ACTIVE SPECKLE PHOTOGRAPHY METHOD USING FOURIER TRANSFORM FOR MEASURING THE THICKNESS OF A TRANSPARENT PLATE

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ABSTRACT

Speckle photography offers a simple and fast technique for measuring the thickness of a transparent plate. It is a two-stage process. In the first stage, two states of the transmitted waves through the transparent plate (one before and one after the rotation of the plate) are recorded. In the second stage, the two recorded images are added, and by applying a Fourier transform, fringes can be obtained and processed to estimate the thickness of the plate. Uncertainty of thickness measurement is investigated. The method is investigated, and its viability is demonstrated by experimental results.

KEYWORDS: Speckle Photography, Fourier Transform, Thickness Measurement

INTRODUCTION

Micrometers and dial gauges are useful for measuring the thickness of a plate to an accuracy of ± 10.0 μm; ± 1.0 μm accuracy is achievable with interferometry.\(^1\) A common technique for measuring the thickness of a transparent plate is to employ a spectrometer such as a Michelson interferometer to measure transmitted light as a function of wavelength. The resultant spacing of interference fringes may be simply related to the so-called optical thickness.\(^2\) Francon\(^2\) has presented an interferometric technique not requiring a spectrometer, by measuring the angular diameters (at infinity) of the first and \(n\)th fringes.

A Fourier transform spectrometer or variant thereof is typically used to make broadband measurements of the reflection and transmission of a transparent plate.\(^3\) By looking at the spacing of interference fringes versus wave number the optical thickness can be estimated. Measuring fringes versus both angle and wave number then provides a way to estimate both the absolute index of refraction and thickness of a plate. Such an approach may be complementary to the technique of Okada et al.,\(^4\) who measure surface shape and refractive index inhomogeneity using a tunable diode laser and a Twyman-Green interferometer.

Speckle photography, which was originally developed as a method for measuring the lateral, or in-plane displacement,\(^5\)-\(^7\) and refractive index of a transparent plate,\(^8\) is a simpler technique than other, having the advantages that it does not require such a high degree of stability of the optical system. It can therefore be more readily used outside the laboratory.

The purpose of this work is to extend the application of Fourier transform method to speckle photography, to measure the thickness of the transparent plate, and detect the major uncertainty components for improving the performance of the thickness estimation.\(^9\)-\(^12\) A relation between the thickness of a transparent plate and the angle of incidence of the collimated light is derived theoretically, and experimentally confirmed.

THEORETICAL ANALYSIS

Let first the transmission of a plane wave through a transparent plate of refractive index \(\mu\) and thickness \(d\)
surrounded by free space (Figure 1).

Let $U(x,y,d)$ and $U(x,y,0)$ are the complex amplitudes of the plane wave after and before transmission respectively. Therefore the ratio $t(x,y) = U(x,y,d) / U(x,y,0)$ represent the complex amplitude transmittance of the plate.

$$U(x,y,d) = ae^{i\theta} e^{-jkd}$$

(1)

Where $a$ and $\varphi$ are the amplitude and phase of the incident wave respectively. $n$ is the number of the transmittance plates.

The transmitted wave illuminates a scattering surface lies in the $(\zeta,\eta)$ plane. Any point on the observation plane $(X,Y)$ is placed at a distance $Z$ in front of this surface will receive contributions from all points on the surface.

Assume that $P_s$ is any point on the scattering surface plane, $P_o$ is any point on the observation plane and $r_{os}$ is the distance between $P_s$ and $P_o$ making an angle $\theta$ with $Z$.

