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HAAR WAVELET QUASI-LINEARIZATION APPROACH FOR SOLVING LANE EMDEN EQUATIONS

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ABSTRACT

The purpose of this work is to investigate the Lane Emden equation by using Haar wavelet quasi-linearization approach defined over the interval [0,1]. The Lane Emden equation is widely studied and challenging equation in the theory of stellar structure for the gravitational potential of a self gravitating, spherically symmetric polytropic fluid which models the thermal behaviour of a spherical cloud of gas acting under the mutual attraction of its molecules and subject to the classical laws of thermodynamics.The proposed method is based on the replacement of unknown function through a truncated series of Haar wavelet series of the function. The method is shown to be very reliable for any finite polytropic index p and for other linear and nonlinear cases of Lane Emden equations.

KEYWORDS: Lane Emden Equation(LEE), Isothermal Gas Sphere, Emden Fowler Equation, Haar Wavelets, Quasi-Linearization Technique

INTRODUCTION

Lane-Emden equation is one of the basic equations in the theory of stellar structure and models many phenomena in mathematical physics and astrophysics [1,2,3]. It is a nonlinear differential equation which describes the equilibrium density distribution in self-gravitating sphere of polytrophic isothermal gas, has a regular singularity at the origin. This equation was first studied by the astrophysicists Jonathan Homer Lane and Robert Emden which considered the temperature variation of a spherical gas cloud under the mutual attraction of its molecules and subject to the laws of classical thermodynamic [4,5].

The polytropic theory of stars essentially follows from thermodynamic considerations that deal with the issue of energy transport, through the transfer of material between different levels of the star and modeling of clusters of galaxies. Mostly problems with regard to the diffusion of heat perpendicular to the surfaces of parallel planes are modelled by the heat equation.

$$\xi^{-\alpha}(\xi^{\alpha}\omega)_{\xi} + f(\xi,\omega) = h(\xi,\omega), \qquad 0 \le \xi \le 1,\ \alpha > 0 \tag{1.1}$$

Where $\omega(\xi)$ is the temperature and $f(\xi,\omega)$ is given function of ξ and ω. For the steady-state case and for $h(\xi,\omega)=0$, this equation becomes a standard Lane Emden equation which is given by

$$(\omega)_{\xi\xi} + \frac{\alpha}{\xi}(\omega)_{\xi} + f(\xi,\omega) = 0, \quad 0 \le \xi \le 1,\ \alpha > 0, \tag{1.2}$$

subject to the initial conditions:

$$\omega(0) = A, \omega^{'}(0) = B \tag{1.3}$$

Recently, many analytic and numerical methods have been used to solve Lane-Emden equations, the main difficulty arises at the singularity of the equation at origin. We discuss here about those methods in brief. Wazwaz [4] applied a non perturbative approximate analytical solution using the modified decomposition method and Yildirim [6] has used the variational iteration method to solve the Emden-Fowler type of equations. Mandelzweig et al.[7] used quasi-linearization approach to solve Eq.(1.2), Parand et.al [8] proposed an approximation algorithm for the solution of using Hermite functions. Singh et. al [9] has used the homotopy analysis method while Ramos [10] presented a series approach on same and made comparisons with homotopy purturbation method. Moreover, in literature wavelet method also used for solving Eq.(1.2). Yousef [11] has applied an integral operator for conversion of Lane-Emden equation to integral equation and solved by Galerkin and collocation methods with Legendre wavelets. But Galerkin method creates numerically complications when nonlinearities are treated in a wavelet sub-space, where integrals of products of wavelets and their derivatives must be computed. This can be done by introducing the connection coefficients [12] and it is applicable only for a narrow class of equations.

In order to mollify this criticism we would like to emphasize on the advantages of the Haar wavelet method, it is possible to detect singularities, local high frequencies, irregular structure and transient phenomena exhibited by the analyzed function. In 1910, Alfred Haar introduced a function which presents an rectangular pulse pair. After that various generalizations and definitions were proposed(state-of-the art about Haar transforms can be found in).Therefore it is not possible to apply the Haar wavelet directly for solving differential equations.

There are some possibilities to come out from this impasse. First the piecewise constant Haar function can be regularized with interpolation splines, this technique has been applied in several papers by Cattani(see e.g. in [13]). The second possibility is to make use of the integral method, by which the highest derivative appearing in the differential equation is expanded into the Haar series. This approximation is integrated while the boundary conditions are incorporated by using integration constants. This approach has been realized by Chen and Hsiao[14]. Cattani observed that computational complexity can be reduced if the interval of integration is divided into some segments. The number of collocation points in each segment is considerably smaller as in the case of Chen and Hsiao method(CHM); Further simplification, it is assumed that the highest derivative is constant in each segment, therefore this method is called "piecewise constant approximation(PCA)". This method has been applied by Hsiao and Wang [15]. Nonlinearity of the system also complicates the solutions, since big systems of nonlinear equations must be solved. Harpreet et. al [16] have proposed the Haar wavelet quasi-linearization technique by using the concept of Chen and Hsiao operational matrix with quasi-linearization process for solving nonlinear boundary value problems.

