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TOPOLOGICAL MANIFOLDS WITH SMOOTH FIBRE BUNDLES

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ABSTRACT

The purpose of this paper is to develop the basic properties of topological manifolds and smooth fibre bundles. If $\mathcal{O}$ is an open covering of a topological manifold *M*, then we have proved that there exists a refinement $\{V_{ij}\}$, where $j \in \mathbf{N}$ and *i* runs through a finite set, such that $V_{ij} \cap V_{ik} = \emptyset$, for each i and $j \neq k$. Finally, smooth fibre bundle (E, π, B, F) is defined; the projection π is a smooth map from the total space E to the base space B and it is shown that every smooth fibre bundle has a finite coordinate representation.

KEYWORDS: Topological manifold, smooth manifold, smooth path, chart, atlas, smooth fibre bundle.

INTRODUCTION

The idea of topological manifold with smooth fibre bundle was introduced by H. Whitney [5, 6] and then it was generalized by A. Dold [1] and P. Olum [2]. A number of significant properties of smooth fibre bundle (E, π, B, F) were obtained by E. H. Spanier [4], G. Wu [7], M. M. Postnikov [3] and others. We begin with the following definition:

An *n-dimensional topological manifold* is a Hausdorff space *M* with a countable basis which satisfies the following condition:

Every point $a \in M$ has a neighbourhood U_a which is homomorphic to an open subset of an *n*-dimensional real vector space *E*.

A *chart* for a topological *n*-manifold *M* is a triple (*U*, *u*, *V*) where *U* is an open subset of *M*, *V* is an open subset of an *n*- dimensional real vector space *E* and $u: U \to V$ is a homeomorphism. Because the chart (*U*, *u*, *V*) is determined by the pair (*U*, *u*), we will denote a chart by (*U*, *u*). An *atlas* on an *n*-manifold *M* is a family of charts $\{(U_\alpha, u_\alpha) \mid \alpha \in \mathcal{I}\}$, where $\mathcal{I}$ is an arbitrary indexing set, such that the sets U_α form a covering of M :

$$M = \bigcup_{\alpha \in \mathcal{I}} U_\alpha .$$