According to Fresnel principle of diffraction, we can write wavefield across the $(X,Y)$ plane as follows

$$U(p_o) = \frac{1}{j\lambda} \int \int ae^{i\theta} e^{-jkd} U(p_s) \frac{e^{jk_{ro}}}{r_{os}} \cos \theta ds$$

(2)

Where $\cos \theta = \frac{Z}{r_{os}}$

This equation can be written as:

$$U(X,Y) = \frac{Z}{j\lambda} ae^{i\theta} \int \int U(\xi,\eta) e^{-jkd} \frac{e^{jk_{ro}}}{r_{os}^2} d\xi d\eta$$

(3)

Where $r_{os} = \sqrt{Z^2 + (X - \xi)^2 + (Y - \eta)^2}$

(4)

Let $m$ be a number less than unity and consider the equation

$$\sqrt{1+m} = 1 + \frac{1}{2} m - \frac{1}{8} m^2 + \ldots \ldots$$

(5)

Then

$$r_{os} = Z \sqrt{1 + \left(\frac{X - \xi}{Z}\right)^2 + \left(\frac{Y - \eta}{Z}\right)^2}$$

(6)

$$r_{os} \approx 1 + \frac{1}{2} \left(\frac{X - \xi}{Z}\right)^2 + \frac{1}{2} \left(\frac{Y - \eta}{Z}\right)^2$$

(7)

$$U(X,Y) = \frac{Z}{j\lambda} ae^{i\theta} \int \int U(\xi,\eta) e^{-jkd} \frac{e^{jk_{ro}}}{Z^2(1 + ((X - \xi)/Z)^2 + ((Y - \eta)/Z)^2)} d\xi d\eta$$

(8)
\[ U(X, Y) = \frac{1}{j\lambda Z} ae^{j\theta} e^{-j\kappa \eta} e^{j\kappa Z} \int \int \hat{U}(\xi, \eta) \frac{e^{j\kappa(X - \xi)^2 + (Y - \eta)^2}}{(1 + ((X - \xi)/Z)^2 + ((Y - \eta)/Z)^2)} d\xi d\eta \]  

(9)

The terms \((X - \xi)^2/Z^2\) and \((Y - \eta)^2/Z^2\) are very small and their effect on the wave amplitude can be neglected, then

\[ U(X, Y) = \frac{1}{j\lambda Z} ae^{j\theta} e^{-j\kappa \eta} e^{j\kappa Z} \int \int \hat{U}(\xi, \eta) \frac{e^{j\kappa(X^2 + Y^2)}}{e^{\frac{\kappa}{2Z}(\xi^2 + \eta^2)}} e^{\frac{-j\kappa X}{Z}} d\xi d\eta \]  

(10)

If \(Z \gg (\xi^2 + \eta^2)/2\) then

\[ U(X, Y) = \frac{1}{j\lambda Z} ae^{j\theta} e^{-j\kappa \eta} e^{j\kappa Z} \int \int U(\xi, \eta) e^{\frac{-j\kappa X}{Z}} d\xi d\eta \]  

(11)

**THICKNESS MEASUREMENT**

To measure the thickness of a transparent plate using the theory of refraction of the incident wave. The plate is putted on a graduated rotatable disc. Spatially coherent light transmitted through the plate and illuminated a rough surface. The obtained speckle is recorded twice, one before the rotation of the transparent plate and other after the rotation and increasing the thickness of the plate. Then the two images are combined digitally “added” and the resultant image will contain a pair of identical speckle pattern separated by a distance \(\Delta \xi \) \(^{14,15}\).

The displacement \(\Delta \xi\) is displaced in the form of fringe pattern by applying FFT to the resultant image. So a bright spot surrounded by a speckle pattern modulated by a cosinusoidal fringes can be observed. The bright central spot is formed by the un-diffracted light. While the cosinusoidal fringes are formed because each pair of corresponding speckles acts as a pair of identical sources of coherent light which form Young’s fringes. These fringes are separated by a distance depending on the value of the rotation and thickness of the transparent plate.

Using equation (11), and put \(\frac{X}{\lambda Z} = \omega_x\) and \(\frac{Y}{\lambda Z} = \omega_y\).

\[ U(X, Y) = \frac{1}{j\lambda Z} ae^{j\theta} e^{-j\kappa \eta} e^{j\kappa Z} \int \int \hat{U}(\xi, \eta) e^{-j2\pi(\omega_x \xi + \omega_y \eta)} d\xi d\eta \]  

(12)

The field \(U_1(X, Y)\) before the rotation of the transparent plate can be represented by:

\[ U_1(X, Y) = \frac{1}{j\lambda Z} ae^{j\theta} e^{-j\kappa \eta} e^{j\kappa Z} \frac{\hat{F}\{U(\xi, \eta)\}}{e^{\frac{\kappa}{2Z}(X^2 + Y^2)}}\]  

(13)

Where \(F\{U(\xi, \eta)\}\) is the Fourier transform function. If the transparent plate rotated with an angle \(\beta\), the incident waves refracted with an angle \(\gamma\).

