AN INVENTORY MODEL WITH TIME DEPENDENT DETERIORATION RATE AND EXPONENTIAL DEMAND RATE UNDER TRADE CREDITS

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ABSTRACT

This paper deals with the optimal policy for the customers to obtain its minimum cost when supplier offers both permissible delay as well as cash discount. In this paper deterioration is considered as time dependent and demand rate is an exponential function of time. Four different cases have been discussed. Truncated Taylor’s series expansion is used to obtain closed form optimal solution. Mathematica software is used for finding optimal solution. Finally, numerical examples and sensitivity analysis are given to validate the purposed model.

KEYWORDS: Inventory, Cash Discount, Trade Credits, Time Dependent Deterioration, Exponential Demand Rate

INTRODUCTION

In classical inventory models the demand rate is assumed to be either constant or time-dependent. In a buyer-seller situation, an inventory model considers a case in which depletion of inventory is caused by constant demand rate. Many research papers have been published concerning the control of deteriorating and non-deteriorating inventory items. Deteriorating items include such items as volatile liquids, medicines; blood, fashion goods, radioactive material and photographic films and non-deteriorating items include such items as dried fruits, wheat and rice. It is a common assumption in too many inventory systems that products have indefinitely long lives. Generally, however, almost all items deteriorate over time but often the rate of deterioration is low and there is little need to consider the deterioration when determining economic lot size.

In past few decades great interest has been shown in developing mathematical models in presence of trade credits. In today’s business translations, it is common to find that the buyers are allowed some credit period before they settle the account with the whole seller. This provides an advantage to the customers, due to the fact that they do not have to pay the whole seller immediately after receiving the product, but delay there payment until the end of allowed period. In this paper, the deterioration rate is considered as time dependent and demand rate is exponential time dependent. At present inflection has become a permanent feature of economy. Lot of researchers has shown inflection effect on inventory.
Goyal [1] derived an EOQ model under the conditions of permissible delay in payment. Aggrawal and Jaggi [2] extended Goyal’s model for deteriorating items. Jammal et al [3] then generalized the model to allow shortages. Hwang & Shim [4] added demand rate is price dependent and developed the optimal price and lot-sizing for a retailer under the condition of a permissible delay in payment. Chung [5] developed an alternative approach to find the EOQ under the conditions of permissible delay in payment being garanteed.EOQ model with trade credit financing for non-decreasing demand is presented by Teng et al.[6].Chung et al. [7] developed an EOQ model for deteriorating items under supplier credits linked to ordering quantity. In this regard, most of researchers like Covert & Philip [8], Dave & Patel [9], Sachan [10], Datta & Pal [11], Goswmi & Chaudhari [12], Raafat [13], Hariga [14], Goyal & Giri [15], Chung & Dye [16], Skouri & Papachristos [17], Ghare & Schrader [18], Skouri et al. [19], considered either a constant or exponential deterioration rate.

In all the mentioned above models the inflations, time value of money and stock is disregarded. Gupta & Vrat [20] first developed a model for consumption environment to minimize the cost with assumption that stock dependent consumption rate is a function of the initial stock level. Padmanabhan & Vrat [21] defined stock-dependent consumption rate as a function of inventory level at any instant of time and developed models for non-sales environment. Inventory models for permissible items with stock-dependent selling rate are developed by Padmanabhan & Vrat [22]. It is assumed that selling rate is a function of current inventory level; Sarker et al. [23] developed an order level lot size inventory model with inventory level dependent demand and deterioration. In paper [23] Sarker assumed that there is the nature of decreasing demand, the replenishment rate is regarded to be finite and the planning horizon is infinite. Balkhi & Benkheronf [24] developed an inventory model for deteriorating items with stock-dependent and time varying demand rate over a finite planning horizon.

It has happened mostly because of the belief that the inflation and the time value of money would not influence the inventory policy to any significant degree. But most of the countries have suffered from large scale inflation and sharp decline in the purchasing power of money last several years. Thus the effects of inflation and time value of money can’t be ignored. Buzacott [25] presented an EOQ model with inflation subject to different types of pricing policies. Mishra [26] developed an EOQ model incorporating inflationary effects. Ray & Chaudhari [27], Chen [28], Sarker et al. [29], Chung & Lui [30] and Wee & Law [31] all have developed the effects of inflation, time value of money and deterioration on inventory model. An EOQ model for deteriorating items under trade credit is developed by Ouyang et al. [32]. Tripathi & Kumar [33] presented credit financing in economic ordering policies of time-dependent deteriorating items. In paper [33] three different cases has been considered and obtained minimum present value of all future cash flow. Tripathi [34] developed an EOQ model with time dependent demand rate and time-dependent holding cost function and minimum total inventory cost is obtained. Tripathi et al. [35] developed a cash flow oriented EOQ model of deteriorating items with time-dependent demand rate under permissible delay in payments.
In this paper demand rate is taken as exponential time dependent and time-dependent deterioration rate. Discount rate is considered and shortages are not allowed in this paper. The main objective of present work is to minimize the total relevant cost of all four different cases. Truncated Taylor’s series expansion is used to determine closed form solution of optimality.

