ADMITTANCE CHARACTERISTICS OF RECTANGULAR WAVEGUIDE COUPLED ON THE EFFECT OF WAVEGUIDE DIMENSIONS IN COMMON NARROW WALL

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ABSTRACT

Waveguide couplers are popular for the applications, where microwave power division is required. Power is coupled to one waveguide to another usually through rectangular slots. The slots are designed in such a way that they are operated at resonance. It is an established fact that coupling depends on slot dimensions and its parameters along with frequency variations. But it also depends on the waveguide dimensions with which the coupler is fabricated. No data is available in literature on such dependence and effects. In view of this investigations are carried out to obtain such vital data. The effects of waveguide dimensions are consolidated in the present paper.

KEYWORDS: Rectangular Waveguide, Admittance, Reactance, Coupling, VSWR

INTRODUCTION

The coupling between two rectangular waveguides through apertures in the common broad wall has been analyzed by quasi-static antenna method, variational method and from the equivalent circuit approach. A shortcoming of quasi-static antenna method [6], its inability to obtain from analysis, the accurate slot wave impedance to be used. The results are of limited application owing to a simplification which is made to permit the use of a well-known result of antenna theory.

It is well known that slot radiators in one of the walls of the waveguide, is very popular due to compactness and space saving considerations. A good amount of papers on such radiators is available. However, most of the papers are centered around infinitesimally thin slots [1-4]. This is with an aim to achieve closed form expressions assuming the width of the slots to be negligible. However, such thin slots cannot withstand in high power applications as there will be electrical breakdown. In view of the above considerations, intensive studies are carried out in the present work to analyze very wide slots and wide slot coupled waveguide junctions. One of the main objectives in the present work is to carry out the analysis for impedance characteristics, VSWR and coupling without any approximations. This has lead us to use analytical approach to some extent and then to numerical evaluation.

In the present paper, for the long axial shunt slot in the common narrow wall of the non-standard waveguides, the self-reaction and discontinuity in modal current are determined [7]. Coupling, VSWR and admittance characteristics are determined from the even and odd mode analysis. Variation of coupling, input VSWR and impedance loading on the primary guide as a function of frequency are presented.

ANALYSIS

Consider the waveguide slot coupler of figure 1, shows two rectangular waveguides coupled through a longitudinal slot of length ‘L’ and width ‘w’ in the common narrow wall between two rectangular waveguides in X-band,
‘a’ and ‘b’ are the narrow wall and broad wall dimensions of the waveguide. The slot is located at a distance ‘d’ from the bottom of the broad wall of waveguides. As the slot considered is of a resonant length, the electric field in the slot can be considered to be sinusoidal. It can be represented by the following expression

\[ E_x = a_x E_0 \sin k \left( \frac{L}{2} - |z| \right) \delta(x - x_0) \delta(y - b) \]  \hspace{1cm} (1)

In order to obtain an expression for self reaction of magnetic current on its own sources it is essential to have the knowledge of electric and magnetic fields.

Figure 1: Rectangular Waveguides Coupled through Longitudinal Slot in the Common Narrow Wall

Here

\[ E_0 \] is the maximum electric field in the slot, d, b, 0 are the coordinates of the center of the slot, and, \( k = \frac{2\pi}{\lambda} \). On the other hand the magnetic field \( H \) can be obtained from the knowledge of vector magnetic potential. The standard relation between magnetic field and vector magnetic potential is given by Marcuvitz [8].

\[ \mu H = \nabla \times A \] \hspace{1cm} (2)

Here, \( A \) is magnetic vector potential. The magnetic current density produced due to vector magnetic potential is related to electric fields in the following form [10]

\[ \mathbf{M} = \mathbf{E} \times a_n \] \hspace{1cm} (3)

As the slot is longitudinal in the direction of z, the magnetic current is z-directional, therefore

\[ \mathbf{M}_z = \mathbf{E}_z \times a_n \] \hspace{1cm} (4)

Here, \( a_n \) is unit vector normal to the aperture plane, the magnetic current distribution is given by

By using the variable separation method the solution for \( A_z^m \) is given by

\[ A_z^m(x, y, z) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \int a_{mn}(x) \cos \left( \frac{n\pi}{a} x \right) \sin \left( \frac{m\pi}{b} y \right) e^{-ixz} dx \] \hspace{1cm} (5)

By calculating the unknown coefficient \( a_{mn}(x) \) is given by

\[ a_{mn}(x) = \frac{\epsilon_n}{\pi ab} \int_{V} M_z^m(x', y', z') A.B.C dv \] \hspace{1cm} (6)
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Here

\[ A = \cos \left( \frac{n\pi}{a} x \right), \quad B = \sin \left( \frac{m\pi}{b} y \right) \]

\[ C = \frac{e^{ixz}}{x^2 + \left( \frac{n\pi}{a} \right)^2 + \left( \frac{m\pi}{b} \right)^2 - k^2} \]