The main aim of this paper is to elaborate the Haar wavelet method and to apply it on linear and nonlinear singular Lane Emden equations which represent important mathematical model in the theory stellar structure [2]. The method reduces the problem to the system of algebraic equations.

This section is devoted to the introduction of different methods for solving LEE's. In section 2, we have discussed the derivation of LEE. Section 3, depicts the fundamentals of construction of Haar wavelets as construction of wavelets, its properties and operational(associative) matrices of derivatives as a working tool. The section 4, reveals that how quasi-linearization works with Haar wavelets on finite intervals. In section 5, we have discussed the convergence of method. In section 6, the applicability of Haar wavelet quasi-linearization method is revealed. Numerical results are compared with available solutions in literature. The conclusion is described in the final section.

FORMULATION OF LANE EMDEN EQUATION

Lane Emden equation(1.1) has been widely studied widely in the literature(see for example Benko et al.[17], Mohan and Al-Bayaty [18], Harley and Momoniat [19]). Dehghan and Shakeri [20] first applied exponential transformation($\xi = e^{\xi}$) to the Lane-Emden equation in order to address the difficulty of a singular point at $\xi = 0.$ For describing it, consider a spherical cloud of gas and denote its hydrostatic pressure at a distance r from the center by pressure P. Let $M(r)$ be the mass of the sphere at radius r, φ is the gravitational potential of the gas and g is the acceleration of the gravity. We simply begin with the Poisson equation and hydrostatic equilibrium condition:

$$g = -\frac{d\varphi}{dr} = \frac{GM(r)}{r^2} \tag{2.4}$$

where G(the gravitational constant) $= 6.668 \times 10^{-8} cgs$ units. We are led to consider a relation of the following form determined φ and $P,$ namely

(i) $dP = -g\rho dr = \rho d\varphi,$ where ρ is the density of gas

(ii) $\nabla^2 \varphi = -4\pi g\rho;$ in which $\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr};$

(iii) The relation of pressure P and density ρ in steller structure and in the polytropic model, is given by $P = k\rho^{\gamma}$ where γ and K are empirical constants. When this value of $\rho = L\varphi^n$ is introduce into (i), we obtain the relation

$$\nabla^2 \varphi = -a^2 \varphi^n \text{ here } a^2 = 4LG. \tag{2.5}$$

If, we let $\varphi = \varphi_0$ where φ_0 is the value of φ at the center of the sphere and let $r = \xi / (a\varphi_0^{\frac{1}{2}(n-1)}),$ then (iii) reduces to the Lane Emden Eq.(1.2) with $f(\xi, \omega) = \omega^p$. The initial conditions specified in Eq.(1.3).

Since we have assumed that at $\varphi = 0$ the surface of the sphere, when $\omega = 0$ and $\varphi = \varphi_0$ at the center of the sphere, when $\omega = 1$, it is clear that we are interested in those values of the solution of Eq.(1.2) between 0 and 1. The radius R of the gas sphere and its total mass $M = M(R)$ are given by the following equations:

$$R = (r)_{\omega=0}, GM = \left(-r^2 \frac{d\varphi}{dr}\right)_{\omega=0} \tag{2.6}$$

and Poisson equation is now replaced by

$$\nabla^2 \varphi = -a^2 e^{\varphi/K}, a^2 = 4\pi\rho_0 G, \tag{2.7}$$

which is generally known as Liouville's equation.

Assuming spherical symmetry, Eq.(2.7) in polar coordinates reduce to the following:

$$\frac{d^2\varphi}{dr^2}+\frac{2}{r}\frac{d\varphi}{dr}+a^2e^{\varphi/K}=0 \tag{2.8}$$

If we let $\varphi=K\omega, r=(\sqrt{K}/a)\xi,$ then (2.8) becomes

$$\frac{d^2\omega}{d\xi^2}+\frac{2}{\xi}\frac{d\omega}{d\xi}+e^{\omega}=0 \tag{2.9}$$

which is Lane Emden Eq. (1.1) with $f(\xi,\omega)=e^{\omega}$ and to be solved subject to the initial conditions: $\omega(0)=\omega'(0)=0.$

We will have various forms of famous Lane-Emden equation and first case is considered by $f(\xi,\omega)=\omega^p(\xi)$

$$\frac{d^2\omega}{d\xi^2}+\frac{2}{\xi}\frac{d\omega}{d\xi}+\omega^p(\xi)=0,\quad \xi\geq 0 \tag{2.10}$$

where p is the polytropic index which is related to the ratio of specific heats of the gas comprising the star.

