COMPUTER AIDED MODELLING AND POSITION ANALYSIS OF CRANK AND SLOTTED LEVER MECHANISM

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ABSTRACT

The paper is discussed about crank and slotted mechanism that converts rotary motion into reciprocating motion at different rate for its two strokes i.e working stroke and return stroke. Time ratio has been calculated for constant length of stroke with specified dimensions. A CAD model has been prepared to simulate the mechanism and to specify the accurate path of the mechanism. Also the analytical method which can be used to define the various position of crank and respective position of slider in quick return mechanism is discussed.

KEYWORDS : Modelling, Position Analysis, Lever Mechanism.

INTRODUCTION

A quick return mechanism is a mechanism that converts rotary motion into reciprocating motion at different rate for its two strokes i.e working stroke and return stroke. When the time required for the working stroke is greater than that of the return stroke, it is a quick return mechanism. It yields a significant improvement in machining productivity. Currently, it is widely used in machine tools, for instance, shaping machines, power-driven saws, and other applications requiring a working stroke with intensive loading, and a return stroke with non-intensive loading. Several quick return mechanisms can be found including the offset crank slider mechanism, the crank-shaper mechanisms, the double crank mechanisms, crank rocker mechanism and Whitworth mechanism. In mechanical design, the designer often has need of a linkage that provides a certain type of motion for the application in designing. Therefore, the purpose of this project is to synthesize quick-return mechanism that converts rotational to translational motion.

Crank and Slotted Lever Quick Return Mechanism

This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines. In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed centre C. A sliding block is attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A.
A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R1R2. The line of stroke of the ram (i.e. R1R2) is perpendicular to AC produced. In the extreme positions, AP1 and AP2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB1 to CB2 (or through an angle $\beta$) in the clockwise direction. The return stroke occurs when the crank rotates from the position CB2 to CB1 (or through angle $\alpha$) in the clockwise direction.

**Computer Aided Modelling of Crank Slotted Mechanism:**

Data required for modeling of crank slotted mechanism is given in table 1. Modeling has been prepared in CATIA V5R17. Dimensions of the mechanism given in the table are useful to calculate length of stroke and time ratio.

**Table 1 : Dimensions of Links of Crank Slotted Mechanism**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Links</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Crank ( BC )</td>
<td>100 mm</td>
</tr>
<tr>
<td>2</td>
<td>Distance between pivots (AC)</td>
<td>250 mm</td>
</tr>
<tr>
<td>3</td>
<td>Slotted Bar ( AP )</td>
<td>650 mm</td>
</tr>
</tbody>
</table>
Computer Aided Modelling and Position Analysis of Crank and Slotted Lever Mechanism

Determination of Length of Stroke and Time Ratio

Let $\angle CAB_1 = $ Inclination of the slotted bar with the vertical.

We know that

$$\sin \angle CAB_1 = \sin \left( 90^\circ - \frac{\alpha}{2} \right)$$

$$= \frac{BLC}{AL} = 0.4$$

$$\angle CAB_1 = \left( 90^\circ - \frac{\alpha}{2} \right) = 23.58^\circ$$

Again $\angle CAB_1 = \left( 90^\circ - \frac{\alpha}{2} \right)$

$$\alpha = 132.84^\circ$$

Time ratio = \frac{Time of cutting stroke}{Time of return stroke} = \frac{550 - x}{x} = 1.71

Length of stroke = $R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin \left( 90^\circ - \frac{\alpha}{2} \right) = 520$ mm

Looking at Figure 1 the Crank and Slotted lever Quick Return Mechanism can be broken up into multiple vectors and two loops. Utilizing these two loops, the following sections will go through the kinematic analysis of the Crank and slotted lever Quick Return Mechanism.

Figure 1 : CAD model of Crank Slotted Mechanism
POSITION ANALYSIS

Figure 2: Vector representation of Crank and slotted lever Quick Return Mechanism

For the Crank and slotted lever Quick Return Mechanism shown in Figure 1, the displacement analysis can be formulated by the following loop-closer equations

\[ l_1 + l_2 = l_3 \]  \hspace{1cm} (1)
\[ l_3 + l_5 + l_7 + l_8 = l_6 + l_7 \]  \hspace{1cm} (2)

Using complex numbers, Equations 1, 2 become

\[ l_1 e^{i\theta_1} + l_2 e^{i\theta_2} = l_3 e^{i\theta_3} \]  \hspace{1cm} (3)
\[ l_3 e^{i\theta_3} + l_5 e^{i\theta_5} + l_6 e^{i\theta_6} = l_7 e^{i\theta_7} \]  \hspace{1cm} (4)

where the link lengths \( l_1, l_2, l_3, l_7 \) and angular positions \( \theta_1, \theta_6, \theta_7 \) are constants. Angular position \( \theta_2 \) is an independent variable; angular positions \( \theta_3, \theta_4, \theta_5 \) are dependent variables.

