ROUTE TO CHAOS AND NONRADIATIVE RECOMBINATION IN LASER DIODE

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ABSTRACT

This paper investigates influence of nonradiative recombination lifetime on the route to chaos of semiconductor laser and associated operation states over a wide range of injection current. The study is based on numerical solutions of an improved time delay model of semiconductor laser subject to optical feedback. The simulation results show that nonradiative recombination lifetime causes significant changes in the route to chaos and the laser dynamic. The feedback strength when the laser transits from continues wave to periodic oscillation or period doubling or chaos state decreases with the increase in the nonradiative recombination lifetime. We identify the route to chaos states of SLs in three distinct operating regions, namely, periodic oscillation, sub-harmonic, and period doubling, which are depending on the value of the nonradiative recombination lifetime and injection current ratio. At higher levels of the injection current, lowest nonradiative recombination lifetime value stabilizes the laser operation and stimulating the laser to operate in periodic oscillation or continues wave.

KEYWORDS: Semiconductor Laser, Nonradiative Recombination, Optical Feedback, Periodic Oscillation, Period Doubling

INTRODUCTION

InGaAsP/InP laser diodes are widely used as light sources for long-distance optical communication systems operating in the 1.2-1.6 μm [1]. The dynamics and route to chaos of such lasers are strongly modified by optical feedback (OFB) from an external reflector. The effect of OFB depends on various factors, including the OFB strength, external cavity length and injection current. While weak OFB levels result in line-width narrowing and improved frequency stability, moderate to strong OFB levels might induce the laser to switch to a state of significant spectral broadening and dynamical complexity, which has been called “coherence collapsed state” [1-4].

However, the operation and dynamic characteristics of the semiconductor laser (SL) are influenced by the nonradiative recombination lifetime $\tau_{nr}$ (NRRLT) parameter, which has strong effect on the carrier life time, threshold current, spontaneous emission factor, output saturation and turn-on time delay [5-7]. A good understanding of the carrier recombination mechanisms in SLs is essential for designing laser diodes with high static and dynamic performance. In particular, it is important to minimize the NRRLT processes that reduce the efficiency of these devices [7].

The route to chaos is one of the interesting nonlinear phenomena associated with OFB; it may include dynamics with period-doubling (PD), sub-harmonics (SH) or quasi-periodicity [8-13]. PD is observed in the limit of short-external cavity [9,11-13]; it follows a region of periodic oscillation and is characterized by the periodic oscillation frequency and its half harmonic. Kao et al. [11] predicted that this frequency should correspond to Hopf-bifurcation point (the onset of periodic oscillation). The quasi-periodic (QP) route to chaos dominates the long-external-cavity dynamics and is a manifest of competition between resonances of the laser cavity and the external cavity [8, 10-13]. Kao et al. [11] reported that QP is
characterized by the Hopf-bifurcation frequency with a beaten component of nearly the external-cavity resonance frequency. SH transitions to chaos were predicted as an intermediate state from PD to QP associated with increasing the external-cavity length [11]. It is characterized by the Hopf-bifurcation frequency and one of its rational SH, and it is followed by mode locking with these frequencies [11]. Ahmed et al. [14] used the ratio of the relaxation frequency to the external-cavity resonance frequency to classify the route to chaos in semiconductor lasers. They showed that the route is PD when the frequency ratio is less than unity. The route becomes sub-harmonic with the periodic oscillation frequency when the ratio is slightly higher than unity and less than 2.25. When the ratio exceeds 2.3, the route is QP characterized by the compound-cavity frequency and the relaxation frequency as well as their frequency difference [14].