Second kind of Lane Emden equation is defined as

$$\frac{d^2\omega}{d\xi^2}+\frac{\alpha}{\xi}\frac{d\omega}{d\xi}+\xi^v f(\xi,\omega)=h(\xi),\quad \xi\geq 0 \tag{2.11}$$

subject to the boundary conditions: $\omega(0)=1,\quad \omega'(0)=0$

FUNDAMENTALS OF HAAR WAVELETS AND OPERATION MATRIX OF INTEGRATION

The structure of Haar wavelet family is based on multiresolution analysis [21,22].A multiresolution analysis $K=\{V_j\subset L_2 \mid j\in J\subset Z\}$ of X consisting of a sequences of nested spaces on $V_j\subseteq V_{j+1}$ different levels j whose union is dense in $L^2(X)$. Let $L_2(X)$ be the space of functions with finite energy defined over a domain $X\subseteq R^n$, and let $\langle .,.\rangle$ be an inner product on X . Bases of the spaces Vj are formed by the sets of scaling basis functions. Bases of the spaces V_j are formed by the sets of scaling basis functions $\{\varphi_{j,k}\mid k\in\kappa(j)\}$, in complete orthonormal system [23],where $\kappa(j)$ is an index set defined over all basis functions on level j. The strictly nested structure of the V_j implies the existence of difference spaces W_j such that $V_j\oplus W_j=V_{j+1}$. The W_j are spanned by sets of Haar wavelet basis functions $\{h_{j,k}\mid k\in K(j)\}$. The basic and simplest form of Haar wavelet is the Haar scaling function that appears in the form of a square wave over the interval $[0,1]$, denoted with $h_i(\xi)$ and generally written as

$$h_1(\xi) = \begin{cases} 1 & \xi \in [0,1) \\ 0 & \text{elsewhere} \end{cases} \tag{3.12}$$

The above expression, called Haar father wavelet, is the zeroth level wavelet which has no displacement and dilation of unit magnitude.

According to the concept of MRA[24] as an example the space V_j can be defined like

$$V_j = sp\{h_{j,k}\}_{j=0,1,2,....,2^j-1} = W_{j-1} \oplus V_{j-1} = W_{j-1} \oplus W_{j-2} \oplus V_{j-2} \oplus .... = \oplus_{j=1}^{J+1} W_j \oplus V_0, \tag{3.13}$$

The Haar mother wavelet is the first level Haar wavelet can be written as the linear combination of the Haar scaling function as

$$h_2(\xi) = h_1(2\xi) + h_1(2\xi - 1) \tag{3.14}$$

Each Haar wavelet is composed of a couple of constant steps of opposite sign during its subinterval and is zero else where.The term wavelet is used to refer to a set of orthonormal basis functions generated by dilation and translation of a compactly supported scaling function $h_1(\xi)$ (father wavelet) and a mother wavelet $h_2(\xi)$ associated with an multiresolution analysis of $L^2(R)$. Thus we can write out the Haar wavelet family as

$$h_i(\xi) = \begin{cases} 1 & \frac{k}{2^j} \le \xi < \frac{k+0.5}{2^j} \\ -1 & \frac{k+0.5}{2^j} \le \xi \le \frac{k+1}{2^j} \\ 0 & \text{elsewhere} \end{cases} \tag{3.15}$$

for $i \ge 2,\ i = 2^j + k + 1,\ j \ge 0,\ 0 \le k \le 2^j - 1$ and $\xi_l = \frac{l - \frac{1}{2}}{2m},\ l = 1,2,....,2m.$ Here m is the level of the wavelet, we assume the maximum level of resolution of index J, then $m = 2^j, (j = 0,1,2,....,J);$ in case of minimal values $m = 1, k = 0$ then $i = 2$. For any fixed level m, there are m series of i to fill the interval $[0,1)$ corresponding to that level and for a provided J, the index number i can reach the maximum value $M = 2^{J+1}$,when including all levels of wavelets. The operational matrix $p_{i,n}(\xi)$ of order $2m \times 2m$ is derived from integration of Haar wavelet family with the aid of following formula:

$$p_{i,2}(\xi)=\begin{cases}\frac{1}{2}\left(\xi-\frac{k}{m}\right)^2 & \xi\,\varepsilon\left[\frac{k}{m},\frac{k+0.5}{m}\right) \\ \frac{1}{4m^2}-\frac{1}{2}\left(\frac{k+1}{m}-\xi\right)^2 & \xi\,\varepsilon\left[\frac{k+0.5}{m},\frac{k+1}{m}\right) \\ \frac{1}{4m^2} & \xi\,\varepsilon\left[\frac{k+1}{m},1\right) \\ 0 & \text{elsewhere}\end{cases} \tag{3.16}$$