The complex amplitude inside the plate is now proportional to \(\exp(-j\kappa nd/\cos\gamma)\). So that the complex amplitude transmittance of the plate:
\[ t(x, y) = \exp(-jkd/\cos\gamma) \]  \hspace{1cm} (14)

The field \( U_2(X, Y) \) after rotation can be represented by:

\[
U_2(X, Y) = \frac{1}{j\lambda Z} a e^{i\phi} e^{\cos(\gamma)} e^{jkZ(x^2+y^2)} \int \int U(\xi - \Delta\xi, \eta) e^{-j2\pi(\xi, \xi - \Delta\xi, \eta, \eta)} d\xi d\eta \]  \hspace{1cm} (15)

\[
U_2(X, Y) = \frac{1}{j\lambda Z} a e^{i\phi} e^{\cos(\gamma)} e^{jkZ(x^2+y^2)} e^{j2\pi\Delta\xi, \eta} \int \int U(\xi - \Delta\xi, \eta) e^{j2\pi((\xi, \xi - \Delta\xi, \eta, \eta)} d\xi d\eta \]  \hspace{1cm} (16)

Assume that the displacement is in one direction (\( \xi \)-axis). Hence;

\[
U_2(X, Y) = \frac{1}{j\lambda Z} a e^{i\phi} e^{\cos(\gamma)} e^{jkZ(x^2+y^2)} e^{j2\pi\Delta\xi, \eta} F[U(\xi, \eta)] \]  \hspace{1cm} (17)

By adding the two Equations (13) and (17)

\[
U_1(X, Y) + U_2(X, Y) = \frac{1}{j\lambda Z} a e^{i\phi} e^{\cos(\gamma)} e^{jkZ(x^2+y^2)} F[U(\xi, \eta)](e^{-jkd} + e^{j2\pi\Delta\xi, \eta} e^{\cos(\gamma)}) \]  \hspace{1cm} (18)

The intensity can be written as

\[
I = \left( \frac{1}{\lambda Z} \right)^2 F^2 \{ U(\xi, \eta) \} \{ e^{-jkd} + e^{j2\pi\Delta\xi, \eta} e^{\cos(\gamma)} \}^2 \]  \hspace{1cm} (19)

\[
I = \left( \frac{1}{\lambda Z} \right)^2 F^2 \{ U(\xi, \eta) \} \{ 2 + 2\cos(knd - \frac{knd}{\cos\gamma} + 2\pi\Delta\xi, \eta) \} \]  \hspace{1cm} (20)

At maximum intensity \( knd - \frac{knd}{\cos\gamma} + 2\pi\Delta\xi, \eta = 2m\pi \)

\[
knd - \frac{knd}{\cos\gamma} + 2\pi\Delta\xi, \eta = 2m\pi \]  \hspace{1cm} (21)

Let \( m = 1 \), we get:

\[
\Delta\xi, \eta = 1 + \frac{nd}{\lambda} \left( \frac{1}{\cos\gamma} - 1 \right) \]  \hspace{1cm} (22)

Put \( \omega_s = \frac{2\pi}{\lambda_s} \)

\[
\Delta\xi, \eta = 1 + \frac{nd}{\lambda} \left( \frac{1}{\cos\gamma} - 1 \right) \]  \hspace{1cm} (23)