\( x, m, n \) are prescribed values of \( x, m \) and \( n \). Using the equations (1) and (4), the final expression for the magnetic vector potential reduces to the following form

\[
A_z = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{imQ}}{\gamma_{ma} a b} V_m Q P \cos n \frac{d}{a} \left( \sin \frac{n\pi w}{2a} \right)
\]

\[
= \frac{K}{K^2 + \gamma^2} \left[ e^{-\gamma H} \cos K \left( \frac{L}{2} - e^{-\gamma L} \cosh \frac{L}{2} - \frac{\gamma L}{k} \sin k \left( \frac{L}{2} - |z| \right) \right) \right]
\]

Here

\[ P = \cos \left( \frac{n\pi}{a} \right), \quad Q = \cos \left( \frac{m\pi}{b} \right) \]

\[ V_m = E_0 w \quad \text{and} \quad \gamma = \sqrt{\left( \frac{n\pi}{a} \right)^2 + \left( \frac{m\pi}{b} \right)^2 - k^2} \]

The magnetic field \( H \) in \( z \)-direction is obtained from the vector potential using Maxwell’s equation as

\[
H_z = \frac{1}{j\omega\mu_0} \left[ k^2 + \frac{\partial^2}{\partial z^2} \right] A_z
\]

Using the concept of self-reaction and discontinuity in modal current is has been possible to obtain impedance loading. Self-reaction is evaluated from the knowledge of magnetic field and magnetic current as defined by V.H. Rumsey [11], which is given by

\[ \langle a, a \rangle = \int \vec{H}_z \cdot \vec{M}_z'' dv. \]

Using the above expression the expression for self-reaction is simplified to the following form [5]

The self-reaction in the primary waveguide \( \langle a, a \rangle \) is given by

\[ \langle a, a \rangle \left[ \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{2 j e_n e_m V_m^2 k^2 \cos^2 m\pi}{\omega \mu_0 a_b \gamma^2 \left( k^2 + \gamma^2 \right)} D_1 \right. \]
\[
0.5 \left[ 1 + e^{-2jL/2} \right] - \cos k \frac{L}{2} \left( 2e^{-jL/2} - \cos k \frac{L}{2} + \sin k \frac{L}{2} \right)
\]

The self-reaction in the secondary waveguide \( \langle a, a \rangle \) is given by

\[
\langle a, a \rangle = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{2j e_n e_m V_m^2 k^2 \cos^2 \frac{m\pi}{2}}{\omega \mu_n a_2 b_2 \gamma (k^2 + \gamma^2)} D_2.
\]

\[
0.5 \left[ 1 + e^{-2jL/2} \right] - \cos k \frac{L}{2} \left( 2e^{-jL/2} - \cos k \frac{L}{2} + \sin k \frac{L}{2} \right)
\]

Here \( D_1 = \cos^2 \left( \frac{n\pi d}{a_1} \right) \left( \frac{\sin^2 \frac{n\pi w}{2a_1}}{n\pi w} \right) \), \( D_2 = \cos^2 \left( \frac{n\pi d}{a_2} \right) \left( \frac{\sin^2 \frac{n\pi w}{2a_2}}{n\pi w} \right) \).

The total self-reaction \( \langle a, a \rangle \) is given by

\[
\langle a, a \rangle = \langle a, a \rangle_1 + \langle a, a \rangle_2
\]

The structure considered in the present work has an equivalent network parameter which is a shunt element. A longitudinal slot in the waveguide wall produces a discontinuity in the modal current giving rise to shunt type of equivalent network parameter \( B_z = \frac{1}{X_z} \), where \( X_z \) given by \([10], [12]\).

\[
X_z = -\frac{\langle a, a \rangle}{I_I}
\]

**Discontinuity in Modal Current**

For the determination of network parameter as expressed in the equation (12) it is required to evaluate the discontinuity in modal current for dominant mode in primary guide. The expression for discontinuity in modal current is given by Marcuvitz and Schwinger [7]

\[
I = jY_{01} \int_{slot} a_n \times E_n \left( h_{01} \sin \beta_{01} z + j h_{201} \cos \beta_{201} z \right) ds
\]

Here \( h_{01} \) and \( h_{201} \) are transverse and longitudinal modal vector functions respectively. \( Y_{01} \) is Characteristic wave admittance and \( \beta_{01} \) is propagation constant. These are given by

\[
h_{01} = \left( \frac{2}{ab} \right)^{1/2} \sin \frac{\pi y}{b} a_x \quad h_{201} = j \left( \frac{2}{ab} \right)^{1/2} \frac{\pi}{b \beta_{01}} \cos \frac{m\pi}{b} a_z
\]
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\[ Y_{01} = \frac{\beta_{01}}{\omega \mu_{01}} \quad \text{and} \quad \beta_{01} = \sqrt{k^2 - \left(\frac{\pi}{b}\right)^2} \]

