

DESIGN OF DISCRETE HIGH PASS FILTER FOR HIGH SPEED OPERATION

ROHITASHWARASTOGI, PARAS & ABHISKEKTOMAR

Department of Electronics & Communication Engineering, College of Technology,
G. B. Pant University of Ag & Technology, Pantnagar, Uttarakhand, India

ABSTRACT

This paper presents a comparison of all discretization methods involved in design of Digital recursive filter and proposes a method to design a High Pass Filter with flat magnitude response in pass band for High Speed Application. The optimization is achieved by Correction of Filter Coefficients. The performance of three methods of filter designs have been compared.

KEYWORDS: Discrete Filter, Discretization, Parameter Correction, Step Response, LPF(Low Pass Filter), HPF (High Pass Filter)

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INTRODUCTION

The transfer function for a discrete filter(IIR)can be obtained by two major techniques that are: Approximation Method and Sampling Method. In approximation method an approximation is taken as the property of the an along filter. In major we opt for area under the curve and the slope of the curve. While in Sampling Method we directly sample the impulse response or step response of the an agog filter. Techniques have been given to design digital filters in [1]. Three methods have been used to design the LPF and performances have been shown [2]. The transfer function of an along filter is characterized by 's', in the approximation method. Following are the mainly used Integrators

Implicit Euler Method

$$S \rightarrow \frac{z-1}{Tz} \quad (1a)$$

Explicit Euler Method

$$S \rightarrow \frac{z-1}{T} \quad (1b)$$

Trapezium Method

$$S \rightarrow \frac{z-1}{T(z+1)} \quad (1c)$$

In general the equation for analog High pass filter is given by

$$H(\omega) = \frac{s^2}{BT_1s^2 + Bs + 1} = \frac{s^2}{s^2 + 2\zeta\omega_0s + \omega_0^2} \quad (2)$$

Where B is the Time Constant of the second order link integration, ω_0 is the frequency of free oscillations, and ζ is the damping coefficient.

$$\omega_0^2 = \frac{1}{BT_1}, \zeta = \sqrt{\frac{B}{4T_1}} \quad (3)$$

$$\text{At, } B = 2T_1 \quad \text{or } \zeta = \frac{1}{\sqrt{2}}$$

The filter becomes a Butterworth High Pass Filter

In general, the transfer function of the digital filter of the second order has the form

$$H(z) = \frac{B_2z^2 + B_1z + B_0}{A_2z^2 + A_1z + A_0} \quad (4)$$

And while filter implementation the difference equation is

$$Y(n) = \frac{1}{A_2} [B_2X(n) + B_1X(n-1)] + B_0X(n-2) - A_1Y(n-1) - A_0Y(n-2) \quad (5)$$

METHOD OF ANALYSIS

Often the magnitude response of a digital LPF is not flat in pass band but the optimized magnitude response of a digital LPF can be obtained as shown in Figure 1. To achieve such flat magnitude response in pass band, the transfer function $|H(j\omega)|$ should follow the condition for flatness. But as the Function $|H(j\omega)|$ is an even function of ω , then we can say if it is continuous and differentiable at $\omega \rightarrow 0$, then all odd derivatives of $|H(j\omega)|$ should be zero.

Hence for flat response:

$$\lim_{\omega \rightarrow 0} \frac{\partial^k}{\partial \omega^k} |H(j\omega)| = 0 \quad \text{For all values of } k \quad (6)$$