Where \( \lambda \) is the wavelength of the used light, and \( \lambda_s \) represent the fringe spacing in the \( \xi \)-direction.
From the Snell’s law, \( \mu' \sin \beta = \mu \sin \gamma \), where \( \mu' = 1 \) is the refractive index of the air, and \( \mu \) is the refractive index of the glass plate. By substitution from the Snell’s law into Equation (23). The thickness of a transparent plate can be given by:

\[
d = \left( \Delta \xi \frac{2\pi}{\lambda_s} - 1 \right) \frac{\lambda}{n} \left[ \frac{1}{\cos \left( \sin^{-1} \left( \frac{\sin \beta}{\mu} \right) \right)} - 1 \right]
\]  

(24)

**EXPERIMENTAL ARRANGEMENT, PROCEDURES AND RESULTS**

The optical setup used to obtain the thickness of the transparent plate is shown in Figure (2). A 7 mW He-Ne Laser (\( \lambda = 0.6328 \mu \text{m} \)) was used for the experiments. The laser light was illuminating a transparent rough surface with rms 9.6 \( \lambda \text{m} \). For estimating the thickness of the glass plate, the glass plate is putted on a graduated rotatable disc in front of the collimated light. Having a vernier, the smallest graduation can be read to 0.1 degree. Different spatial carriers can be accomplished by the non-rotation and rotation of the transparent plate, which have a definite thickness. The plate was tilted with an angle 0.5 degree to be constant during the experiment. A first exposure is made by illuminating the rough object in case of the non-rotation of the transparent plate, and received using a webcam camera attached to a computer. A second exposure is made by illuminating the rough surface in case of the rotation of the transparent plate with an angle 0.5 degree, the thickness of the plate was 5.8 mm. The unit to be measured on monitor is pixel. So we must calibrate the magnification before the test. The calibration process of the experiment made by putting the rough surface on a travelling micrometer and move it axially, and measuring the values of \( \Delta \xi \) and \( \lambda_s \) with the corresponding values of the micrometer readings. Fig. 3 shows the calibration curve for measuring the value of \( \lambda_s \) in \( \lambda \text{m} \). Fig. 4 shows the calibration curve for measuring the value of \( \Delta \xi \) in \( \lambda \text{m} \). The values \( \Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5, \) and \( \Lambda_6 \) which are shown in the equations of the calibrations of \( \lambda_s \) and \( \Delta \xi \) are considered as constants depending on the calibration process. From the recorded two images and by using the software program (Image J Program), we can obtain the value of \( \Delta \xi \). Combining the two images into one and computing their two-dimensional Fourier transform, we get the fringe image in the spectral field (Fig. 5) for angle of rotation 0.5 degree and thickness of 5.8 mm. Scanning Fig. 5 along the \( \lambda_s \)-axis (\( \lambda_s = 0 \)), we can get the distribution of the light intensity, so the value of \( \lambda_s \) can be measured. After measuring the value of \( \lambda_s \) and \( \Delta \xi \), we substitute in Eq. 24, we can get the thickness of the glass plate. Figure 6(a, b) show the fringe images in the spectral field for different thicknesses of values 11.6 mm, and 40.6 mm respectively. Figure (7) shows the dependence of the ratio \( \frac{\Delta \xi}{\lambda_s} \) (ratio of the distance between the two identical speckle pattern and the fringe spacing in the x-direction) on the angle of incidence \( \beta \). From the figure we can conclude that the ratio \( \frac{\Delta \xi}{\lambda_s} \) approximately have the same value for different thicknesses at the angle of incidence 0.5\(^\circ\). When the angle of incidence increase, the ratio for the different thicknesses getting away from each other, and it increases gradually with increasing the angle of incidence as shown in the figure. This mean that the high accuracy
of measurements can be obtained at the small angle of incidence, when the angle of incidence increases, the accuracy of measurements decreases. Figure 8(a,b) show the images in the spectral field for different thickness of values 11.6 mm and 40.6 mm respectively, and the angle of incidence was 2°. For the investigation of the dependence of the measured uncertainty on the changing of wavelengths, different filters with wavelengths 0.5780, 0.5460, 0.4360, 0.4050, and 0.3600 μm respectively were used. Figure 9(a,b,) show the images in the spectral field for wavelengths 0.5460 and 0.3600 μm respectively. The angle of rotation was 0.5 degree, and the thickness was 11.6 mm. As we can see the visibility decrease with decreasing the used wavelengths.