The rest of the paper organized as follows: In the next section 2. Notations and assumptions is given followed by mathematical formulation in section 3. In section 4, numerical examples are given followed by sensitivity analysis is in section 5. Finally concluding remarks and future research direction is given in the last section 6.

**NOTATIONS AND ASSUMPTIONS**

The following notations are used throughout the paper

- \( p \): selling price per unit
- \( c \): the unit purchasing cost with \( p > c \)
- \( I_e \): the interest charged per dollar in stock per year by the supplier
- \( I_d \): the interest earned per dollar per year
- \( s \): the ordering cost per order
- \( Q \): the order quantity
- \( r \): the cash discount rate \( 0 < r < 1 \)
- \( h \): the unit holding cost per year excluding the interest charges
- \( M_1 \): the period of cash discount
- \( M_2 \): the period of permissible delay in settling account with \( M_2 > M_1 \)
- \( T \): the replenishment time interval
- \( I(t) \): the level of inventory at time \( t \), \( 0 \leq t \leq T \)
- \( T_1, T_2, T_3, and T_4 \): the optimal replenishment time for case I, II, III and IV respectively
- \( Z(T) \): the total relevant cost per year
- \( Z_1(T), Z_2(T), Z_3(T)\) and \( Z_4(T) \): the total relevant cost per year for case I, II, III and IV respectively
- \( Z^*(T_1), Z^*(T_2), Z^*(T_3)\) and \( Z^*(T_4) \): Optimal total relevant cost per year for case I, II, III and IV respectively
- \( Q^*(T_1), Q^*(T_2), Q^*(T_3)\) and \( Q^*(T_4) \): the optimal order quantity for case I, II, III and IV respectively
In addition following assumption is being made:

(i) the demand rate is exponential time dependent i.e. \( D(t) = \lambda_0 e^{\alpha t} \) where \( \lambda_0 \) is the initial demand and \( 0 < \alpha < 1 \)

(ii) the deterioration rate is time dependent i.e. \( \theta = \theta(t) = \theta t \), \( 0 < \theta < 1 \)

(iii) lead time is zero

(iv) time horizon is infinite

(v) shortages are not allowed

(vi) when the account is not settled the generated sells revenue is deposited in an interest bearing account. At the end of \( M_1 \) or \( M_2 \) the account is settled as well as the buyer pays of all units sold and starts paying for the interest charges on the items in the stock.

**MATHEMATICAL FORMULATIONS**

The variation of inventory with respect to time can be described by the following differential equation

\[
\frac{dI(t)}{dt} + \theta I(t) = -\lambda_0 e^{\alpha t}, 0 \leq t \leq T
\]

(1)

With boundary conditions \( I(0) = Q, I(T) = 0 \). The solution of (1) is given by

\[
I(t) = \lambda_0 \left\{ (T - t) + \frac{\alpha}{2} (T^2 - t^2) + \frac{\theta + \alpha^2}{6} (T^3 - t^3) \right\} e^{\frac{\theta t}{T}}, 0 \leq t \leq T
\]

(2)

The order quantity is

\[
Q = \lambda_0 \left\{ T + \frac{\alpha}{2} T^2 + \frac{\theta + \alpha^2}{6} T^3 \right\}
\]

(3)

Total demand during one cycle is

\[
\lambda_0 \left\{ T + \frac{\alpha}{2} T^2 + \frac{\theta + \alpha^2}{6} T^3 \right\}
\]

The total relevant cost per year consists of the following elements:

(a) cost of placing order \( = \frac{s}{T} \)
(b) cost of purchasing  
\[ \frac{\text{Q}}{T} = c \lambda_o \left\{ 1 + \frac{\alpha}{2} T + \frac{(\theta + \alpha^3)}{6} T^2 \right\} \]  
(5)

(c) cost of carrying inventory  
\[ \frac{h}{T} \int_0^T I(t) \, dt = h \lambda_o T \left( \frac{1}{2} + \frac{\alpha}{3} T + \frac{\alpha^2}{6} T^2 \right) \]  
(approx.)  
(6)

**Case I.**  \[ T \geq M_1 \], since the payment is made at time  \( M_1 \) the customer save rcQ per cycle due to price discount. From (2) we know that the discount per year is given by

\[ \frac{rcQ}{T} = c \lambda_o r \left\{ 1 + \frac{\alpha}{2} T + \frac{(\theta + \alpha^3)}{6} T^2 \right\} \]  
(7)

According to the assumption (d) the customer pays off all units ordered at time  \( M_1 \) to obtain the cash discount. Consequently, the items in stock have to be financed (at interest rate \( I_\alpha \)) after time  \( M_1 \) and hence the interest payable per year is

\[ \frac{c(1-r)I_\alpha}{T} \int_0^T I(t) \, dt = \frac{c(1-r) \lambda_o (T-M_1)}{T} \left[ 1 - \frac{(T^2 + M_1^2 + TM_1)\theta}{6} \right] \left[ 1 + \frac{\alpha}{2} T^2 + \frac{(\theta + \alpha^3)}{6} T^3 \right] - \frac{(T + M_1) - (T^2 + M_1^2 + TM_1)\alpha}{6} \]  
(8)