For all odd values of k this function is automatically zero. Putting $k=2$; we have the relation

$$b_2(4b_0 + b_1) + b_1b_0 = a_2(4a_0 + a_1) + a_1a_0 \quad (7)$$

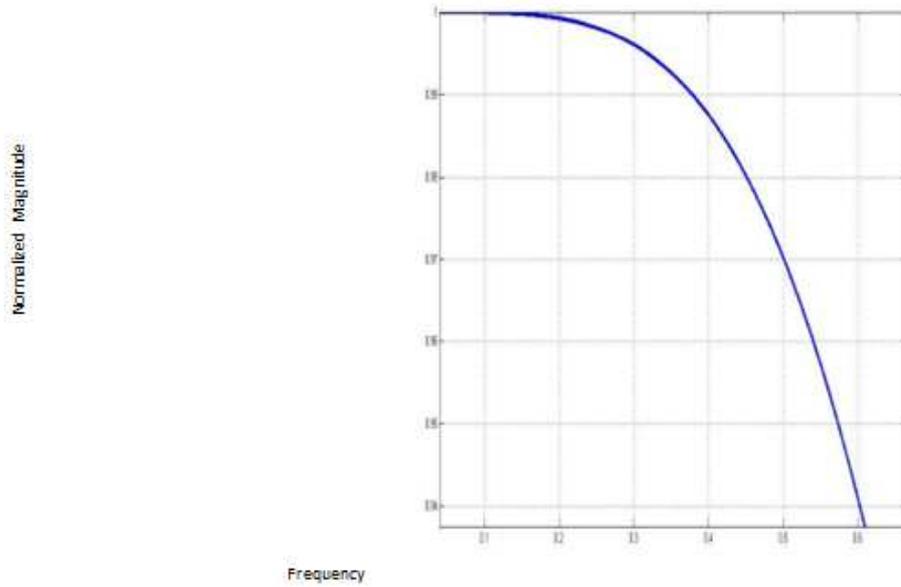


Figure 1: Magnitude Plot of a Butterworth LPF

As our goal is to design a digital high pass filter with flat magnitude response, we can derive an equation for this by using equation (7). Various filters have been compared in [3].

Figure 2 shows the magnitude responses of a Digital HPF from which we can easily observe that the magnitude response of Digital HPF is a shifted version of that of Digital LPF, hence if we produce a respective shift in the graph, we will have to apply in equation (2) to get the constraint for HPF.

Now shifting the graph by π , the new coefficients will be:

$$H(j\omega T - \pi T) = \frac{b_2 e^{2(j\omega T - \pi T)} + b_1 e^{(j\omega T - \pi T)} + b_0}{a_2 e^{2(j\omega T - \pi T)} + a_1 e^{(j\omega T - \pi T)} + a_0} \tag{8}$$

$$H(j\omega T - \pi T) = \frac{b_2 e^{-2j\pi T} e^{2j\omega T} + b_1 e^{-j\pi T} e^{j\omega T} + b_0}{a_2 e^{-2j\pi T} e^{2j\omega T} + a_1 e^{-j\pi T} e^{j\omega T} + a_0} \tag{9}$$

$$H(j\omega T - \pi T) = \frac{B_2 e^{2j\omega T} + B_1 e^{j\omega T} + B_0}{A_2 e^{2j\omega T} + A_1 e^{j\omega T} + A_0} \tag{10}$$

Where

$$B_2 = b_2 e^{-2j\pi T}, B_1 = b_1 e^{-j\pi T}, B_0 = b_0$$

$$A_2 = a_2 e^{-2j\pi T}, A_1 = a_1 e^{-j\pi T}, A_0 = a_0$$

Assuming $T=1$, the relation of coefficients will be $B_2 = b_2$, $B_1 = -b_1$, $B_0 = b_0$ and $A_2 = a_2$, $A_1 = a_1$, $A_0 = a_0$ and the relation for flat response will be

$$b_2(4b_0 - b_1) - b_1b_0 = a_2(4a_0 - a_1) - a_1a_0 \tag{11}$$

Now applying different methods of discretization we will find a suitable method for the high speed design.