Abdulrhmann et al. [15] investigated the influence of NRRLT on the quantum noise in 1550-nm InGaAsP/InP SL in reference [15]. However, based on our knowledge, there is no report that studied the effect of NRRLT lifetime $\tau_{nr}$, OFB strength and injection current ratio on the route to chaos characteristics and associated states of operation for semiconductor lasers. In this article, we apply the time-delay rate equation model in [8, 14] of OFB in semiconductor lasers to study influence of NRRLT on the route to chaos and associated laser operation. In such a model, OFB is treated as time delay of the laser radiation due to multiple roundtrips between the laser front facet and the external mirror. Intensive computer simulations are run to investigate the operation and dynamics of 1550-nm InGaAsP/InP SL with a short external cavity using a wide range of injection current. The route to chaos and associated laser operation is classified in terms of the bifurcation diagrams of the peak value of the photon number at each feedback strength and injection current. We identify the route to chaos states of SLs in three distinct operating regions, namely, periodic oscillation (PO), sub-harmonic (SH), and period doubling (PD) depending on the value of the NRRLT and injection current ratio. The results show that NRRLT induces significant changes in the route to chaos and associated laser dynamics. The OFB strength corresponding to the transition from continuous wave (CW) to PO or PD or chaos occur decreases with the increase in the NRRLT. Under higher values of injection current and by decreasing NRRLT parameter, the laser route to chaos changes from SH to PO state. At higher values of the injection current, lowest NRRLT works in such a way to induce PO operation.

The paper is organized as follows; in the next section the time-delay rate equation model of SL with OFB is introduced. These rate equations are solved numerically in the following section. In the results and discussions section, the bifurcation diagrams of the photon number of SL over wide ranges of NRRLT and injection current are shown. The time variations, fast Fourier transforms (FFT) of the photon numbers and phase portrait of the induced route to chaos states of operation are also presented. The conclusions are presented at the end of the article.

**THEORETICAL MODEL OF SIMULATION**

The time-delay rate equations of the photon number $S(t)$, optical phase $\theta(t)$ and electron number $N(t)$ when counting the NRRLT and which describing the SL dynamics are formulated as [8, 14]:

$$dS/dt = \left( (a_\xi/V)(N - N_g) - BS - G_{th} + (c/n_p L_p) \ln \left| \frac{\tau'}{\tau} \right| S + (a_\xi/V)N \right) + \left( \frac{a_\xi}{V} \right) \left( N - N_g \right) S - N \left[ (1/\tau_e) + (1/\tau_{\text{nr}}) \right] + (I/e), \tag{1}$$

$$d\theta/dt = \left( \left( \alpha a_\xi/2V \right)(N - \tilde{N}) - \left( c/2n_p L_p \right) (\varphi - \tilde{\varphi}) \right), \tag{2}$$

$$dN/dt = -\left( a_\xi/V \right)(N - N_g) S - N \left[ \left( 1/\tau_\varphi \right) + \left( 1/\tau_\text{nr} \right) \right], \tag{3}$$

where $a_\xi(N-N_g)/V$ is the linear gain coefficient with $a$ and $N_g$ as material constants and $\xi$ as the confinement factor of the optical field into the active region of volume $V$, $\alpha$ is the line-width enhancement factor, and $I$ is the injection current and $\tilde{N}$ is the corresponding time averaged electron number. $G_{th}$ is the threshold gain level, which is determined by the
loss coefficient $k$ of the laser and mirror loss:

$$G_{s0} = \left( c/n_e \right) \left\{ k + (1/2L)\ln \left( 1/R_eR_f \right) \right\}.$$ (4)

Various models have been advanced to account for gain nonlinearities based on the density-matrix theory [15, 16]. In this paper, the gain saturation is included in the rate equations in terms of coefficient $B$, which is given in the third-order perturbation theory of gain by [15, 16].

$$B = \left( 9\hbar\omega / 4e, n_e^2 \right) \left( \tau_e / (1 - \tau_e) \right)^{m-1} \exp \left\{ -j\omega \tau \right\} \left( S(t - m\tau) / S(t) \right) = |T| \exp \left\{ -j\phi \right\}.$$ (5)

Where, $R_c$, is the dipole moment, $\tau_{in}$ is the intra-band relaxation time and $N_i$ is an injection level characterizing the saturation coefficient. The radiative recombination of the electrons is counted in the rate equations (1)-(3) in terms of the corresponding radiative lifetime $\tau_r$. On the other hand, NRRLT processes such as nonradiative contribution arising from crystal impurities and Auger re-combination is expressed by NRRLT lifetime $\tau_{nr}$ [6, 15].