MEASUREMENT UNCERTAINTIES OF METHOD

The uncertainty of d was estimated by combining the standard uncertainties of the parameters λ, λc, n, Δξ, and γ.

Generally, the result of measurement is determined from the relation between the result d and the values of the input parameters, and can be expressed by a model:

\[ d = f(\lambda, \lambda_c, n, \Delta \xi, \gamma) \]  

(25)

where \( \lambda, \lambda_c, n, \Delta \xi \), and γ represent model input parameters. The uncertainty of the result \( u(d) \) depends on the uncertainty of the input parameters and is described by the equation:

\[ u(d)^2 = \sum_{i=1}^{N} \left( \frac{\partial d}{\partial x_i} \right)^2 u(x_i)^2 \]  

(26)

Where \( x_1, ..., x_N \) represent model input parameters, \( u(x_i) \) are the standard uncertainties of the input parameters, and \( \frac{\partial d}{\partial x_i} \) is the sensitivity coefficient. The sensitivity coefficient describes how the measurement result varies with changes in the value of input estimates. Eq. (26) is valid for measurement where there is no correlation between input parameters.

According to equations (25) and (26), \( u(d)^2 \) can be expressed in the form:

\[ u^2(d) = \frac{\lambda^2}{n^2} \left[ \frac{2\pi\Delta \xi}{\lambda^2} \left( \frac{1}{\cos \gamma} - 1 \right) \right]^2 u^2(\lambda) + \left( \frac{2\pi/\lambda}{\lambda_c} \right)^2 u^2(\Delta \xi) + \frac{\sin^2 \gamma}{(1-\cos \gamma)^2} \left( \frac{2\pi\Delta \xi}{\lambda_\gamma} - 1 \right)^2 u^2(\gamma) \]  

(27)

To calculate the expanded uncertainty of the result of measurement at the 95% confidence level, the result for combined uncertainty was multiplied by a coverage factor of 2.
Relative uncertainty contributions are used to illustrate the relative impact of different uncertainty components. The relative contribution \( r_i \) of an uncertainty component \( x_i \) to the combined standard uncertainty is defined here as:

\[
 r_i = \frac{\left( \frac{\partial d}{\partial x_i} \right)^2}{u(d)^2} u(x_i)^2
\]  

(28)

Where \( d \) is the model equation \( d = f(x_1, x_2, \ldots, x_i, \ldots, x_N) \), \( x_i \) are the input parameters \( (\Delta \xi, \lambda, \lambda_s, n, \gamma) \), and \( u(d)^2 \) is the combined uncertainty calculated according to Eq. (26).

The values input estimates with their respective standard uncertainties, sensitivity coefficients, relative contributions, and types of standard uncertainty evaluations are given in Table 1. Figure (10) shows the dependence of the uncertainty of measurement on the changing of wavelengths for different thicknesses. As we can see, the uncertainty decrease with increasing the wavelength of the used light, and decreasing the thickness of the transparent plates. More accurate results can be obtained by using a light of wavelength 0.6328 μm and angle of rotation 0.5°. The present method can be more active for the measurement of the small thicknesses of the transparent plates.

CONCLUSIONS

A simple digital method for processing speckle photography is used for measuring the thickness of a transparent plate by applying FFT function to the double exposure speckle pattern. Fourier transform is applied to determine the results and theoretical analysis show good agreement with the experimental results. Uncertainty of measurements is evaluated. For more accurate and precise measurement, it should be used a small angle of rotation 0.5°, and a laser light of wavelength 0.6328 μm.

REFERENCES


APPENDICES

Figure 1: The Construction of the Speckle Pattern

Figure 2: Interferometric Technique for Measuring Thickness of a Transparent Plate
Active Speckle Photography Method Using Fourier Transform for Measuring the Thickness of a Transparent Plate

Figure 3: The Calibration Curve for Measuring the Fringe Spacing; $\lambda_x$ in the x-Axis Direction

Figure 4: The Calibration Curve for Measuring the Separation between a Pair of Identical Speckle Pattern $\Delta \xi$

Figure 5: Image of Fringes in the Spectral Field: The Thickness of the Transparent Plate was 5.8 mm and the Angle of its Rotation was 0.5°
Figure 6: (a, b) Images of Fringes in the Spectral Field: The Thickness of the Transparent Plates was 11.6 mm, and 40.6 mm Respectively, the Plates Tilted by an Angle 0.5°.