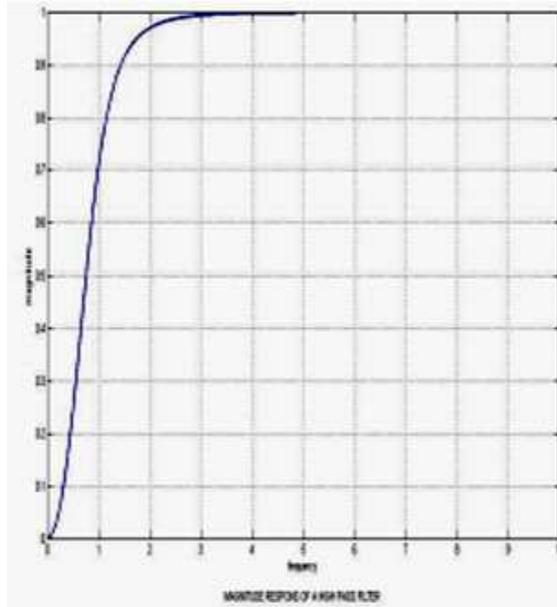


Figure 2: Magnitude Plot of a Butterworth HPF

Now as we know a constraint for a flat magnitude response as per the equation (11). We shall analyse different Integration method for the design of IIR filter and observe their stability as well as step response.

DESIGN OF FILTERS USING DIFFERENT APPROXIMATIONS

Applying Back ward Euler Method (Case A)

We have

$$H(z) = \frac{z^2 - 2z + 1}{(2B + 1)z^2 - 3Bz + B} \tag{12}$$

And applying constraint equatic

$$(2B + 1)(4B + 3B) + 3B^2 = 8$$

Accepting positive value of B, we have $B = 0.51$ and applying this value in filter equation we have

$$H(z) = \frac{z^2 - 2z + 1}{2z^2 - 1.5z + 0.5} \tag{13}$$

Now by analyzing this equation we can check that the pole so f above filter are lying inside the Unit circle $|Z|=1$; hence the filter is table (Figure 3). Figure 4 shows the magnitude response of this filter.

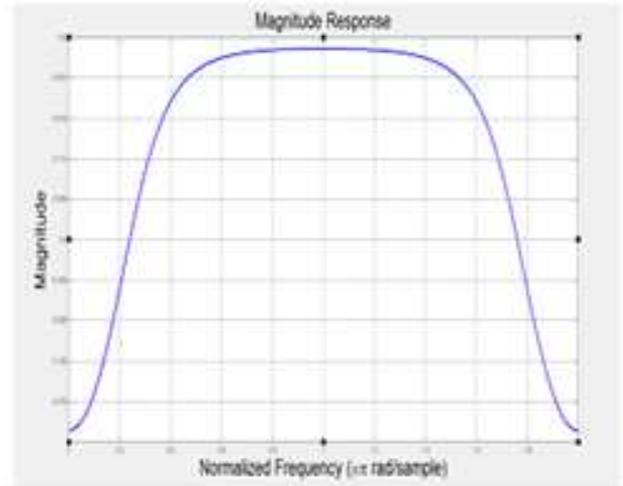
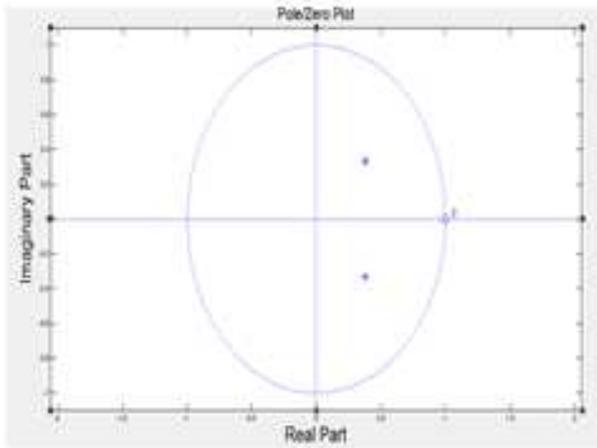


Figure 3: Pole Zero PLOT of Digital IIR Filter (Case A)

Figure 4: Magnitude Plot of Digital IIR Filter (Case A)

Applying Forward Euler Method (Case B)

We have filter

$$H(z) = \frac{z^2 - 2z + 1}{Bz^2 - Bz + 1} \tag{14}$$

And applying constraint equation

$$B(4+B) + B = 8$$

Accepting the positive values of B, we have B = 1.28 and applying this value in filter equation (14), we have

$$H(z) = \frac{z^2 - 2z + 1}{1.28z^2 - 1.28z + 1} \tag{15}$$

The poles of above filter are lying inside the Unit circle $|Z|=1$; hence the filter is stable (Figure 5).