The complex coefficient $T$ describes the influence of OFB on the threshold conditions are given by [14]:

$$T = 1 - \sum_{m=1}^{\infty} (K_{ex})^m \left( R_x / (1 - R_x) \right) \exp \left\{ -j\omega \tau \right\} \left( S(t - m\tau) / S(t) \right) = |T| \exp \left\{ -j\phi \right\}.$$ (7)

Where, $m$ is an index for the round-trip, $\psi$ is a phase term combining the phase changes due to reflection by the external mirror, $R_{ex}$, $n_{ex}$ and $L_{ex}$ are the external mirror power reflectivity, refractive index and external cavity length, respectively.

The combined phase $\psi$ is the phase retarded from the external mirror $\phi_{ex}$, front facet, $\phi_f$, and due to a round-trip in the external cavity $\omega \tau$, as $\psi = \phi_{ex} + \phi_f + \omega \tau$, where $\omega$ is the emission circular frequency and $\tau = 2\pi n_{ex} L_{ex} / c$ is the roundtrip time. The strength of OFB is measured by the coefficient $K_{ex}$, which is determined by the ratio of the external reflectivity $R_{ex}$ to the front-facet reflectivity $R_f$ as,

$$K_{ex} = (1 - R_f) \sqrt{\eta R_{ex}/R_f}.$$ (8)

Where, $\eta$ is the coupling ratio of the injected light into the laser cavity. The argument $\phi$ of the complex OFB function $T$ is obviously given by:

$$\phi = -\tan^{-1} \left\{ \text{Im}[T] / \text{Re}[T] \right\} + \ell \pi$$ (9)

Where, $\ell$ is an integer. Determining the value of $\phi$ in the two-dimensional space ($\text{Re}[T]$-$\text{Im}[T]$) depends on both the signs and magnitudes of $\text{Re}[T]$ and $\text{Im}[T]$.

By considering the effect of the external OFB the equation of the laser threshold current can be written as

$$I_{th} = e \left\{ 1/\tau_e + 1/\tau_{nr} \right\} \left[ N_s + \sqrt{V/\alpha} \left[ G_{s0} - (c/n_e L) \ln(T) \right] \right].$$ (10)

The present model of OFB is regarded as generalization of the Lang and Kobayashi model [17], in which a single round-trip ($m=1$) is assumed. In [8, 9], the authors showed that the present model approaches to that of Lang and Kobayashi under low values of $K_{ex}$ (weak to moderate OFB). In this case the OFB function $T$ is reduced to

$$\ln T = -K_{ex} \exp \left\{ -j\omega \tau \right\} \left| S(t - \tau) / S(t) \right|$$ (11)
Numerical comparison of both models was also presented in [8 and 9] confirming agreement of the simulated results only in the regimes of weak and intermediate OFB.

Characteristics of the Solitary Laser

The solitary laser is assumed to operate under CW operation. The steady-state characteristics are obtained by neglecting the OFB terms in Equations (1)–(3), equating the right-hand sides with zero and then solving the algebraic equations for the steady-state values of the photon number and electron numb

\[
BS_0^3 + \left[ \frac{BV}{(a\xi(1/\tau_r + (1/\tau_m)))} + G_{th0} \right] S_0^2 - \left( \frac{1}{e} - (N_{th0}/\tau_r) + (a\xi/V)N_e \right)S_0 - \frac{1}{e} = 0 \tag{12}
\]

While \( \bar{N} \) is determined from the equation

\[
\bar{N} = \left[ \left( a\xi/V \right) N_e S_0 + 1/e \right] \left( (a\xi/V) S_0 + (1/\tau_r + (1/\tau_m)) \right) \tag{13}
\]