From figure \( \theta_6 = \theta_3 = \theta_4 \) and \( l_4 = l_3 + l_8 \)

Substituting and rearranging Equations 1 and 2:

\[ l_1 e^{i\theta_1} = l_3 e^{i\theta_3} + l_2 e^{i\theta_2} \]  \hspace{1cm} (5)
\[ l_3 e^{i\theta_3} + l_5 e^{i\theta_5} + l_6 e^{i\theta_6} = l_7 e^{i\theta_7} \]  \hspace{1cm} (6)

As equation 5 has 2 unknowns and equation 6 has 3 unknowns,

Utilizing Euler’s equation, \( e^{i\theta} = \cos \theta + i \sin \theta \)

\[ l_3 \cos \theta_4 + i \sin \theta_4 = l_1 \cos \theta_1 + i \sin \theta_1 + l_2 \cos \theta_2 + i \sin \theta_2 \]

Separate this equation in real numbers and imaginary numbers.

\[ l_3 \cos \theta_4 = l_1 \cos \theta_1 + l_2 \cos \theta_2 \]  \hspace{1cm} (7)
\(l_1 \sin \theta_2 = l_1 \sin \theta_1 + l_2 \sin \theta_2 \) \hspace{1cm} \text{----------(8)}

Squaring Equations 7, 8 and adding them together;

\[l_3 = \sqrt{(l_1 \cos \theta_2 + l_2 \cos \theta_2)^2 + (l_1 \sin \theta_2 + l_2 \sin \theta_2)^2} \] \hspace{1cm} \text{----------(9)}

Dividing Equation 8 by Equation 7 and simplifying gives

\[\theta_4 = \tan^{-1} \left( \frac{l_1 \sin \theta_2 + l_2 \sin \theta_2}{l_1 \cos \theta_2 + l_2 \cos \theta_2} \right) \] \hspace{1cm} \text{---------- (10)}

By knowing the value of \(\theta_4\), Equation 6 has only 2 unknown values,

\[l_0 e^{i\phi_6} - l_0 e^{i\phi_5} = l_1 e^{i\phi_4} - l_1 e^{i\phi_7} \] \hspace{1cm} \text{---------- (11)}

Since right hand side of equation 11 is constant

Let, \(l_0 e^{i\phi} = l_1 e^{i\phi_4} - l_1 e^{i\phi_7}\)

Now equation 11 becomes,

\[l_0 e^{i\phi_6} - l_0 e^{i\phi_5} = l_0 e^{i\phi} \] \hspace{1cm} \text{---------- (12)}

Again break the equation into real and imaginary part,

\[l_0 \cos \phi_6 - l_1 \cos \phi_5 = l \cos \phi \] \hspace{1cm} \text{---------- (13)}
\[l_0 \sin \phi_6 - l_1 \sin \phi_5 = l \sin \phi \] \hspace{1cm} \text{---------- (14)}

By solving equation 13 and 14,

\[L_6 = \left( \frac{l \cos \phi + l_1 \cos \phi_5}{\cos \phi_6} \right) \] \hspace{1cm} \text{---------- (15)}
\[L_6 = \left( \frac{l \sin \phi + l_1 \sin \phi_5}{\sin \phi_6} \right) \] \hspace{1cm} \text{---------- (16)}

Where equation 15 is used when \(\cos \phi_6 > 0\) and equation 16 is used when \(\cos \phi_6 = 0\)

Put equation (15)in equation (14)

\[\left( \frac{l \cos \phi + l_1 \cos \phi_5}{\cos \phi_6} \right) \times \sin \phi_6 - l_2 \sin \phi_2 = l \sin \phi \]

\[\sin(\phi_6 - \phi_2) = \frac{l \cos \phi \sin \phi_6 - l_1 \cos \phi_5 \sin \phi_2}{l_6} \] \hspace{1cm} \text{---------- (17)}

Solving for \(\phi_2\), we get

\[\phi_{2a} = \phi_2 + \sin^{-1} \left( \frac{l \cos \phi \sin \phi_6 - l_1 \cos \phi_5 \sin \phi_2}{l_6} \right) \] \hspace{1cm} \text{---------- (18)}
\[\phi_{2b} = \phi_2 + \pi - \sin^{-1} \left( \frac{l \sin \phi \cos \phi_6 - l_1 \sin \phi_5 \cos \phi_2}{l_6} \right) \] \hspace{1cm} \text{---------- (19)}
By knowing all of the angular positions and the length of $l_6$, we can find the position of the output slider C by using
\[ P_c = l_4 + l_5 \quad \text{------------------ (20)} \]

CONCLUSIONS

From the given dimensions of links of the mechanism, time ratio has been calculated which is equal to 1.71 for constant stroke length of 520 mm. Also, by knowing all of the angular positions and the length of $l_6$, the position of the output slider C can be defined. The paper describes the analytical method which can be used to define the various position of crank and respective position of slider in quick return mechanism.

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