Finally during [0 ,  \( M_1 \)] period, the costumer sells the product and deposits the revenue into an account that earns interest \( I_\alpha \) per dollar per year. Therefore, the interest earned per year is

\[ \frac{pI_\alpha}{T} \int_0^{M_1} \lambda_o e^{\alpha t} \, dt = \frac{pI_\alpha \lambda_o}{\alpha T} \left[ e^{\alpha M_1} (M_1 - \frac{1}{\alpha}) + \frac{1}{\alpha} \right] \]  
(9)

Total relevant cost is given by

\[ Z_1(T) = \text{cost of placing order} + \text{cost of purchasing} + \text{cost of carrying inventory} + \text{interest payable per year} - \text{interest earned per year} \]

\[ = \frac{s}{T} + c \lambda_o \left\{ 1 + \frac{\alpha}{2} T + \frac{(\theta + \alpha^3)}{6} T^2 \right\} + h \lambda_o T \left( \frac{1}{2} + \frac{\alpha}{3} T + \frac{\alpha^2}{6} T^2 \right) + \]

\[ \frac{c(1-r) \lambda_o (T-M_1)}{T} \left[ 1 - \frac{(T^2 + M_1^2 + TM_1)\theta}{6} \right] \left[ 1 + \frac{\alpha}{2} T^2 + \frac{(\theta + \alpha^3)}{6} T^3 \right] - \frac{(T + M_1) - (T^2 + M_1^2 + TM_1)\alpha}{6} \]

\[ - \frac{pI_\alpha \lambda_o}{\alpha T} \left[ e^{\alpha M_1} (M_1 - \frac{1}{\alpha}) + \frac{1}{\alpha} \right] \]  
(10)

**Case II.** \( T < M_1 \), In this case the customers sells \( \lambda_o e^{\alpha t} . T \) units in total at time T and has \( c(1-r) \lambda_o e^{\alpha t} . T \) to pay the supplier in full at time  \( M_1 \) consequently there is no interest payable, while the cash discount is the same as that in case (1). However, the interest earned per year is
\[
\frac{pI_d}{T} \left[ \int_0^r \lambda_0 e^{at} dt + (M_1 - T) \int_0^r \lambda_0 e^{at} dt \right] = \frac{pI_d \lambda_0}{\alpha T} \left[ e^{at} (T - \frac{1}{\alpha}) + \frac{1}{\alpha} + (M_1 - T)(e^{at} - 1) \right]
\] (11)

As a result, the total relevant cost per year \( Z_2(T) \) is given by
\[
Z_2(T) = \frac{s}{T} + c \lambda r \left[ 1 + \frac{\alpha}{2} T \left( \frac{\theta + \alpha^2}{6} \right) \right] + h \lambda_0 T \left( \frac{1}{2} + \frac{\alpha}{3} T + \frac{\alpha^2}{6} T^2 \right) - \frac{pI_d \lambda_0}{\alpha T} \left[ e^{at} (T - \frac{1}{\alpha}) + \frac{1}{\alpha} + (M_1 - T)(e^{at} - 1) \right]
\] (12)

**Case III.** \( T \geq M_2 \) since the payment is made at time \( M_2 \) there is no discount in this case i.e. \( r = 0 \). The interest payable per year is given by
\[
\frac{c I}{T} \left[ I(t) dt = \frac{c \lambda I_s (T - M_2)}{T} \left[ 1 - \left( \frac{T^2 + M_2^2 + TM_2 \theta}{6} \right) \right] \left\{ T + \frac{\alpha}{2} T^2 + \left( \frac{\theta + \alpha^2}{6} \right) T^3 \right\} - \frac{(T + M_2)}{2} - \frac{(T^2 + M_2^2 + TM_2 \alpha)}{6} \right]
\] (13)

The interest earned per year is
\[
\frac{pI_d}{T} \left[ \int_0^{M_2} \lambda_0 e^{at} dt \right] = \frac{pI_d \lambda_0}{\alpha T} \left[ e^{aM_2} (M_2 - 1) + \frac{1}{\alpha} \right]
\] (14)

The total relevant cost is given by
\[
Z_3(T) = \frac{s}{T} + c \lambda r \left[ 1 + \frac{\alpha}{2} T \left( \frac{\theta + \alpha^2}{6} \right) \right] + h \lambda_0 T \left( \frac{1}{2} + \frac{\alpha}{3} T + \frac{\alpha^2}{6} T^2 \right) +
\frac{c \lambda I_s (T - M_2)}{T} \left[ 1 - \left( \frac{T^2 + M_2^2 + TM_2 \theta}{6} \right) \right] \left\{ T + \frac{\alpha}{2} T^2 + \left( \frac{\theta + \alpha^2}{6} \right) T^3 \right\} - \frac{(T + M_2)}{2} - \frac{(T^2 + M_2^2 + TM_2 \alpha)}{6}\right]
\frac{pI_d \lambda_0}{\alpha T} \left[ e^{aM_2} (M_2 - 1) + \frac{1}{\alpha} \right]
\] (15)

**Case IV.** \( T < M_2 \) In this case there is no interest charged. The earned per year is
\[
\frac{pI_d}{T} \left[ \int_0^r \lambda_0 e^{at} dt + (M_2 - T) \int_0^r \lambda_0 e^{at} dt \right] = \frac{pI_d \lambda_0}{\alpha T} \left[ e^{at} (T - \frac{1}{\alpha}) + \frac{1}{\alpha} + (M_2 - T)(e^{at} - 1) \right]
\] (16)