HAAR WAVELET APPROXIMATION WITH QUASI-LINEARIZATION APPROACH TO LANE EMDEN EQUATION

The Haar basis has the very important property of multiresolution analysis that $V_{j+1}=V_j\oplus W_j$. The orthogonality property puts a strong limitation on the construction of wavelets and allows us to transform any square integral function on the interval time $[0,1)$ into Haar wavelets series as

$$\omega(\xi)=a_0h_0(\xi)+\sum_{j=0}^{\infty}\sum_{k=0}^{2^j-1}a_{2^j+k}h_{2^j+k}(\xi),\qquad \xi\in[0,1] \tag{4.17}$$

Similarly the highest derivative can be written as wavelet series $\sum_{i=-\infty}^{\infty}a_ih_i(\xi)$. In applications, Haar series are always truncated to 2m terms[25] and written as

$$\sum_{i=0}^{2m}a_ih_i(\xi)=a^TH_{2m}(\xi) \tag{4.18}$$

Then we have used the quasi-linearization process. The quasi-linearization process is an application of the Newton Raphson Kantrovich approximation method in function space given by Bellman and Kalaba[26]. Consider an nth order nonlinear ordinary differential equation

$$L^{(n)}\omega(\xi)=f(\omega(\xi),\omega^{(1)}(\xi),\omega^{(2)}(\xi),\omega^{(3)}(\xi)......,\omega^{(n-1)}(\xi),\xi) \tag{4.19}$$

with the initial conditions

$$\omega(0)=\lambda_0,\omega^{(1)}(0)=\lambda_1,\omega^{(2)}(0)=\lambda_2,\omega^{(3)}(0)=\lambda_3,......,\omega^{(n)}(0)=\lambda_n \tag{4.20}$$

Here $L^{(n)}$ is the linear n^{th} order ordinary differential operator, f is nonlinear functions of $\omega(\xi)$ and its $(n-1)$ derivatives are $\omega^{(s)}(\xi), s=0,1,2,......,n-1.$

The quasi-linearization prescription determines the $(r+1)^{th}$ iterative approximation to the solution of Eq. (4.20) and its linearized form is given by Eq.(4.22)

$$L^{(n)}\omega_{r+1}(\xi)=f(\omega_r(\xi),\omega_r^{(1)}(\xi),\omega_r^{(2)}(\xi),\omega_r^{(3)}(\xi)......,\omega_r^{(n-1)}(\xi),\xi)+\sum_{s=0}^{n-1}(\omega_{r+1}^{(s)}(\xi)-\omega_r^{(s)}(\xi))$$
$$f_{\omega}^{(s)}(\omega_r(\xi),\omega_r^{(1)}(\xi),\omega_r^{(2)}(\xi),\omega_r^{(3)}(\xi)......,\omega_r^{(n-1)}(\xi),\xi) \tag{4.21}$$

where $\omega_r^{(0)}(\xi)=\omega_r(\xi)$. The functions $f_{\omega^{(s)}}=\dfrac{\partial f}{\partial \omega^s}$ are functional derivatives of the functions. The zeroth approximation $\omega_0(\xi)$ is chosen from mathematical or physical considerations. When $f(\xi,\omega)$ is nonlinear function in Eq. (1.2), we linearize by using quasi-linearization process and followed by Eq. (4.22), Eq.(1.2) can be written as

$$\omega_{r+1}^{(2)}(\xi)+\frac{\alpha}{\xi}\omega_{r+1}^{(1)}(\xi)+f(\omega_r(\xi),\omega_r^{(1)}(\xi),\omega_r^{(2)}(\xi),\omega_r^{(3)}(\xi)......,\omega_r^{(n-1)}(\xi),\xi)+$$
$$\sum_{s=0}^{n-1}(\omega_{r+1}^{(s)}(\xi)-\omega_r^{(s)}(\xi))f_{\omega}^{(s)}(\omega_r(\xi),\omega_r^{(1)}(\xi),\omega_r^{(2)}(\xi),\omega_r^{(3)}(\xi)......,\omega_r^{(n-1)}(\xi),\xi)=0 \tag{4.22}$$

