus, condition (5) is satisfied. Next, we verify that condition (3) holds.

$$\overline{\overline{C}^R \pi_1}^L S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Consequently, since condition (3) is satisfied, it is possible to form the matrix of the coefficients of the controller (6), for which it is necessary to determine the matrices entering into this formula:

$$\left(\overline{\overline{C}^R \pi_1}^L B\right)^{\sim} = \begin{bmatrix} 0 & 0.4799 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \ \left(\overline{C}^R \pi_1\right)^{\sim} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \overline{\overline{\overline{C}^R \pi_1}^L B}^R = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Substituting the obtained numerical values of the matrices in (6), we obtain:

$$\{K\}_{\gamma,\chi} = -\left(\overline{\overline{C}^R \pi}^L B\right)^{\sim} \overline{\overline{C}^R \pi}^L A \overline{C}^R \pi \left(\overline{C}^R \pi\right)^{\sim} + \overline{\overline{\overline{C}^R \pi}^L B}^K \chi + \gamma \overline{\overline{C}^R \pi}^L =$$

$$= \begin{bmatrix} \gamma_{11} & -13.21 & -16.04 & \gamma_{12} & 0 & 0 \\ (\gamma_{21} + \chi_1) & \chi_2 & \chi_3 & (\gamma_{22} + \chi_4) & \chi_5 & \chi_6 \\ \gamma_{31} & 0 & 0 & \gamma_{32} & 0 & 0 \end{bmatrix}, \qquad (13)$$

where the forming matrices χ and γ, with arbitrary numerical values are taken in the form:

$$\chi = [\chi_1 \quad \chi_2 \quad \chi_3 \quad \chi_4 \quad \chi_5 \quad \chi_6] \text{ and } \gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \\ \gamma_{31} & \gamma_{32} \end{bmatrix}.$$

In [3], a regulator was justified, which for the three-generator electric system has the form:

$$K = \begin{bmatrix} k_{E_{q1}}^{\Delta\delta_1} & 0 & 0 & k_{E_{q1}}^{\Delta s_1} & 0 & 0 \\ 0 & k_{E_{q2}}^{\Delta\delta_2} & 0 & 0 & k_{E_{q2}}^{\Delta s_2} & 0 \\ 0 & 0 & k_{E_{q3}}^{\Delta\delta_3} & 0 & 0 & k_{E_{q3}}^{\Delta s_3} \end{bmatrix}, \quad (14)$$

withthecoefficientsoftheparametersoftheelectricalsystem.

The synthesized regulator (14) with matrices χ and γ with arbitrary numerical values of elements should be designed in such a way that necessary technical requirements are provided in the dynamic system: stability, damping of low-frequency oscillations [12-13].

Note that (13) must be coordinated with (14), namely, we can assume $k_{E_{q1}}^{\Delta\delta_1}=\gamma_{11}$, $k_{E_{q1}}^{\Delta s_1}=\gamma_{12}$, $k_{E_{q2}}^{\Delta\delta_2}=\chi_2$, $k_{E_{q2}}^{\Delta s_2}=\chi_5$,and the remaining elements are equal to zero and finally the matrix of the coefficients of the controller (13) will have the form:

$$\{K\}_{\gamma,\chi} = \begin{bmatrix} \gamma_{11} & 0 & 0 & \gamma_{12} & 0 & 0 \\ 0 & \chi_2 & 0 & 0 & \chi_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (15)$$

Characteristically, with the chosen output matrix of the system *C* under study and the perturbation matrix *S*, the third regulator is not involved in regulating the regime of a complex electrical system.

Now let check the influence of the regulator (15) on the spectrum of the intrinsic dynamics of a regulated electrical system having a matrix

$$A_y=A+BK. \quad (16)$$

Choose $k_{E_{q1}}^{\Delta\delta_1}=\gamma_{11}=-10$, $k_{E_{q1}}^{\Delta s_1}=\gamma_{12}=-2$, $k_{E_{q2}}^{\Delta\delta_2}=\chi_2=8$, $k_{E_{q2}}^{\Delta s_2}=\chi_5=1$, under which the matrix (16) is equal to:

$$A_y = A + BK = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -78.037 & 23.53 & 33.42 & -4.167 & 0 & 0 \\ 39.33 & -108.62 & 50.53 & 0 & -2.0678 & 0 \\ 23.61 & 50.53 & -95.15 & 0 & 0 & 0 \end{bmatrix},$$

with the spectrum: -1.0790 ± 4.2951i; -0.6007 ± 12.2361i; -1.4379 ± 10.3156i.