Figure 7: The Dependence of the Ratio $\frac{\Delta \xi}{\lambda_x}$ on the Angle of Incidence $\beta$.

Figure 8: (a, b) Images of Fringes in the Spectral Field: The Thickness of the Transparent Plates was 11.6 mm, and 40.6 mm Respectively, the Plates Tilted by an Angle 2°.
Active Speckle Photography Method Using Fourier Transform for Measuring the Thickness of a Transparent Plate

Figure 9: (a, b). Images of Fringes in the Spectral Field: The Wavelength of used Light was 0.5460 and 0.3600 μm Respectively. The Thickness of the Transparent Plates was 11.6 mm, and the Plate Tilted by an Angle 0.5°

Figure 10: Dependence of Uncertainty of Measurements on the Changing of Wavelengths for Different Thicknesses
Table 1: Uncertainty Components and their Relative Standard Uncertainties

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Probability Distribution</th>
<th>Standard Uncertainty ($\mu$m)</th>
<th>Sensitivity Coefficient</th>
<th>Uncertainty Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fringe spacing in x-direction ($\lambda_x$)</td>
<td>Rectangular Type B</td>
<td>$4.32 \times 10^{-4}$</td>
<td>$3.375 \times 10^{3}$</td>
<td>0.0631</td>
</tr>
<tr>
<td>Separation between a pair of speckle ($\Delta \zeta$)</td>
<td>Rectangular Type B</td>
<td>$3.56 \times 10^{-4}$</td>
<td>$1.472 \times 10^{4}$</td>
<td>0.814</td>
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<tr>
<td>Refraction angle ($\gamma$)</td>
<td>Rectangular Type B</td>
<td>$1.12 \times 10^{-6}$</td>
<td>$5.82 \times 10^{6}$</td>
<td>1.263</td>
</tr>
<tr>
<td>Number of the transparent plates ($n$)</td>
<td>Rectangular Type B</td>
<td>$1.3 \times 10^{-4}$</td>
<td>$1.676 \times 10^{4}$</td>
<td>0.14</td>
</tr>
<tr>
<td>Wavelength($\lambda$)</td>
<td>Rectangular Type B</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.6328 $\mu$m</td>
<td>-</td>
<td>$2.356 \times 10^{-5}$</td>
<td>$9.149 \times 10^{3}$</td>
<td>0.014</td>
</tr>
<tr>
<td>0.5780 $\mu$m</td>
<td>-</td>
<td>$6.25 \times 10^{-5}$</td>
<td>$1.001 \times 10^{4}$</td>
<td>0.0116</td>
</tr>
<tr>
<td>0.5460 $\mu$m</td>
<td>-</td>
<td>$9.42 \times 10^{-5}$</td>
<td>$1.058 \times 10^{4}$</td>
<td>0.0296</td>
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<tr>
<td>0.4360 $\mu$m</td>
<td>-</td>
<td>$1.89 \times 10^{-4}$</td>
<td>$1.361 \times 10^{4}$</td>
<td>0.076</td>
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<tr>
<td>0.4050 $\mu$m</td>
<td>-</td>
<td>$1.2 \times 10^{-4}$</td>
<td>$1.43 \times 10^{4}$</td>
<td>0.0875</td>
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<tr>
<td>0.3600 $\mu$m</td>
<td>-</td>
<td>$1.62 \times 10^{-4}$</td>
<td>$1.58 \times 10^{4}$</td>
<td>0.195</td>
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<tr>
<td>Combined Standard Uncertainty = 5.682 $\mu$m</td>
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<tr>
<td>Expanded Uncertainty (k) = 11.724 $\mu$m</td>
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