Any $\omega(\xi)\in L_2([0,1])$ can be expressed in the form of Haar wavelet series as

$$\omega_{r+1}^{(n)}(\xi)=\sum_{i=0}^{2m}a_i h_i(\xi) \tag{4.23}$$

then using the concept of operational matrix and Haar wavelet technique, we can obtain the derivatives from following equation

$$\omega_{r+1}^{(n-n)}(\xi)=\sum_{i=0}^{2m}a_i p_{i,n}(\xi)+\xi^n\omega_r^{(n-1)}(0)+\xi^{n-1}\omega_r^{(n-2)}(0)+......+\omega_r^{(n-n)}(0) \tag{4.24}$$

If $f(\xi,\omega)$ is nonlinear function then by Eq's(4.23) and (4.25), Eq.(1.2) becomes

$$\sum_{i=0}^{2m}a_i h_i(\xi_l)+\left(\frac{\alpha}{\xi_l}\right)\sum_{i=0}^{2m}a_i p_{i,1}(\xi_l)+f(\omega_r(\xi_l),\omega_r^{(1)}(\xi_l),.....,\omega_r^{(n-1)}(\xi_l),\xi_l)+\sum_{s=0}^{n-1}(\omega_{r+1}^{(s)}(\xi_l)-\omega_r^{(s)}(\xi_l))$$
$$f_{\omega}^{(s)}(\omega_r(\xi_l),\omega_r^{(1)}(\xi_l),\omega_r^{(2)}(\xi_l),.....,\omega_r^{(n-1)}(\xi_l),\xi_l)\sum_{i=0}^{2m}a_i p_{i,2}(\xi_l)=-(\omega^{'}(0)+\xi_l\omega^{'}(0)+\omega(0)) \tag{4.25}$$

Finally get the Haar wavelet solution (HWS) of Eq. (1.2) by substituting the value of $a_i^{'}s$ in Eq.(4.25).

NUMERICAL RESULTS AND DISCUSSIONS

This section presents the numerical results and discussion of the proposed method for solving different cases of Lane Emden equation. Some special cases in Eq.(1.2) are considered to demonstrate the efficiency of method. All results are computed by C++ and MATLAB R2007b and are reported in Figures1-6 and Tables1-4.

HOMOGENEOUS LANE EMDEN TYPE EQUATIONS

Case 6.1.1 : If $f(\xi,\omega)=\omega^p,\ h(\xi)=0,\alpha=2,A=1,B=0$ in Eqs.(1.2 and 1.3), obtain standard Lane Emden equation with polytropic index p as

$$\frac{d^2\omega}{d\xi^2}+\frac{2}{\xi}\frac{d\omega}{d\xi}+\omega^p(\xi)=0,\quad \alpha,\xi\geq 0 \tag{6.26}$$

which play an important role in the polytropic models and discussed by Chandrasekhar in [1]. However, the polytrope of index p = 0 also terminates at a finite radius just as is observed in the relation for a polytrope of index p = 1. Though these two solutions for p = 0 and p = 1 share

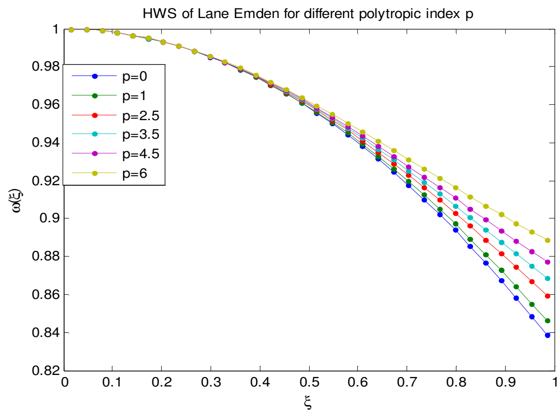

Figure 1: Plot of Lane Emden Equation for different Polytropic Index P and M = 32 by Present Method

Many characteristics, the solution for the polytrope of index p = 6 contains some radically different and unexpected characteristics. Here the density of the star initially decreases rapidly as radius increases but slows rapidly. Though it may not be apparent on the graphic provided, the function never reaches 0. It is, therefore, evident that a polytropic star of index p = 5 has an infinite radius, and in reality cannot exist. Finally the solution of Eq.(6.26) has been obtained by substituting the value of $a_i^{'}s$ in approximate Haar wavelet series solution. It has been claimed in the literature that exact solution is available only for p = 0,1 and 5. We can find HWS for any finite p. Numerical results by proposed method for p = 0, 1.5, 2.5,3.25,3.5,4.5,6 are shown in Figure1 and Table1.

Case 6.1.2 : The White-dwarf equation

$$\frac{d^2\omega}{d\xi^2}+\frac{\alpha}{\xi}\frac{d\omega}{d\xi}+(\omega^2(\xi)-C)^{3/2}=0, \quad \xi>0 \tag{6.27}$$

with the supplementry conditions: $\omega(0)=1, \quad \omega'(0)=0$.