With the selected parameters of the controller (15), the electrical system became stable, with one electromechanical frequency of 0.6839 Hz, and two electromagnetic frequencies, respectively 1.6426 and 1.9484 Hz. (Figure 2).

It is obvious that in the presence of A_y, it is possible to comprehensively investigate the dynamic properties of a complex regulated electric system by varying the parameters of the regulator (15), including determining the conditions for the invariance of the output of the system under investigation to perturbations arising in the system under study.

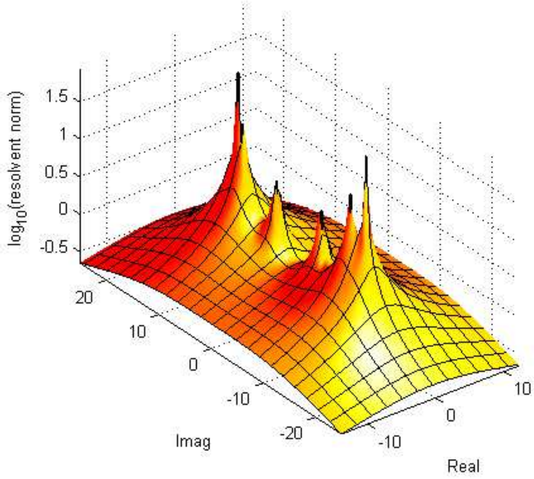

Figure 2: 3D-Visualization of the Roots of the Electrical System under Study with the Spectrum: -1.079 ±4.2951i; -0.6007±12.2361i; -1.4379±10.3156i

The horizontal axes in Figure 2 correspond to the axes of the complex plane, the logarithm of the norm of the resolvent function is laid along the vertical axis, the peaks localize the eigenvalues of the matrix.

Figure 2 shows the characteristics of the change in the deviation of the angle of the first generator $\Delta\delta_1$=f(t), the stable, regulated (16) electric system (Figure 3, A), with the synthesized parameters of the regulator (15), and the unstable, unregulated power plant (10), (Figure 3, B). The process decays rapidly enough and has an almost aperiodic character.

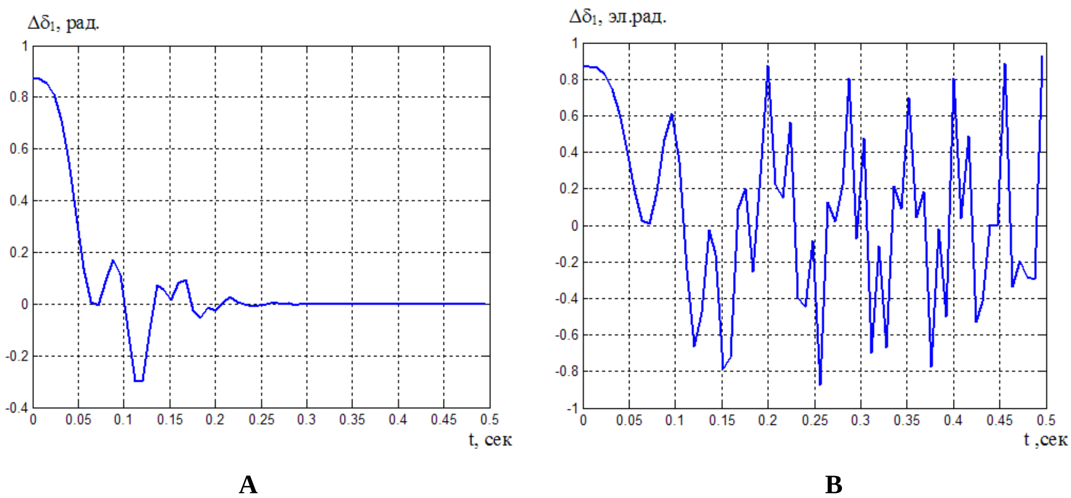

Figure 3: Characteristics of the Change in the Angle of the First Generator $\Delta\delta_1$=f(t) three-Generator Electrical System

4. CONCLUSIONS

The above procedure for the synthesis of the electrical system regulator is quite simple, since it is based on the modern theory of matrices adapted for computer applications, and therefore it is computationally efficient and can be recommended for the study of complex, automatically controlled complex EPS.

Thus, it can be noted that on the basis of the method of canonization of matrices, which is the basis of the technology of embedding systems, the conditions for the invariance of the output of the electrical system to arbitrary external disturbances are justified. To solve this problem, an appropriate regulator has been synthesized.

The main difference between the application of the technology of investment of systems is the reduction of the number of computational costs since this technology is based on the matrix analysis for which rich software complexes have been developed. In addition, an analytical description of the class of regulators ensuring the required dynamic properties of the systems under study, including invariance, is also very important.

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