The total relevant cost \( Z_4(T) \) is given by
\[
Z_4(T) = \frac{s}{T} + c \lambda r \left[ 1 + \frac{\alpha}{2} T \left( \frac{\theta + \alpha^2}{2} \right) \right] + h \lambda_0 T \left( \frac{1}{2} + \frac{\alpha}{3} T + \frac{\alpha^2}{6} T^2 \right) - \frac{pI_d \lambda_0}{\alpha T} \left[ e^{at} (T - \frac{1}{\alpha}) + \frac{1}{\alpha} + (M_2 - T)(e^{at} - 1) \right]
\] (17)
THEORETICAL RESULTS

From equation (10), (12), (15) & (17), it is difficult to obtain optimal solutions explicitly. Thus, the model is solved approximately using a truncated Taylor’s series expansion for the exponential terms, i.e. $e^{\alpha T}, e^{\alpha M_1}, etc$. Which is a valid approximation for the smaller values of $\alpha T, \alpha M_1, etc$.

With the above approximation, the total relevant costs $Z_i(T) ; i = 1,2,3,4$ is given by

\[ Z_1(T) \equiv \frac{s}{T} + c \lambda_n \left\{ 1 + \frac{\alpha}{2} T + \left( \frac{\alpha + \alpha^2}{6} \right) T^2 \right\} + h \lambda_n T \left( \frac{1}{2} + \frac{\alpha^2}{3} T + \frac{\alpha^2}{6} T^2 \right) + c(1 - r) \lambda_n \frac{1}{T} (T - M_1) \]

\[ \left[ 1 - \frac{T^2 + M_1^2 + TM_1 \theta}{6} \right] \left\{ T + \frac{\alpha}{2} T^2 + \left( \frac{\alpha + \alpha^2}{6} \right) T^3 \right\} - \frac{(T + M_1)}{2} - \frac{(T^2 + M_1^2 + TM_1 \alpha)}{6} \right] - \frac{p I_s \lambda_n M_1^2}{T} \]

(18)

\[ Z_2(T) \equiv \frac{s}{T} + c \lambda_n \left\{ 1 + \frac{\alpha}{2} T + \left( \frac{\alpha + \alpha^2}{6} \right) T^2 \right\} + h \lambda_n T \left( \frac{1}{2} + \frac{\alpha^2}{3} T + \frac{\alpha^2}{6} T^2 \right) - p I_s \lambda_n M_1 \]

(19)

\[ Z_3(T) \equiv \frac{s}{T} + c \lambda_n \left\{ 1 + \frac{\alpha}{2} T + \left( \frac{\alpha + \alpha^2}{6} \right) T^2 \right\} + h \lambda_n T \left( \frac{1}{2} + \frac{\alpha^2}{3} T + \frac{\alpha^2}{6} T^2 \right) + \frac{c I_n \lambda_n (T - M_1)}{T} \]

\[ \left[ 1 - \frac{T^2 + M_1^2 + TM_1 \theta}{6} \right] \left\{ T + \frac{\alpha}{2} T^2 + \left( \frac{\alpha + \alpha^2}{6} \right) T^3 \right\} - \frac{(T + M_1)}{2} - \frac{(T^2 + M_1^2 + TM_1 \alpha)}{6} \right] - \frac{p I_s \lambda_n M_1^2}{T} \]

(20)

\[ Z_4(T) \equiv \frac{s}{T} + c \lambda_n \left\{ 1 + \frac{\alpha}{2} T + \left( \frac{\theta + \alpha^2}{6} \right) T^2 \right\} + h \lambda_n T \left( \frac{1}{2} + \frac{\alpha^2}{3} T + \frac{\alpha^2}{6} T^2 \right) - p I_s \lambda_n M_2 \]

(21)

Differentiating (18), (19), (20), & (21) with respect to $T$ two times, we get

\[ \frac{dZ_1(T)}{dT} = -\frac{s}{T^2} + c \lambda_n \left\{ \frac{\alpha}{2} + \left( \frac{\theta + \alpha^2}{3} \right) T \right\} + \lambda_n h \left( \frac{1}{2} + \frac{2\alpha^2}{3} T + \frac{\alpha^2}{2} T^2 \right) + c(1 - r) I_t \lambda_n \]

\[ \left[ -\left( \frac{M_1}{2} + \frac{\alpha}{6} M_1 \right) \frac{1}{T^2} + \left( \frac{1}{2} - \frac{\alpha}{2} T + \frac{\theta \alpha}{12} M_1^2 \right) + \left[ \frac{\alpha}{3} - \left( \frac{\theta + \alpha^2}{6} \right) M_1 + \frac{\theta (\theta + \alpha^2)}{36} M_1^2 \right] \right] \left\{ T + \frac{\alpha}{2} T^2 + \left( \frac{\alpha + \alpha^2}{6} \right) T^3 \right\} + \frac{p I_s \lambda_n M_1^2}{T^2} \]