This equation is introduced by Chandrasekhar in his study of the gravitational potential of the degenerate White-dwarf stars can be modelled by so called White-dwarf equation. It is clear that Eq. (6.27) is of Lane Emden type where $f(\xi,\omega)=(\omega^2-C)^{3/2}$. If $C=0$, it reduces to Lane Emden equation of polytropic index $p=3$. For a thorough discussion of the White-dwarf, see [28]. By our purposed method, Eq.(6.27) yields the solution C=0,0.1,0.2,0.3 which is shown in Figure 2 and Table 2.

Case 6.1.3a: Second order nonlinear Emden Fowler Equation is in the following form as

$$\frac{d^2\omega}{d\xi^2}+\frac{2}{\xi}\frac{d\omega}{d\xi}+\beta^r(\xi)\omega^p=0, \quad \xi\geq 0 \tag{6.28}$$

subject to the initial conditions: $\omega(0)=1, \quad \omega'(0)=0$. Haar wavelet solution is obtained by proposed technique for m=32 and is shown by Figure 3 and Table3.

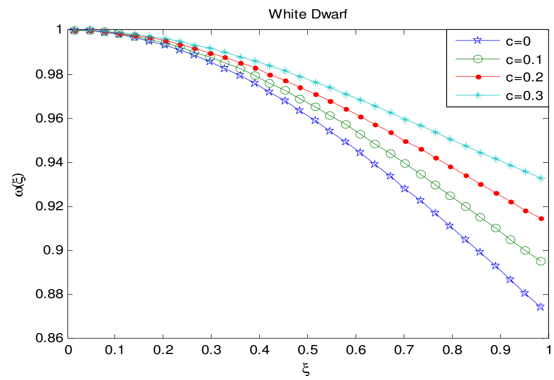

Figure2: Plot of Haar Wavelet Solution of White-Dwarf for m = 32

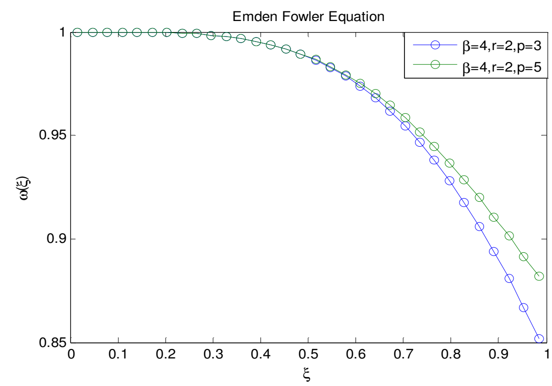

Figure 3: Plot of HWS for Case 6.1.3a by Present Method,m = 32.

Case 6.1.3b : If $f(\xi,\omega)=ae^{(-\omega)}\ln^2(\xi)$ in Eq.(1.2) and consider $a=-1$ with conditions: $\omega(0)=0,\omega'(0)=0$, get the following form of nonlinear Emden Fowler equation[28] :

$$\frac{d^2\omega}{d\xi^2}+\frac{2}{\xi}\frac{d\omega}{d\xi}+ae^{(-\omega)}\ln^2(\xi)=0,\xi\geq 0 \tag{6.29}$$

By proposed technique, Haar wavelet solution of Eq.(6.29) for a = -1 and m = 32 is dipicted in Table 3.

Case 6.1.4: Isothermal Gas Spheres Equation

Isothermal Gas Spheres equation modeled by Davis [2] and for $f(\xi,\omega)=e^{\omega}$, Eq.(1.2) can have the form

$$\frac{d^2\omega}{d\xi^2}+\frac{2}{\xi}\frac{d\omega}{d\xi}+e^{\omega}=0,\ \xi\geq 0 \tag{6.30}$$

with initial conditions: $\omega(0)=0,\omega'(0)=0$ .This equation describes a model known as the singular isothermal spheres [29]. Unfortunately, the singular isothermal sphere has infinite density at $\xi=0$. In order to understand the behavior of the solutions there, a Haar series expansion is used. For large radius where the effect of the central conditions is very weak the solution should asymptotically approach the singular isothermal solution. Comparison of Haar wavelet solution with series solution is shown by Figure 4 and Table 4.