These steady-state values are then employed to determine the relaxation frequency \( f_r \) of the solitary laser via a small-signal analysis as

\[
f_r \approx \left( \frac{1}{2\pi} \right) \sqrt{\left( a\xi/V \right) \left( N_e - S_0 \right) + BS_0} \] \tag{14}

PROCEDURES OF NUMERICAL CALCULATIONS

Equations (1)–(3) are solved numerically with the fourth-order Runge-Kutta method. The simulation model is applied to InGaAsP/InP lasers emitting in the wavelength of 1550 nm. The time step of integration is set as \( \Delta t = 5 \text{ ps} \), which is so short that the cutoff frequency of the Fourier transform of the laser signal is much higher than both the relaxation frequency \( f_r = 4.5 \text{ GHz} \) at \( I/I_{th0} = 2.0 \) and the external cavity resonance frequency \( f_{ext} = 2.5 \text{ GHz} \). It is also short enough to get good resolution of the time variation of the photon number. The integration was carried out over a period of 6\( \mu \text{s} \), which is long enough to achieve stable dynamics of the laser. The injection current is varied between \( I_{th0} \) and \( 6I_{th0} \), where \( I_{th0} = 10.8 \text{ mA} \) is the threshold current of the solitary laser. The influence of OFB is taken in terms of the strength of OFB \( K_{ex} \).

The integration was first made without OFB (the case of the solitary laser) from time \( t = 0 \) until the first round trip time \( \tau \), i.e., putting \( K_{ex} = 0 \) and \( \varphi = 0 \) in the rate equations. The calculated values of \( S(t) \) and \( \theta(t) \) are then stored for use as time delayed values \( S(t-\tau) \) and \( \theta(t-\tau) \) for the further integration of rate equations (1)–(3) including the feedback terms. We include three round trips (\( m = 3 \)), which is large enough to account for the case of strong OFB. The calculated values of \( S(t=0 \text{ to } \tau) \) and \( \theta(t=0 \text{ to } \tau) \) are then stored for use as time delayed values \( S(t-\tau) \) and \( \theta(t-\tau) \) for the further integration of rate equations over the period \( t=2\tau \) to \( 3\tau \) including OFB terms. Then the calculated values \( S(t-\tau), S(t-2\tau), \theta(t-\tau), \) and \( \theta(t-2\tau) \) are used as time delayed values for integration over the period \( t=2\tau \) to \( 3\tau \). Starting from this time, the integration is proceeded over a long period of time \( T=4-6 \mu\text{s} \) by considering all terms of \( S(t-m\tau) \) and \( \theta(t-m\tau), m=0,1,2,3 \), as time delayed values.

The value of the combined phase change \( \psi \) is set to be \( \psi = \omega \tau \) and the value of the external cavity length is set to be \( L_{ex} = 0.06 \text{ m} \) in the present calculations. The phase \( \varphi \) of the feedback light was chosen to vary continuously for time evolution because the solution of arc tangent is limited in the range of \( -\pi/2 \) to \( \pi/2 \) in the computer work. The averaged values \( \bar{\varphi} \) are set as zero in this paper. The applied numerical values of InGaAsP/InP lasers are given in table 1 in Ref. [18].

RESULTS AND DISCUSSIONS

In order to understand the effect of the NRRLT on the behavior of the SL subject to wide range of OFB including
strong OFB, we simulate the laser dynamics over wide ranges of the NRRLT, the OFB strength (in terms of $K_{ex}$) and injection current ratio.

### Bifurcation Diagrams in Terms of OFB Strength

Investigation of the influence of NRRLT on the route to chaos of the laser is done by simulating the bifurcation diagram of the photon numbers $S(t)$ in terms of the OFB strength $K_{ex}$ and injection current ratio $(I-I_{th})/I_{th0}$.