(22)

\[ \frac{dZ_2(T)}{dT} = -\frac{s}{T^2} + c \lambda_n \left\{ \frac{\alpha}{2} + \left( \frac{\theta + \alpha^2}{3} \right) T \right\} + \lambda_n h \left( \frac{1}{2} + \frac{2\alpha^2}{3} T + \frac{\alpha^2}{2} T^2 \right) \]

(23)

\[ \frac{dZ_3(T)}{dT} = -\frac{s}{T^2} + c \lambda_n \left\{ \frac{\alpha}{2} + \left( \frac{\theta + \alpha^2}{3} \right) T \right\} + \lambda_n h \left( \frac{1}{2} + \frac{2\alpha^2}{3} T + \frac{\alpha^2}{2} T^2 \right) + c I_t \lambda_n \left[ -\left( \frac{M_1}{2} + \frac{\alpha}{6} M_1 \right) \frac{1}{T^2} + \left( \frac{1}{2} - \frac{\alpha}{2} T + \frac{\theta \alpha}{12} M_1^2 \right) + \left[ \frac{\alpha}{3} - \left( \frac{\theta + \alpha^2}{6} \right) M_1 + \frac{\theta (\theta + \alpha^2)}{36} M_1^2 \right] \right] \left\{ T + \frac{\alpha}{2} T^2 + \left( \frac{\alpha + \alpha^2}{6} \right) T^3 \right\} + \frac{p I_s \lambda_n M_1^2}{T^2} \]

(24)

\[ \frac{dZ_4(T)}{dT} = -\frac{s}{T^2} + c \lambda_n \left\{ \frac{\alpha}{2} + \left( \frac{\theta + \alpha^2}{3} \right) T \right\} + \lambda_n h \left( \frac{1}{2} + \frac{2\alpha^2}{3} T + \frac{\alpha^2}{2} T^2 \right) \]

(25)
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\[
\frac{d^2 Z_1(T)}{dT^2} = \frac{2s}{T^3} + \frac{c_\alpha r}{3} + \lambda_\alpha h \left( \frac{2\alpha}{3} + \alpha^2 T \right) + c(1-r)I_r \lambda_\alpha.
\]

(26)

\[
\frac{d^2 Z_2(T)}{dT^2} = \frac{2s}{T^3} + \frac{c_\alpha r}{3} + \lambda_\alpha h \left( \frac{2\alpha}{3} + \alpha^2 T \right) > 0
\]

(27)

\[
\frac{d^2 Z_3(T)}{dT^2} = \frac{2s}{T^3} + \frac{c_\alpha r}{3} + \lambda_\alpha h \left( \frac{2\alpha}{3} + \alpha^2 T \right) > 0
\]

(28)

\[
\frac{d^2 Z_4(T)}{dT^2} = \frac{2s}{T^3} + \frac{c_\alpha r}{3} + \lambda_\alpha h \left( \frac{2\alpha}{3} + \alpha^2 T \right) > 0
\]

(29)

The optimal (minimum) value of \( T = T_i^* \); \( i = 1, 2, 3, 4 \) obtained by solving \( \frac{dZ_i}{dT} = 0; i = 1, 2, 3, 4 \), we have

\[
18\lambda_\alpha h \alpha^2 T^4 + \left[ 12c_\alpha r \left( \theta + \alpha^2 \right) + 24\lambda_\alpha h \alpha + c(1-r)I_r \lambda_\alpha \left( 2\alpha - 12(\theta + \alpha^2)M_i + 2\theta(\theta + \alpha^2)M_i^3 \right) \right] T^3 + 3\lambda_\alpha h \alpha^2 T^4 + \lambda_\alpha \left( 2c_\alpha r \left( \theta + \alpha^2 \right) + 4h\alpha \right) T^3 + 3\left( c_r \alpha + h \right) \lambda_\alpha T^2 - 6S = 0
\]

(30)

\[
18\lambda_\alpha h \alpha^2 T^4 + \left[ 12c_\alpha r \left( \theta + \alpha^2 \right) + 24\lambda_\alpha h \alpha + c(1-r)I_r \lambda_\alpha \left( 2\alpha - 12(\theta + \alpha^2)M_i + 2\theta(\theta + \alpha^2)M_i^3 \right) \right] T^3 + 3\lambda_\alpha h \alpha^2 T^4 + \lambda_\alpha \left( 2c_\alpha r \left( \theta + \alpha^2 \right) + 4h\alpha \right) T^3 + 3\left( c_r \alpha + h \right) \lambda_\alpha T^2 - 6S = 0
\]

(31)

\[
18\lambda_\alpha h \alpha^2 T^4 + \left[ 12c_\alpha r \left( \theta + \alpha^2 \right) + 24\lambda_\alpha h \alpha + c(1-r)I_r \lambda_\alpha \left( 2\alpha - 12(\theta + \alpha^2)M_i + 2\theta(\theta + \alpha^2)M_i^3 \right) \right] T^3 + 3\lambda_\alpha h \alpha^2 T^4 + \lambda_\alpha \left( 2c_\alpha r \left( \theta + \alpha^2 \right) + 4h\alpha \right) T^3 + 3\left( c_r \alpha + h \right) \lambda_\alpha T^2 - 6S = 0
\]