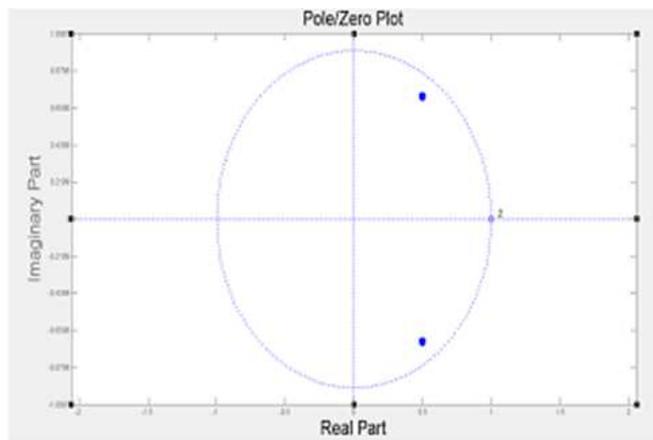


Figure 5: Pole Zero of Filter (Case B)

Figure 6 shows the magnitude plot for the High Pass filter characterized by the above method of integration.

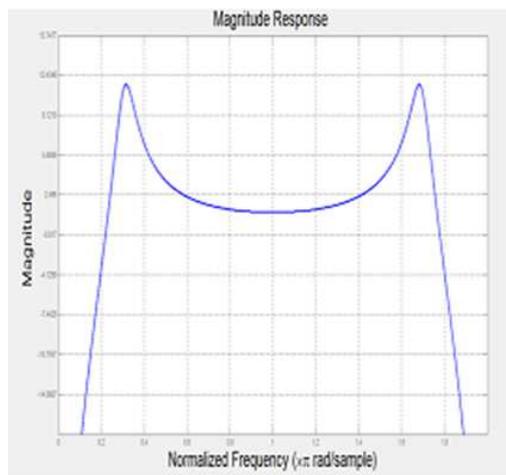


Figure 6: Magnitude Plot of Digital IIR Filter (Case B)

Applying Trapezium Method (Case C)

We have filter equation as

$$H(z) = \frac{4z^2 - 8z + 4}{(6B+1)z^2 + (-8B+2)z + (2B+1)} \tag{16}$$

$$112B^2 + 14B - 131 = 0$$

Accepting positive value of B, we have B=1.021 and applying this value in filter equation we have

$$H(z) = \frac{4z^2 - 8z + 4}{7z^2 - 6z + 3} \tag{17}$$

Here also the poles of above filter are lying inside the Unit circle $|Z|=1$; hence the filter is stable (Figure 3). Figure 4 shows the magnitude response of this filter.

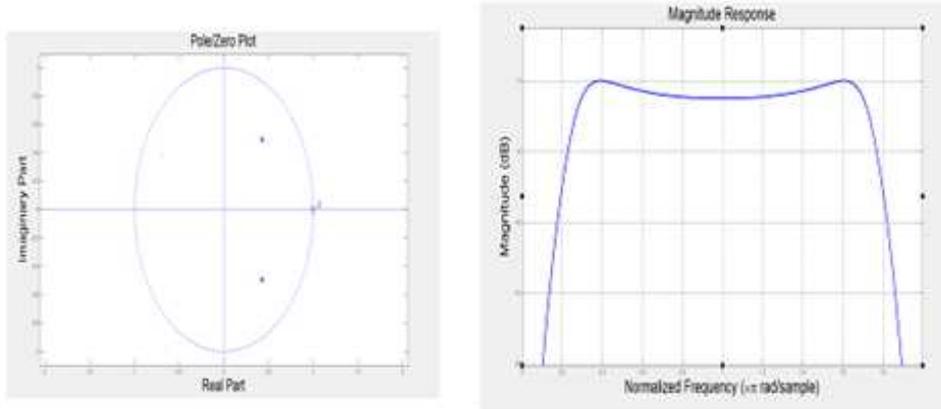


Figure 7: Pole Zero Plot of Digital Filter (Case c) Figure 8: Magnitude Plot of Digital Filter (Case c)

PERFORMANCE ANALYSIS OF PROPOSED FILTERS

As in previous section we have applied all the major method of Discretization for IIR filter design. Now we shall study their step response for convergence time.