The bifurcation diagrams at four values of the injection current ratio, $(I-I_{th})/I_{th0}=0.05$, 0.1 (near the threshold), 1.0 and 2.0 (well above the threshold) and at four values of NRRLT parameter, $\tau_{nr} = 0.5 \tau$, (blue solid diamond), 1.0 $\tau$, (green solid up triangle), 5.0 $\tau$, (red solid circle) and 10.0 $\tau$, (black empty square), are plotted in figures 1(a) – (d), respectively. In these figures, we are interested in exploring an overview of the SL operation over a wide range of the NRRLT, injection current and OFB strength.

Under weak OFB, the solution of the rate equations is still stationary and the laser operates in CW as shown in the figures. This means that the operation is stable at all values of NRRLT parameter. By increasing the OFB strength, $K_{ex}$, this stationary solution bifurcates first into a stable PO, which characterized by undamped relaxation oscillation.

The starting point of the bifurcation is called a Hopf-bifurcation (HB) point which, and as indicated in the Figures 1(a) – (d) by arrows, is moved to higher values of the OFB strength $K_{ex}$ by the decreasing the value of the NRRLT parameter; it increase from 0.002 to 0.004 by decreasing NRRLT parameter $\tau_{nr}$ from 10.0 to 0.5, respectively as shown in Figures 1 (a) – (d).

With further increase in OFB, the results in Figure 1 shows that the solution of the rate equations bifurcates into a torus (multiple bifurcation points) followed by a chaotic state and this bifurcations depend on the NRRLT values and injection current.

In Figure 1(a), when $(I-I_{th})/I_{th0}=0.05$, and $\tau_{nr} = 10.0 \tau$, PO is followed by four bifurcation points referring to noisier transition to chaos. Such a route to chaos is SH as mentioned in Ref. [11], and in this case we called it 4SH route to chaos. By decreasing the NRRLT to $\tau_{nr} = 5.0 \tau$, the torus is PD in which PO bifurcates first into two branches, where the trajectory of $S(t)$ has two peaks of different heights in every two successive periods.

With the increase in $K_{ex}$, the PO is multiplied to more than twice and the laser is attracted to transition to chaos. When $\tau_{nr} = 1.0 \tau$, the torus still PD and by decreasing the NRRLT to $\tau_{nr} = 0.5 \tau$, the torus change to 4SH.

In Figure 1(b), when $(I-I_{th})/I_{th0}=0.1$, and $\tau_{nr} = 10.0 \tau$, PO is followed by multiple bifurcation points referring to multiple SH route to chaos. By decreasing the NRRLT to $\tau_{nr} = 5.0 \tau$, the torus transformed to 3SH, when $\tau_{nr} = 1.0 \tau$, the 3SH changed to PO route to chaos, and by decreasing the NRRLT to $\tau_{nr} = 0.5 \tau$, the torus moved to PD route to chaos.

By increasing the injection current ratio to the region well above the threshold at $(I-I_{th})/I_{th0}=1.0$, and $\tau_{nr} = 10.0 \tau$ and 5.0 $\tau$, PO is followed by PD route to chaos as shown in Figure 1(c).

When $\tau_{nr} = 1.0 \tau$ and 0.5 $\tau$, multiple bifurcation points are appeared, which means that the route to chaos is 3SH. Figure 1(d), shows that, the bifurcations diagram when $(I-I_{th})/I_{th0}=2.0$ (very far from threshold), when $\tau_{nr} = 10.0 \tau$, 5.0 $\tau$, and 1.0 $\tau$, PO is followed by 3SH route to chaos and by decreasing the NRRLT to $\tau_{nr} = 0.5 \tau$, the route to chaos is PO. That is means, the route to chaos of the SL various from SH, PD to PO route to chaos depending on the value of the NRRLT parameter and the injection current ratio.
Figure 1: Bifurcation Diagrams of a SL under OFB when $\tau_{ar} = 10.0\tau_r, 5.0\tau_r, 1.0\tau_r$, and $0.5\tau_r$ and (a) $(I-I_{thc})/I_{th0} = 0.05$, (b) $(I-I_{thc})/I_{th0} = 0.1$, (c) $(I-I_{thc})/I_{th0} = 1.0$ and (d) $(I-I_{thc})/I_{th0} = 2.0$