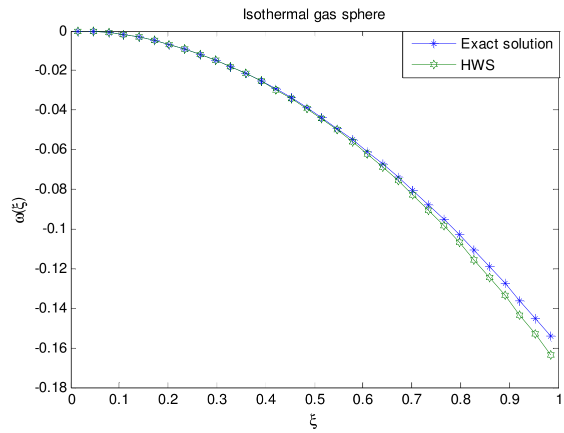

Figure 4: Plot of Comparison of HWS for Isothermal Gas Spheres, m = 32

Case 6.1.5: If in Eq.(1.1) $f(\xi,\omega)=2(2\xi^2+3)\omega(\xi)$ get the following case

$$\frac{d^2\omega}{d\xi^2}+\frac{2}{\xi}\frac{d\omega}{d\xi}+\omega(\xi)=2(2\xi^2+3)\omega(\xi), \quad \xi\ge 0 \tag{6.31}$$

subject to the boundary conditions: $\omega(0)=1, \quad \omega'(1)=0$. The exact solution for this problem is

$\omega(\xi)=e^{\xi^2}$ and comparison of HWS with exact one is shown in Figure 5 and Table 4.

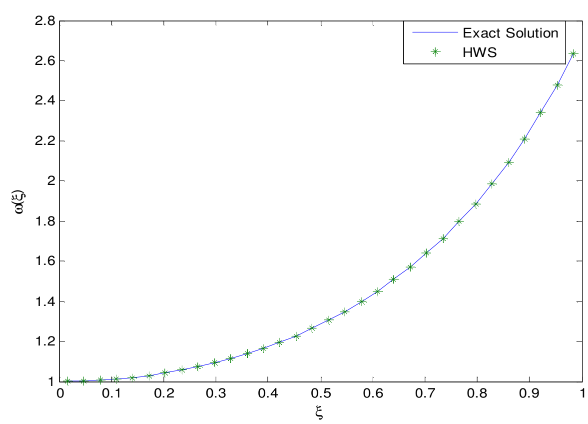

Figure 5: Plot of Comparison of HWS for Case 6.1.5, m = 32

NON-HOMOGENEOUS LANE EMDEN TYPE EQUATIONS

Case 6.2.1: Consider the Differential Equation

$$\frac{d^2\omega}{d\xi^2}+\frac{8}{\xi}\frac{d\omega}{d\xi}+\xi\omega(\xi)=\xi^5-\xi^4+44\xi^2-30\xi, \quad \xi\ge 0 \tag{6.32}$$

subject to the boundary conditions: $\omega(0)=0, \quad \omega'(0)=0$,which has analytic solution $\omega(\xi)=\xi^4-\xi^3$ and comparison of HWS with analytic is shown in Figure6 and Table4.

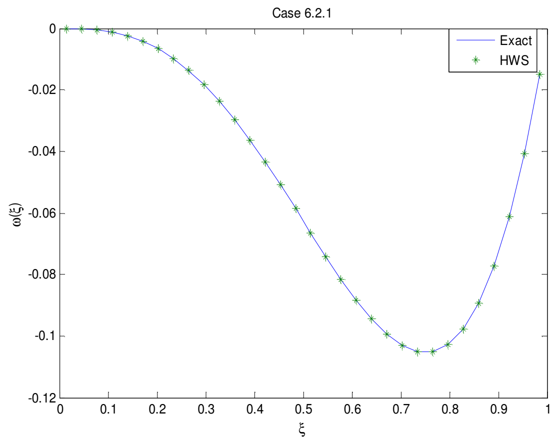

Figure 6: Plot of Comparison of HWS for m = 32

Table1: Comparison of Present Method Solution with Exact One and [32] for Case 6.1.1

ξ	EXACT p=0	HWS p=0 m=8	EXACT p=1	HWS p=1 m=16	EXACT p=5	HWS p=5 m=32	HWS p=2.5 m=32	HWS p=3.25 m=32	HWS p=6 m=32
0.1	**0.998333**	**0.998333**	**0.998334**	**0.998334**	**0.998337**	**0.998337**	**1.00000**	**1.00000**	**0.998338**
0.2	**0.993333**	**0.993334**	**0.993347**	**0.993346**	**0.993399**	**0.993389**	**0.998335**	**0.998336**	**0.993412**
0.3	**0.985001**	**0.985001**	**0.985067**	**0.985068**	**0.985336**	**0.985346**	**0.993366**	**0.993377**	**0.985414**
0.4	**0.973333**	**0.973336**	**0.973546**	**0.973547**	**0.974355**	**0.974448**	**0.98517**	**0.985223**	**0.974677**
0.5	**0.958333**	**0.958337**	**0.958851**	**0.958854**	**0.960769**	**0.961108**	**0.973877**	**0.974046**	**0.961689**
0.6	**0.940005**	**0.940006**	**0.941071**	**0.940837**	**0.944911**	**0.944546**	**0.959672**	**0.960096**	**0.944042**
0.7	**0.918333**	**0.918342**	**0.920311**	**0.919361**	**0.927146**	**0.924088**	**0.942095**	**0.921597**	**0.945626**
0.8	**0.893333**	**0.893334**	**0.896695**	**0.894552**	**0.907841**	**0.900428**	**0.896007**	**0.897038**	**0.925762**
0.9	**0.865013**	**0.865013**	**0.870363**	**0.866548**	**0.887357**	**0.874265**	**0.86832**	**0.869669**	**0.902853**