(32)

\[
18\lambda_\alpha h \alpha^2 T^4 + \lambda_\alpha \left( 2c_\alpha r \left( \theta + \alpha^2 \right) + 4h\alpha \right) T^3 + 3\left( c_r \alpha + h \right) \lambda_\alpha T^2 - 6S = 0
\]

(33)

**NUMERICAL EXAMPLES**

**Example 1 (case I).** Let \( \alpha = 0.01; h = $4/unit/year; I_c = 0.09/year; I_d = 0.06/year; c = $20/unit; p = $35/unit; \theta = 0.02; r = 0.02; M_i = 0.041095 \text{ year}; \lambda_\alpha = 500; s = $15/order.** Substituting these values in equation (30) and solving; we get optimal \( T = T_i^* = 0.098371 \text{ years}; \) and optimal economic ordered quantity \( Q^*(T_i) = 49.21129 \text{ units}; \) and total relevant cost \( Z^*(T_i) = $447.70968, \) which verified case 1 i.e. \( T \geq M_i \)

**Example 2 (case II).** Let \( \alpha = 0.01; h = $4/unit/year; c = $20/unit; \theta = 0.02; r = 0.02; 0.041095 \text{ year}; \lambda_\alpha = 200; s = $5/order.** Substituting these values in equation (31) and solving; we get optimal \( T = T_2 = 0.0353285 \text{ years}, \)
and optimal economic ordered quantity $Q^*(T_2) = 7.06698$ units; and total relevant cost $Z^*(T_2) = $ 218.11776, which verified case II i.e. $T < M_1$

**Example 3 (case III).** Let $\alpha = 0.01; h = $ 4/unit/year; $I_c = 0.09$/year; $I_d = 0.06$/year; c= $ 20/unit; p= $35/unit; $\theta=0.02; r=0.02; M_2= 0.082191$ year, $\lambda_0 = 100; s= $5/order. Substituting these values in equation (32) and solving; we get optimal $T=T_3= 0.120068$ years; and optimal economic ordered quantity $Q^*(T_3) = 12.01459$ units; and total relevant cost $Z^*(T_3) = $94.96095, which verified case III i.e. $T \geq M_2$

**Example 4 (case IV).** Let $\alpha = 0.01; h = $ 4/unit/year; c = $ 20/unit; $\theta=0.02; r=0.02; M_2= 0.082191$ year; $\lambda_0 = 500; S= $3/order. Substituting these values in equation (33) and solving; we get optimal $T=T_4= 0.0546937$ years; and optimal economic ordered quantity $Q^*(T_4) = 27.3546$ units; and total relevant cost $Z^*(T_4) = $423.37542, which verified case IV i.e. $T < M_2$

**SENSITIVITY ANALYSIS**

We have performed sensitivity analysis by changing $s$, $I_c$, $I_d$ & $r$ and keeping the remaining parameters at their original values. The corresponding variations is the replenishment cycle time, economic order quantity and total relevant cost are exhibited in Table 1 (1.a, 1.b. 1.c,) for case (1) Table 2 (Table 2.a, 2.b) for case (2), Table 3 (Table 3.a, 3.b, 3.c, ) for case (3) and Table 4 (Table 4.a, 4.b, ) for case (4) respectively

Case I Table 1,

**Table 1.a Variation of $s$ keeping all the parameters same as in Ex.1.**

<table>
<thead>
<tr>
<th>$s$</th>
<th>Replenishment cycle time $T_i$ (in years)</th>
<th>Economic order quantity $Q^*(T_i)$</th>
<th>Total relevant cost $Z^*(T_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.098371</td>
<td>49.21129</td>
<td>447.70968</td>
</tr>
<tr>
<td>20</td>
<td>0.114616</td>
<td>57.34336</td>
<td>494.6583</td>
</tr>
<tr>
<td>25</td>
<td>0.128825</td>
<td>64.45757</td>
<td>535.73357</td>
</tr>
<tr>
<td>30</td>
<td>0.141611</td>
<td>70.86039</td>
<td>572.70849</td>
</tr>
</tbody>
</table>

**Table 1.b Variation of $I_c$ keeping all the parameters same as in Ex.1.**

<table>
<thead>
<tr>
<th>$I_c$</th>
<th>Replenishment cycle time $T_i$ (in years)</th>
<th>Economic order quantity $Q^*(T_i)$</th>
<th>Total relevant cost $Z^*(T_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.054035</td>
<td>27.02511</td>
<td>314.58469</td>
</tr>
<tr>
<td>0.06</td>
<td>0.053602</td>
<td>26.80874</td>
<td>314.73214</td>
</tr>
<tr>
<td>0.07</td>
<td>0.053198</td>
<td>26.60658</td>
<td>314.87113</td>
</tr>
<tr>
<td>0.08</td>
<td>0.05282</td>
<td>26.41722</td>
<td>315.0024</td>
</tr>
</tbody>
</table>
An Inventory Model with Time Dependent Deterioration Rate and Exponential Demand Rate Under Trade Credits

Table 1.c Variation of $r$ keeping all the parameters same as in Ex.1.