Figure 2: Dependencies of the Optical Feedback Strength $K_{rx}$ at HB Point on the NRRLT $\tau_{ar}/\tau_r$ at Several Values of Injection Current Ratio

The feedback level when a PO appears at HB point calculated from the numerical simulation of the rate equations using bifurcation diagrams shown in Figure 1, as a function of the NRRLT ratio $\tau_{ar}/\tau_r$, is shown in Figure 2 at several values of injection current ratio. Notice that they are almost linear functions of $\tau_{ar}/\tau_r$ with negative slope, which means that lowering the NRRLT increase the feedback level required to destabilize a CW, PO, PD, SH or a chaos state. When the injection current ratio $(I-I_{thc})/I_{th0}$ near threshold, the variation of the feedback level of the HB point with NRRLT is very slight and almost constant at higher values of $\tau_{ar}/\tau_r$. By increasing the injection current ratio $(I-I_{thc})/I_{th0}$ to be far from threshold the variation becomes faster and it is clear from Figures 1 and 2 the feedback level at HB point increase by increasing the current ratio at constant value of $\tau_{ar}/\tau_r$. Figures 1 and 2, demonstrate that, lowering the NRRLT parameter
renders a steady state more stable by increasing the feedback level above which the relaxation oscillations become undamped.

By decreasing the NRRLT, the level of the OFB strength corresponding to HB point and route to chaos increase, this is because by decreasing the value of the NRRLT, the laser threshold current is increased according to eq. (10). On the other hand, as shown in Figures 1 and 2, the effect of injection current ratio for increasing the level of the OFB strength corresponding to HB point and route to chaos is much more significant as compared with the effect of NRRLT. We believe that, from the result shown in Figures 1 and 2, the instability of the SL can be reduced by reducing the NRRLT parameter and increasing the injection current ratio to values far from the threshold, which helps to design a laser diode with high static and dynamic performance.

**Bifurcation Diagrams in Terms of Injection Current**

Figures 3(a) and (b) plot the bifurcation diagrams as a function of the injection current ratio \((I-I_{thc})/I_{th0}\) at two values of OFB strength \(K_{ex} = 0.01\) and 0.02 (route to chaos) and four values of the NRRLT parameter \(\tau_{nr} = 10.0\tau_r, 5.0\tau_r, 1.0\tau_r\) and \(0.5\tau_r\), respectively. In these figures, we are interested in exploring an overview of the SL operation over a wide range of the NRRLT and injection current ratio \((I-I_{thc})/I_{th0}\). Near to the threshold current, the laser operates in chaos as shown in Figure 3(a), which means that no dynamic stability is obtained at all values of the NRRLT parameter \(\tau_{nr}\). By increasing the injection current, the laser operates in PO and is then changed to CW. When \((I-I_{thc})/I_{th0} > 1.1\), the laser keeps oscillating until reaching to CW operation. With increasing the OFB strength \(K_{ex}\) to 0.02, Figure 3(b) shows that the laser operates in a chaotic state (chaos) near above to the threshold current. By increasing the injection current ratio between 0.3 and 0.5 and the NRRLT parameter to \(\tau_{nr} = 0.5\), the unstable state of chaos operation becomes a PD in which chaos bifurcates first into two branches, where the trajectory of \(S(t)\) has two peaks of different heights in every two successive periods. With the increase in \((I-I_{thc})/I_{th0}\), PD state appeared when \(\tau_{nr} = 1.0\tau_r, 5.0\tau_r,\) and \(10.0\tau_r\) at different values of \((I-I_{thc})/I_{th0}\). When \((I-I_{thc})/I_{th0}\) higher than 1.0, the laser operation moved to PO at all values of the NRRLT parameter \(\tau_{nr}\), and then the laser is attracted to transition to CW operation at higher values of injection current \((I-I_{thc})/I_{th0} > 2.8\).