Table 2: Haar Wavelet Solution for White Dwarf Case 6.1.2

ξ	C=0	C=0.1	C=0.2	C=0.3
0	**1.00000**	**1.00000**	**1.00000**	**1.00000**
0.1	**0.998338**	**0.998581**	**0.99858**	**0.999028**
0.2	**0.993411**	**0.994379**	**0.995296**	**0.996154**
0.3	**0.985393**	**0.987552**	**0.989595**	**0.99151**
0.4	**0.974551**	**0.978345**	**0.981931**	**0.985295**
0.5	**0.96123**	**0.967073**	**0.972591**	**0.977767**
0.6	**0.944937**	**0.953273**	**0.961128**	**0.968464**
0.7	**0.925193**	**0.936497**	**0.947116**	**0.956942**
0.8	**0.902402**	**0.917117**	**0.930901**	**0.943532**
0.9	**0.876936**	**0.895488**	**0.912833**	**0.928599**

Table 3: Performance of the Present Method for Case 6.1.3a, 6.1.3b

	Case:6.1.3*a*		Case:6.1.3b
ξ	*HWS* $\beta=4, r=2, p=3$	*HWS* $\beta=4, r=2, p=5$	HWS $a=-1$
0.1	**0.999981**	**0.999981**	**0.0174218**
0.2	**0.999682**	**0.999682**	**0.042658**
0.3	**0.998388**	**0.998391**	**0.0675273**
0.4	**0.994932**	**0.994989**	**0.0896878**
0.5	**0.98777**	**0.988102**	**0.108385**
0.6	**0.978188**	**0.977812**	**0.126458**
0.7	**0.967531**	**0.964653**	**0.147124**
0.8	**0.95451**	**0.958547**	**0.170919**
0.9	**0.938587**	**0.930351**	**0.198392**

Table 4: Performance of the Method for Case 6.2.4

	Isothermal Gas Sphere		Case:6.1.5		Case:6.2.1	
ξ	Exact[32]	HWS	Exact	HWS	Exact	HWS
1/64	**-4.07e-05**	**-4.07e-05**	**1.0002**	**1.0002**	**0.0000**	**-6.21E-06**
11/64	**-0.0049172**	**-.0049262**	**1.0300**	**1.0300**	**-0.0042**	**-0.0041739**
19/64	**-0.014628**	**-0.014705**	**1.0921**	**1.0921**	**-0.0184**	**-0.0183589**
27/64	**-0.0294091**	**-0.029731**	**1.1948**	**1.1948**	**-0.0434**	**-0.0433754**
35/64	**-0.0491268**	**-0.050055**	**1.3486**	**1.3486**	**-0.0741**	**-0.0740964**
43/64	**-0.0736069**	**-0.075738**	**1.5705**	**1.5705**	**-0.0995**	**-0.0995347**
51/64	**-0.102639**	**-0.106826**	**1.8870**	**1.8870**	**-0.1028**	**-0.102844**
59/64	**-0.135982**	**-0.143295**	**2.3393**	**2.3393**	**-0.0612**	**-0.0613196**
63/64	**-0.154188**	**-0.163499**	**2.6353**	**2.6353**	**-0.0149**	**-0.015047**

CONCLUSIONS

In above discussion, the Lane Emden equation was solved by Haar wavelet quasi-linearization method. The main advantage of our proposed method is its direct application to all types of differential equations, whether they are linear or nonlinear, homogeneous or inhomogeneous, and with constant coefficients or with variable coefficients. Summarizes the application of present method by Haar wavelet approximation for solving nonlinear singular Lane-Emden equation and a comparison is made with existing results in the literature.

The advantage of quasi-linearization is that one does not have to apply iterative procedure. Quasi-linearization is iterative process in numerical analysis but our proposed technique gives excellent numerical results for solving nonlinear phenomena without any iteration on selecting collocation points.

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