<table>
<thead>
<tr>
<th>$r$</th>
<th>Replenishment cycle time $T_1$ (in years)</th>
<th>Economic order quantity $Q^*(T_1)$</th>
<th>Total relevant cost $Z^*(T_1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.052486</td>
<td>26.25053</td>
<td>415.14261</td>
</tr>
<tr>
<td>0.04</td>
<td>0.052508</td>
<td>26.26154</td>
<td>515.15863</td>
</tr>
<tr>
<td>0.05</td>
<td>0.052531</td>
<td>26.27264</td>
<td>615.17462</td>
</tr>
<tr>
<td>0.06</td>
<td>0.052533</td>
<td>26.2838</td>
<td>715.19059</td>
</tr>
</tbody>
</table>

Case II Table 2. Table 2.a Variation of $s$ keeping all the parameters same as in Ex.2.

<table>
<thead>
<tr>
<th>$s$</th>
<th>Replenishment cycle time $T_2$ (in years)</th>
<th>Economic order quantity $Q^*(T_2)$</th>
<th>Total relevant cost $Z^*(T_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.027367</td>
<td>5.47416</td>
<td>183.32092</td>
</tr>
<tr>
<td>4</td>
<td>0.031599</td>
<td>6.32096</td>
<td>201.97876</td>
</tr>
<tr>
<td>5</td>
<td>0.035328</td>
<td>7.06698</td>
<td>218.41776</td>
</tr>
<tr>
<td>6</td>
<td>0.038699</td>
<td>7.74144</td>
<td>233.28014</td>
</tr>
</tbody>
</table>

Table 2.b Variation of $r$ keeping all the parameters same as in Ex.2.

<table>
<thead>
<tr>
<th>$r$</th>
<th>Replenishment cycle time $T_2$ (in years)</th>
<th>Economic order quantity $Q^*(T_2)$</th>
<th>Total relevant cost $Z^*(T_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.035328</td>
<td>7.06698</td>
<td>218.11776</td>
</tr>
<tr>
<td>0.03</td>
<td>0.035319</td>
<td>7.06514</td>
<td>258.45817</td>
</tr>
<tr>
<td>0.04</td>
<td>0.035310</td>
<td>7.0633</td>
<td>298.49859</td>
</tr>
<tr>
<td>0.05</td>
<td>0.035300</td>
<td>7.06146</td>
<td>338.53904</td>
</tr>
</tbody>
</table>

Case III Table 3. Table 3.a Variation of $s$ keeping all the parameters same as in Ex.3.

<table>
<thead>
<tr>
<th>$s$</th>
<th>Replenishment cycle time $T_3$ (in years)</th>
<th>Economic order quantity $Q^*(T_3)$</th>
<th>Total relevant cost $Z^*(T_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.120068</td>
<td>12.01459</td>
<td>94.96095</td>
</tr>
<tr>
<td>10</td>
<td>0.177753</td>
<td>17.79298</td>
<td>128.53464</td>
</tr>
<tr>
<td>15</td>
<td>0.220813</td>
<td>22.10929</td>
<td>153.62194</td>
</tr>
<tr>
<td>20</td>
<td>0.256722</td>
<td>25.71082</td>
<td>174.56035</td>
</tr>
</tbody>
</table>

Table 3.b Variation of $I_c$ keeping all the parameters same as in Ex.3.

<table>
<thead>
<tr>
<th>$I_c$</th>
<th>Replenishment cycle time $T_3$ (in years)</th>
<th>Economic order quantity $Q^*(T_3)$</th>
<th>Total relevant cost $Z^*(T_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.125057</td>
<td>12.51417</td>
<td>94.43049</td>
</tr>
<tr>
<td>0.06</td>
<td>0.123686</td>
<td>12.37688</td>
<td>94.57366</td>
</tr>
<tr>
<td>0.07</td>
<td>0.122403</td>
<td>12.41133</td>
<td>94.70942</td>
</tr>
<tr>
<td>0.08</td>
<td>0.121199</td>
<td>12.12784</td>
<td>94.83834</td>
</tr>
</tbody>
</table>
Table 3.c Variation of $d$ keeping all the parameters same as in Ex.3.

<table>
<thead>
<tr>
<th>$d$</th>
<th>Replenishment cycle time $T_1$ (in years)</th>
<th>Economic order quantity $Q^*(T_1)$</th>
<th>Total relevant cost $Z^*(T_1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.132919</td>
<td>13.30152</td>
<td>102.43716</td>
</tr>
<tr>
<td>0.03</td>
<td>0.129826</td>
<td>12.99176</td>
<td>100.63752</td>
</tr>
<tr>
<td>0.04</td>
<td>0.126657</td>
<td>12.6744</td>
<td>98.79394</td>
</tr>
<tr>
<td>0.05</td>
<td>0.123406</td>
<td>12.34884</td>
<td>96.90303</td>
</tr>
</tbody>
</table>

Case IV Table 4.

Table 4.a Variation of $s$ keeping all the parameters same as in Ex.4.

<table>
<thead>
<tr>
<th>$s$</th>
<th>Replenishment cycle time $T_2$ (in years)</th>
<th>Economic order quantity $Q^*(T_2)$</th>
<th>Total relevant cost $Z^*(T_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.054693</td>
<td>27.3546</td>
<td>423.37542</td>
</tr>
<tr>
<td>4</td>
<td>0.063150</td>
<td>31.58564</td>
<td>440.34628</td>
</tr>
<tr>
<td>5</td>
<td>0.070600</td>
<td>35.34308</td>
<td>455.29877</td>
</tr>
<tr>
<td>6</td>
<td>0.077334</td>
<td>38.68298</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.b Variation of $r$ keeping all the parameters same as in Ex.4.