![Figure 3: Bifurcation Diagrams of a SL under OFB](image)

To explain the results that is shown in Figures. 1, 2 and 3, we used eq. (10) to demonstrate the effect of the NRRLT parameter, OFB strength and injection current on the laser threshold current characteristics. Any decrease in the NRRLT leads to increase in the threshold current as referred in eq. (10). And any increase in the OFB strength reduces the total cavity loss of the laser diode [19], which caused to increase the photon lifetime and increase the threshold carrier.
density, which increase the laser threshold current. Thus, from the physical and practical point of view, any increase in the laser threshold current leads to increase in the laser stability. These results support our conclusion that, at lower values of NRRLT and when the injection current be well above the threshold the laser stability is improved and moves toward stable operations.

**Time Variation and FFT of S(t) and the Phase Portrait for Various States of the SL**

In this section, we examine the time variation and FFT of S(t) and phase portrait of SL of the operation states that characterize the routes to chose shown in Figures. 1(a) - (d) at four values of the NRRLT parameter $\tau_{nr}=10.0\,\tau_{r},\ 5.0\,\tau_{r},\ 1.0\,\tau_{r}$ and $0.5\,\tau_{r}$. The examination is taken over a wide range of the injection current ratio $(I-I_{th0})/I_{th0}=0.05,\ 0.1,\ 1.0$ and $2.0$.

The simulated time variation of the photon number $S(t)$, frequency FFT spectrum and phase portrait are shown in Figure 4 when $(I-I_{th0})/I_{th0}=0.05$. The laser exhibits oscillations with four peaks route to chaos when $\tau_{nr}=10.0\,\tau_{r}$ and $K_{ex}=0.01$ (Figure 4 a), and by increasing the NRRLT parameter to $\tau_{nr}=5.0\,\tau_{r}$ and $1.0\,\tau_{r}$ and $K_{ex}=0.0104$ and $0.0106$ (Figures 4 d and g) these 4 peaks oscillations change to oscillations with two peaks which is changed to 4 peaks oscillations at $\tau_{nr}=0.5\,\tau_{r}$ and $K_{ex}=0.012$ (Figure 4 j). Figures 4. (b, e, h, and k) plot the FFT spectra of oscillations at the route to chaos point when $\tau_{nr}=10.0\,\tau_{r},\ 5.0\,\tau_{r},\ 1.0\,\tau_{r}$ and $0.5\,\tau_{r}$ and $K_{ex}=0.01,\ 0.0104,\ 0.0106$ and $0.012$, respectively. The spectrum is characterized by the relaxation frequency $f_r$. The oscillations shown in Figures. 4 (a, d, g and j) are confirmed by the 4SH attractor that characterizes the phase portrait of Figures. 4 (c, f, i and l).

By increasing $(I-I_{th0})/I_{th0}$ to 0.1, the laser operation becomes more complicated and could be considered a kind of periodic variation operation such as SH, PO and PD oscillations. These three types of the route to chaos appear in Figures. 5 (a, d, g and j). When $\tau_{nr}=10.0\,\tau_{r}$ and $K_{ex}=0.0095$, the multiple SH route to chaos is indicated by the multiple different peaks (Figure 5 (a)) which changes to three peaks when $\tau_{nr}=5.0\,\tau_{r}$ and $K_{ex}=0.01$ (Figure. 5 (d)).

These features represent the 3SH route to chaos as discussed on Figures. 1(a) and (b). Figure 5 (g), ($\tau_{nr}=1.0\,\tau_{r}$ and $K_{ex}=0.012$) shows the case that the system nonlinearity decreases the irregularities in $S(t)$ attracting the laser oscillations into PO and when $\tau_{nr}=0.5\,\tau_{r}$ and $K_{ex}=0.016$ the PD route to chaos is indicated by the two peaks (Figure 5 (j)).