Backward Euler Method

By backward Euler Method we have got the filter expression as equation (13).

$$H(z) = \frac{z^2 - 2z + 1}{2z^2 - 1.5z + 0.5}$$

Figure 9 shows the comparison of step response of an analog HPF and Digital HPF.

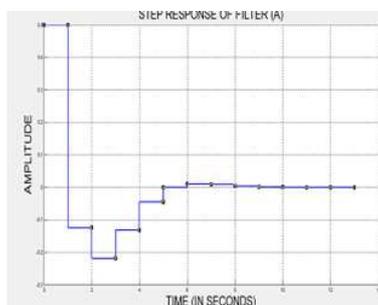


Figure 9: Filter (Case A), Settling Time: 07 Seconde

Forward Euler Method

By back ward Euler Method we have got the filter expression as equation (15). Figure 10 shows the comparison of step response of an analog HPF and Digital HPF.

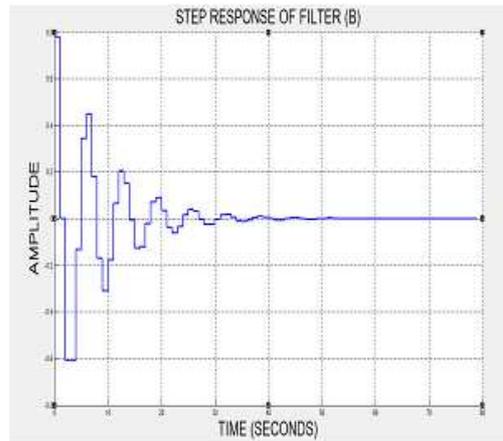


Figure 10: Performane of Proposed Filter (B), Settling Time: 33 Seconde

Trapezium Method

By Trapezium Method we have got the filter expression as in equation (17). **Figure 11** showsthecomparisonofstepresponseof an along HPF and Digital HPF.

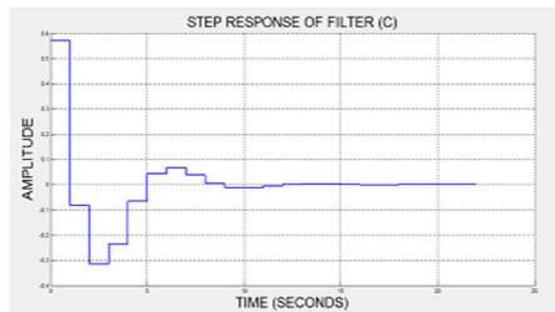


Figure 11: Filter (c), Settling Time: 11 Seconde

RESULTS COMPARISION

Hence the different methods of integrations give different High Pass Filters without the resonance peak in their pass band. The settling time for the different High Pass filters have been studied and the following results are compared as shown in Table 1.

Table 1: Comparison of Settling Time (Speed) of the Proposed Filters

Integrator	Filter Expression	Settling Time
Backward Euler Integrator	$H(z) = \frac{z^2 - 2z + 1}{2z^2 - 1.5z + 0.5}$	07 seconds
Forward Euler Integrator	$H(z) = \frac{z^2 - 2z + 1}{1.28z^2 - 1.28z + 1}$	33 seconds
Trapezoid Integrator	$H(z) = \frac{4z^2 - 8z + 4}{7z^2 - 6z + 3}$	11 seconds

CONCLUSIONS

As shown in table above, we observe that the settling is minimum for the design by Back ward Euler Method. Hence, it has a very fast converging filter.

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