<table>
<thead>
<tr>
<th>$r$</th>
<th>Replenishment cycle time $T_3$ (in years)</th>
<th>Economic order quantity $Q^*(T_3)$</th>
<th>Total relevant cost $Z^*(T_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.070638</td>
<td>35.33246</td>
<td>255.22481</td>
</tr>
<tr>
<td>0.04</td>
<td>0.070600</td>
<td>35.31315</td>
<td>455.29877</td>
</tr>
<tr>
<td>0.05</td>
<td>0.070561</td>
<td>35.29394</td>
<td>555.33575</td>
</tr>
</tbody>
</table>

The all above results can be summed up as follows:

(A). (i).Case I from Table 1(a). An increase of $s$ results increase of replenishment cycle time $T_1$, economic order quantity $Q^*(T_1)$ and Total relevant cost $Z^*(T_1)$, keeping all other parameters same.

(ii). From Table 1(b). An increase if $I_c$ results decrease of replenishment cycle time $T_1$, economic order quantity $Q^*(T_1)$ and slight increase of Total relevant cost $Z^*(T_1)$, keeping all other parameters same.

(iii) From Table 1(c). An increase of $r$ results, slight increase of replenishment cycle time $T_1$, economic order quantity $Q^*(T_1)$ and increase of Total relevant cost $Z^*(T_1)$, keeping all other parameters same.

(B). (iv). From Table 2(a). An increase of $s$ results increase of replenishment cycle time $T_2$, economic order quantity $Q^*(T_2)$ and Total relevant cost $Z^*(T_2)$, keeping all other parameters same.

(v). From Table 2(b). An increase of $r$ results, slight decrease of replenishment cycle time $T_2$, economic order quantity $Q^*(T_2)$ and increase of the total relevant cost $Z^*(T_2)$, keeping all other parameters same.

(C). (vi) From Table 3(a). An increase of $s$ results increase of replenishment cycle time $T_3$, economic order quantity $Q^*(T_3)$ and Total relevant cost $Z^*(T_3)$, keeping all other parameters same.
(vii). From Table 3(b). An increase of $I_c$ results, slight decrease of replenishment cycle time $T_3$, economic order quantity $Q^*(T_3)$ and increase of Total relevant cost $Z^*(T_3)$, keeping all other parameters same.

(viii) From Table 3 (c). An increase of $I_d$ results, decrease of replenishment cycle time $T_3$, economic order quantity and Total relevant cost $Z^*(T_3)$, keeping all other parameters same.

(ix) From Table 4(a). An increase of $s$ results, increase of economic order quantity $Q^*(T_4)$ and Total relevant cost $Z^*(T_4)$, keeping all other parameters same.

(x). From Table 4(b). An increase of $r$ results, slight decrease of replenishment cycle time $T_4$, economic order quantity $Q^*(T_4)$ and increase of Total relevant cost $Z^*(T_4)$.

CONCLUSIONS AND FUTURE RESEARCH DIRECTION

This model incorporates some realistic features with some kinds of inventory. First time dependent deterioration over time is a natural feature for goods. Secondly, occurrence of cash flow in inventory is a marketing strategies and natural phenomena in real situation. Thirdly, time dependent demand at that time. It is important to consider the effects of inflation and the time value of money in formulating inventory replenishment policy. The model is very useful in the retail business and industries where the demand is influenced by political factors and natural calamities. We have given a mathematical formulation of the problem presented an optimal procedure for finding optimal replenishment policy. Four different cases have been discussed. We have also verified that the effect of inflation in formulating replenishment policy. Finally, the sensitivity analysis of the solution to change in the values of different parameters has been discussed. It is seen that changes in the order cost ($s$), the cash discount rate ($r$), the interest charge ($I_c$), and the interest earned ($I_d$) lead to significant effect on the order quantity as well as Total relevant cost.

This paper can be extended for several ways. For instance we may extend the paper for stock – dependent demand rate as well as price dependent demand rate. We may also extend this paper for allowing shortages. Finally we could generalize the model for non- deteriorating items.

APPENDIX

The solution of (1) is

$$I(t)e^{\theta t^2/2} = C - \lambda_0 \int e^{(\alpha t + \theta t^2/2)} dt = C - \lambda_0 \int (1 + \alpha t + \frac{(\theta + \alpha^2)}{2} t^2) dt$$

$$e^{(\alpha t + \theta t^2/2)} \cong (1 + \alpha t + \frac{(\theta + \alpha^2)}{2} t^2), \text{Approx.}$$

Where $C$ is a constant of integration which is obtained by using the condition $I(T) = 0$. Therefore solution becomes
\[ I(t) = \lambda_0 \left( (T - t) + \frac{\alpha}{2}(T^2 - t^2) + \frac{(\theta + \alpha^2)}{6}(T^3 - t^3) \right) e^{\frac{-\theta t}{2}} \]

REFERENCES