The spectra show multiple, three, one and two peaks, which correspond to multiple, three, one and two different peaks of $S(t)$ appears in Figures. 5 (b, e, h and k), respectively. In Figures. 5 (c, f, i and l), a tours, which represent the multiple SH, 3SH, LC and PD attractors route to chaos appear in the phase portrait, which confirm the result shown in Figures. 5 (a, d, g, j).

The increase in the $(I-I_{th0})/I_{th0}$ to 1.0 shows a different kind of the route to chaos as shown in Figure. 6. When $\tau_{nr}=10.0\,\tau_{r}$ and $5.0\,\tau_{r}$ and $K_{ex}=0.024$, the PD route to chaos is indicated by the two different peaks (Figure. 6 a and d) which change to three peaks when $\tau_{nr}=1.0\,\tau_{r}$ and $0.5\,\tau_{r}$ and $K_{ex}=0.028$ and 0.048 (Figures 6 g and j).

These features represent the 3SH route to chaos as discussed on Figure 1(c). The spectra show two and three peaks, which correspond to two and three different peaks of $S(t)$ appears in Figures. 6 (b, e, h and k). In Figures. 6 (c, f, i and l), a tours, which represent the PD and 3SH attractors route to chaos appear in the phase portrait, which confirm the result shown in Figures. 6 (a, d, g, j).

Under higher values of $(I-I_{th0})/I_{th0} = 2.0$, and when it becomes well above the threshold, the route to chaos of the SL, when $\tau_{nr}=10.0\,\tau_{r},\ 5.0\,\tau_{r}$ and $1.0\,\tau_{r}$ and $K_{ex}=0.03,\ 0.03$ and $0.05$, is characterized by three different peaks in the time variation of the photon number $S(t)$, as shown in
Figure 4: Time and FFT Variations of Photon Number $S(t)$ and Phase Portraits of a SL at Route to Chaos Point, when $(I-I_{thc})/I_{th0} = 0.05$ and at $\tau_{nr} = 10.0\tau_r, 5.0\tau_r, 1.0\tau_r$ and $0.5\tau_r$.

Figure 5: Time and FFT Variations of Photon Number $S(t)$ and Phase Portraits of a SL at Route to Chaos Point, when $(I-I_{thc})/I_{th0} = 0.1$ and at $\tau_{nr} = 10.0\tau_r, 5.0\tau_r, 1.0\tau_r$ and $0.5\tau_r$.

Figures 7 (a, d and g), respectively. When $\tau_{nr} = 0.5\tau_r$ and $K_{ex} = 0.07$, the route to chaos changes to PO route to chaos with one peak. Figures 7 (b, e, h and k) show the FFT power spectra characterizing the 3SH and PO dynamics. The 3SH and LC attractors representing the SH and PO operation are shown in the phase portrait in Figures 7 (c, f, i and l), respectively. Finally, we can conclude that, the decrease in the NRRLT under higher values of the injection current ratio converts the complicated and unstable system to the stable state PO operation.
CONCLUSIONS

We investigated the influence of NRRLT parameter on the route to chaos of semiconductor lasers. The simulations were applied to 1550-nm InGaAsP/InP lasers over a wide range of the injection current. The route to chaos and the associated laser operations are classified by the bifurcation diagrams of the photon number, time-variations of the photon number and the FFT power spectrum as functions of the NRRLT parameter and injection current. The simulation
results showed that, inclusion of the NRRLT parameter in the rate equations of SLs cause significant changes in the operations and the route to chaos of the lasers. We found that the values of the feedback strength when the transitions from CW to PO or the coherence collapsed state (chaos) occurs decreases with the increase in NRRLT parameter. Including NRRLT parameter in the time-delay rate equation model was found to change the route to chaos of the laser from SH or PD type to the SH or PD or PO type. We identify the route to chaos states of SLs in three distinct operating regions, namely, PO, SH, and PD depending on the value of the NRRLT parameter and injection current ratio. Under higher values of the injection current ratio and by decreasing the NRRLT, the laser route to chaos changes from SH or unstable route to chaos to stable PO route to chaos.

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REFERENCES


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