Here \( a \) and \( b \) are narrow and broad wall dimensions of the waveguide. The integral appearing in (14) is evaluated
over the slot aperture and using the equation (12) and (13), the normalized reactance \( \bar{X}_z \) is obtained in the form of

\[ \bar{X}_z = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left\{ \frac{-1}{2a} \sin^2 \frac{n\pi w}{2a} \right\} \left( \frac{\pi d}{2a} \right)^2 \left[ \begin{array}{c} a \\ b \end{array} \right] \]

Here

\[ D = \cos^2 \left( \frac{n\pi d}{2a} \right) \left( \begin{array}{c} \sin^2 \frac{n\pi w}{2a} \\ \frac{n\pi w}{2a} \end{array} \right) \]

Analyzing the network using even and odd mode equivalent circuit is found that the magnitude of reflection coefficient \( |\rho_{11}| \) at port 1 and coupling coefficient \( |\rho_{12}| \) and \( |\rho_{13}| \) at port 2 and 3 are respectively [13]. The lumped parameter equivalent network for the coupled waveguides looking from port 1 of figure 1, in the equivalent transmission-line representation assumes the corresponding even-odd mode [9] equivalent circuits.

The reflection coefficients of all the ports are

\[ |\rho_{11}| = |\rho_{12}| = |\rho_{13}| = \left| \frac{1}{\bar{X}_z} \right| \]

The variational expression for the equivalent network parameter obtained above is based on assumption that the slot walls are of zero thickness. Hence, coupling in decibels is

\[ \text{Coupling } C = 20 \log \frac{1}{\bar{X}_z} \cdot \text{dB} \]

The normalized shunt admittance is related to normalized shunt impedance by the relation

\[ Y = g_n + jb_n = \frac{1}{Z} = \frac{1}{r + jx} \]
The expression for the normalized admittance in terms of equivalent network parameter as follows

\[ Y = \left(1 + \frac{1}{1 + \frac{1}{X_z}}\right) + \frac{jX_z}{1 + \frac{1}{X_z}} \]  

(18)

The reflection coefficient seen by figure 1 at the reference plane is given by the equation

\[ \rho = \frac{1 - Y_{LN}}{1 + Y_{LN}} \]

Here,

\[ Y_{LN} = 1 + Y \]

The VSWR in terms of reflection coefficient is given by

\[ \text{VSWR} = \frac{1 + |\rho|}{1 - |\rho|} \]  

(19)

RESULTS

Using the above expressions for equivalent network parameter the variation of coupling, VSWR and admittance characteristics as function of frequency is computed. These variations are obtained for different waveguide dimensions and different displacements. Some results are shown in figures (2-9).

Figure 2: Variation of Coupling versus Frequency, Displacement 2mm from Center  
\[ a = 1.016\text{cm and } b = 2.0, 2.1, 2.286 \text{ cm}, \text{Slot Width (w) = 1mm, Slot Length (L) =1.42cm} \]

Figure 3: Variation of VSWR versus Frequency, Displacement 2mm from Center  
\[ a = 1.016\text{cm and } b = 2.0, 2.1, 2.286 \text{ cm}, \text{Slot Width (w) = 1mm, Slot Length (L) =1.42cm} \]
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![Figure 4: Variation of Real Part of Admittance versus Frequency, Displacement 2mm from Center a= 1.016cm and b= 2.0, 2.1, 2.286 cm, Slot Width (w) = 1mm, Slot Length (L) =1.42cm](image1)

![Figure 5: Variation of Imaginary Part of Admittance versus Frequency, Displacement 2mm from Center a= 1.016cm and b= 2.0, 2.1, 2.286 cm, Slot Width (w) = 1mm, Slot Length (L) =1.42cm](image2)

![Figure 6: Variation of Coupling versus Frequency, Displacement 2mm from Center b= 2.286cm and a= 0.9, 1.016, 1.1 cm, Slot Width (w) = 1mm, Slot Length (L) =1.42cm](image3)

![Figure 7: Variation of VSWR versus Frequency, Displacement 2mm from Center b= 2.286cm and a= 0.9, 1.016, 1.1 cm, Slot Width (w) = 1mm, Slot Length (L) =1.42cm](image4)
CONCLUSIONS

It is evident from the results presented, the maximum coupling is about -7dB and it occurs at the center frequency of X-band. It is minimum at frequency away from resonance. VSWR is maximum at the center frequency and it is found to reduce away from center frequency. These are found to depend on waveguide dimensions, slot width and displacement of the slot. When the displacement is increased to 3mm from the center of the common wall, the coupling is found to be increased and it is almost same from 8 to 9.5GHz. It is following exponential towards the end of X-band. VSWR has been similar variations with respect to increased displacement. It has been possible in the present work to control coupling, VSWR and admittance with waveguide dimensions also. This work is useful for the designer of such couplers as more parameters are available, and also the designer can control the parameters of interest.

REFERENCES


